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Some Theoretical Studies on Turbulence and Magneto-Hydrodynamic Turbulence

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SOME THEORETICAL STUDIES ON TURBULENCE AND MAGNETO–HYDRODYNAMIC TURBULENCE

Ph.D. THESIS

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A

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Dedicated to my beloved Parents

Chand Mohammad Munshi

and

Late Rezia Begom

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Certified that the Thesis entitled **"Some Theoretical Studies on Turbulence and Magneto-hydrodynamic Turbulence"** submitted by Mr. Md. Abul kalam Azad in fulfillment of the requirement for the degree of Doctor of Philosophy in Mathematics, University of Rajshahi, Rajshahi- 6205, Bangladesh has been completed under my supervision. I believe that this research work is an original one and it has not been submitted elsewhere for any degree.

I wish him every success in life.

8/4/04

(M. Shamsul Alam Sarker)
Supervisor

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PREFACE

The thesis entitled "Some Theoretical Studies on Turbulence and Magneto-hydrodynamic turbulence" is being presented for the award of the degree of Doctor of Philosophy in Mathematics. It is the out come of my researches conducted in the Department of mathematics, Rajshahi University, Rajshahi, Bangladesh under the guidance of Dr. M. Shamsul Alam Sarker, Professor, Department of Mathematics, Rajshahi University, Rajshahi- 6205, Bangladesh.

The whole thesis has been divided into six chapters.

The first chapter is a general introductory chapter and gives the general idea of Turbulence and Magneto hydrodynamic turbulence and its principal concepts. Some results and theories, which are needed in the subsequent Chapter, have been included in this chapter. A brief review of the past researches related to this thesis has also been given. Throughout the work we have considered the flow of fluids to be isotropic and homogeneous. The notions generally adopted are those used by Batchelor, Chandrasekhar, Deissler, Jain and Lundgren in their research papers. Number inside brackets [ ] refer to the references which are arranged alphabetically at the end of the thesis.

The second chapter consists of three parts:

In part–A, we have studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi- time in a rotating system. In this Chapter we have considered correlations between fluctuating quantities at two and three point. Two and three point correlation equations in a rotating system are obtained and these correlation equations are converted to spectral form by taking their fourier transforms. Finally the energy decay law of temperature fluctuations in homogeneous turbulence at times before the final period for the case of multi-point and multi-time in a rotating system is obtained. This study shows that due to the effect of rotation of fluid in homogeneous turbulence, the temperature energy fluctuations decays more rapidly than the energy for non-rotating fluid for times before the final Period.

In part-B, we have studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in presence of dust particles. In this chapter, at first we have considered correlations between fluctuating quantities at two-point two-time and three-point three-time correlation equations in presence of dust particles. The equations are obtained and the set of equations is made to determinate by neglecting the quadruple correlations in comparison to the second and third order correlations. Then the correlation equations are converted into spectral form by taking their fourier transforms and then the energy decay law of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in presence of dust particles is obtained.

In part-C, we have studied the effect of coriolis force on the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in presence of dust particles and derived an early period decay equation of homogeneous turbulence in a dusty fluid under the effect of rotation. The obtained equation shows that the effect of rotation in homogeneous turbulence the temperature energy fluctuation decays more rapidly than the energy for non-rotating clean fluid for times before the final period. It is the extension work of part-A and part-B of this chapter.

In the third chapter, a hierarchy of distribution functions in the statistical theory for simultaneous velocity, magnetic field, concentration and temperature fluctuations in MHD turbulent flow in a rotating system in presence of dust particles have been studied. Some properties of distribution functions have been discussed. Equations for the evolution of one and two point bivariate distribution functions in MHD turbulent flow in a rotating system in presence of dust particles have been derived and finally a comparison of the equation for one point distribution functions in case of zero viscosity and negligible diffusivity is made with first equation of BBGKY hierarchy in the kinetic theory of gasses.

The fourth chapter also consists of two parts:

In part-A, we have studied the decay of temperature fluctuations in MHD turbulence before the final period in a rotating system and have considered correlations between fluctuating quantities at two and three point. Two-and three-point correlation equations in a rotating system is obtained and the set of equations is made to determinate by neglecting the quadruple correlations in comparison to the second and third order correlations. The correlation equations are converted to spectral form by taking their Fourier-transforms. Finally we have obtained the decay law of temperature fluctuations energy before the final period.

In part-B, we have discussed the decay of temperature fluctuations in dusty fluid MHD turbulence before the final period in a rotating system. The results obtained have been compared with the equations of part-A of this chapter.

Chapter V is again divided into three parts:

In part-A, we have discussed the decay of MHD turbulence before the final period for the case of multi-point and multi-time in a rotating system. Two-and three-point correlation equations in a rotating system have been obtained and the set of equations is made to determinate by neglecting the quadruple correlations in comparison with the second and third order correlations. The correlation equations have been converted into spectral forms by taking their Fourier transforms. Finally integrating the energy spectrum over all wave numbers, the solution is obtained and this solution gives the energy decay law of magnetic energy fluctuations in MHD turbulence before the final period in a rotating system for the case of multi-point and multi-time. The result shows that due to the effect of rotation in the magnetic, field, the turbulent energy decays more rapidly than the energy for non-rotating fluid.

In part-B, we have considered the decay of MHD turbulence before the final period for the case of multi-point and multi-time in presence of dust particles and the decay law of magnetic energy fluctuations of MHD turbulence before the final period for the case of multi-point and multi-time in presence of dust particles is obtained.

In part-C, we have studied the Decay of dusty fluid MHD turbulence before the final period in a rotating system for the case of multi-point and multi-time. We have considered the two and three point correlation equations and solved them after neglecting the fourth order correlations in comparison with the second and third order correlations. Finally the energy decay law of dusty fluid MHD turbulence before the final period in a rotating system for the case of multi-point and multi-time has been obtained. Here due to the effect of rotation in presence of dust particles in the magnetic field, the turbulent energy decays more rapidly than the energy for non-rotating clean fluid. It is also the extension work of part-A and part-B of this chapter.

The six chapter is an over all review of the works with conclusions based on the findings of the thesis.

The following research papers, which are extracted from this thesis, have been accepted and communicated for publication in different national and international journals.

(1) Decay of MHD turbulence before the final period for the case of multi-point and multi-time in presence of dust particles
(Accepted for publication in the journal of "The Bangladesh Journal of Scientific and Industrial Research").

(2) Statistical theory of certain distribution functions in MHD turbulent flow in a rotating system in presence of dust particles
(Accepted for publications in the journal of "Rajshahi University Studies , Part-B").

(3) Decay of MHD turbulence before the final period for the case of multi-point and multi-time in a rotating system
(Accepted for publications in the journal of "Rajshahi University Studies , Part-B").

(4) Decay of dusty fluid MHD turbulence before the final period in a rotating system for the case of multi-point and multi-time
(Presented in the 14th Mathematics conference, 27-29 December, 2003, Department of Mathematics, University of Dhaka, Dhaka and communicated for publication)

(5) Decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in a rotating system (Communicated for publication).

(6) Decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in presence of dust particles (Communicated for publication).

(7) Decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in a rotating system in presence of dust particles (Communicated for publication).

(8) Decay of temperature fluctuations in MHD turbulence before the final period in a rotating system (Communicated for publication).

(9) Decay of temperature fluctuations in dusty fluid MHD turbulence before the final period in a rotating system (Communicated for publication).

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CONTENTS

CHAPTER--I

General Introduction:

CHAPTER –II

Part-A : Decay of Temperature Fluctuations in Homogeneous Turbulence Before the Final Period for the Case of Multi-Point and Multi-Time in a Rotating System

Part-B: Decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi –point and multi-time in presence of dust-particles

Part-C: Decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi—time in a rotating system in presence of dust particles

CHAPTER -III

Statistical theory of certain distribution functions in MHD turbulent flow in a rotating system in presence of dust particles

CHAPTER-IV

Part—A: Decay of temperature fluctuations in MHD turbulence before the final period in a rotating system

Part-B: Decay of temperature fluctuations in dusty fluid MHD turbulence before the final period in a rotating system

CHAPTER-V

Part-A: Decay of MHD turbulence before the final period for the case of multi –point and multi-time in a rotating system

Part-B : Decay of MHD turbulence before the final period for the case of multi-point and multi-time in presence of dust particles

Part-C : Decay of dusty fluid MHD turbulence before the final period in a rotating system for the case of multi-point and multi-time

CHAPTER-VI

CHAPTER-I

General Introduction

1. Basic Concept and Definition of Turbulence:

The theory of turbulent motion has received considerable attention in recent developments of high-speed jet aircraft, plasma physics and chemical engineering. The formation of a turbulent boundary layer is one of the most frequently encountered phenomena in high-speed aerodynamics.

It is common experience that the flow observed in nature such as rivers and winds usually differ from stream flow or laminar flow of a viscous fluid. The mean motion of such flow does not satisfy the Navier-Stockes equations for a viscous fluid. Such flows, which occur at high Reynolds numbers, are often termed turbulent flows. In turbulent flow, the steady motion of the fluid is steady so far as the temporal mean values of velocities and the pressures are concerned where as actually both velocities and the pressures are irregularly fluctuating. The velocity and pressure distributions in turbulent flows as well as the energy losses are determined mainly by turbulent fluctuations. The essential characteristic of turbulent flows is that the turbulent fluctuations are random in nature.

The instability of laminar flow at a high Reynolds numbers, are causes disruption of the laminar pattern of fluid motion. With sufficient disturbances the result is known as turbulence. The irregular, chaotic motion of fluid particle characterizes turbulence. It is far too complicated to be known in complete detail. This fact has been recognized by all theories extant today start with a stochastic formulation of the phenomenon.

At least, the technical people understand the meaning of turbulence. The use of the word "Turbulence" to characterize a certain type of flow, namely, the counterpart of streamline motion is comparatively recent. Reynolds, O. [102] made the first systematic experimental investigation of turbulent flow. The turbulent motion of fluid was described by Reynolds [102], one of the pioneers in the study of turbulent flows as "sinuous motion" because fluid particles in turbulent flow appeared to follow sinusoidal or irregular paths.

Turbulence is rather a familiar notion; yet it is not easy to define in such a way as to cover the detailed characteristic comprehended in it and to make the definition agree with the modern view of it held by professionals in this field of applied science. The word "Turbulence" means: agitation, commotion, disturbance etc. This definition is, however, too general, and does not suffice to characterize turbulent fluid motion in the modern sense. In 1937, Taylor and Vonkarman [128] gave the following definition:

" Turbulence is an irregular motion which in general makes its appearance in fluids, gaseous or liquid, when they flow past solid surface or even when neighbouring streams of the same fluid flow past or over one another".

According to this definition, the flow has to satisfy the condition of irregularity. But this irregularity is a very important feature. Because of irregularity, it is impossible to describe the motion in all details as a function of time and space co-ordinates. But fortunately turbulent motion is irregular in the sense that it is possible to describe it by laws of probability. It appears possible to indicate distinct average values of various quantities, such as velocity, pressure, temperature, etc and this is very important. If turbulent motion were entirely irregular, it would be inaccessible to any mathematical treatment. Therefore, it is not sufficient just to say that turbulence is an irregular motion yet we do not have clear-cut definition of turbulence. This is rather difficult.

Hinze [41] suggested that, "Turbulent fluid motion is an irregular condition of flow in which various quantities show a random variation with time and space co-ordinates, so that statistically distinct average values can be discerned".

The addition "with time and space co-ordinates" is necessary; it is not sufficient to define turbulent motion as irregular in time alone. For instance, the case in which a given quantity of a fluid is moved bodily in an irregular way; the motion of each part of the fluid is then irregular with respect to time to a stationary observer, but not to an observer moving with the fluid. Nor is turbulent motion, a motior at is irregular in space alone, became a steady flow with an irregular flow pattern might th ome under the definition of turbulence.

Turbulence sets in for various reason A sudden change in one of parameters of a flow field, e.g., kinematics viscosity, could ea cause instability viscosity, for example, is responsible for conversion of kinetic energy nto heat, thus causing turbulence to arise. Such phenomena are almost surely found in shearing flow with high Reynolds numbers. As the case for nonlinear stochastic phenomena, the problem of turbulence is still far from being solved.

Physically, turbulence is a manifestation of an inter-active motion of eddies of various sizes, where by an eddy we mean a lump of fluid over which flow properties do not vary substantially.

Different Kinds of Turbulence:

As Taylor and Von Karman have stated in their definition, Turbulence can be generated by the friction forces at fixed walls (fluid flow through conduits, fluid flow past solid surfaces) or by the flow of layers of fluids with different velocities past or over one another.

The above definition indicates that there are two distinct types of turbulence.

(i) Wall turbulence

(ii) Free turbulence

(i) Wall turbulence : Turbulence is generated by the viscous effect due to presence of a solid is called wall turbulence.

(ii) Free turbulence : Turbulence in the absence of walls, generated by the flow of layers of fluids at different velocities is called free turbulence.

In the case of real viscous fluids, viscosity effects will result in the conversion of kinetic energy of flow into heat; thus turbulent flow, like all flow of such fluids, is dissipative in nature. If there is no continuous external source of energy for the continuous generation of the turbulent motion, the motion will decay. Other effects of viscosity are to make the turbulence more homogeneous and to make it less dependent on direction.

Isotropic Turbulence:

The turbulence is called isotropic if its statistical features have no preference for any direction, so that perfect disorder reigns. No average shear stress can occur and consequently, no velocity gradient of the mean velocity. This mean velocity, if it occurs, is constant throughout the field.

Isotropic turbulence is the simplest type of turbulence, since no preference for any specific direction and a minimum number of quantities and relations are required to describe its structure and behaviour. However, it is also a hypothetical type of turbulence, because no actual turbulent flow shows true isotropy, though conditions may be made such that isotropy is more or less closely approached.

From theoretical considerations and experimental evidence it is known that the fine structure of most actual non-isotropic turbulent flows is nearly isotropic (local isotropy). Hence many features of isotropic turbulence may apply to phenomena in actual turbulence that is determined mainly by the fine-scale structure, where local isotropy prevails.

In isotropic turbulence the mean value of any function of the velocity components and their derivatives is unaltered by any rotation or reflection of the axes of references. Thus in particular, $\overline{u^2} = \overline{v^2} = \overline{w^2}$ and $\overline{uv} = \overline{vw} = \overline{wu} = 0$

Isotropy introduce a great simplicity into the calculations. The study of isotropic turbulence may also be of practical importance, since far from solid boundaries it has been observed that $\overline{u_1^2}$, $\overline{u_2^2}$, $\overline{u_3^2}$ tend to become equal to one another, e.g. in the natural winds at a sufficient height above the ground and in a pipe flow the axis.

Homogeneous Turbulence:

The turbulence which has quantitatively the same structure in one parts of the flow field is called homogeneous turbulence. In a homogeneous turbulent flow field the statistical characteristic are invariant for any translation in the space occupied by the fluid.

The conception of homogeneous turbulence is idealized, in that there is no known method of realizing such a motion exactly. The various method of producing turbulent motion in a laboratory or in nature all involves discrimination between different parts of the fluid, so that the average properties of the motion depend on position. However, in certain circumstances this departure from exact independence of position can be made very small, and it is possible to get a close approximation to homogeneous turbulence. Most of the theoretical works in turbulence and MHD turbulence in homogeneous and isotropic field in an incompressible fluid at rest.

Non-isotropic turbulence:

In all other cases where the mean-velocity shows a gradient, the turbulence will be non-isotropic or an isotropic.

Shear flow turbulence:

The gradient in mean velocity is associated with the occurrence of an average shear stress, the expression "shear flow turbulence". It is often used to designate this class of flow. Wall turbulence and an isotropic free turbulence fall into this class.

Reynolds number and its effect on turbulent flow:

Reynolds define a dimensionless quantity which is defined by the ratio of the inertia force to the viscous force is called Reynolds number after his name. This Reynolds number can be used as a measure for indicating the occurrence of laminar and turbulent flow transition between them.

According to definition of Reynolds number

$$\text{Reynolds no.} = \frac{inertia\ \ force}{viscous\ \ force} = \frac{\rho v^2}{\frac{\mu v}{d}} = \frac{\rho v d}{\mu} = \frac{vd}{\nu}$$

v $\rightarrow$ mean velocity of liquid

d $\rightarrow$ diameter of pipe

ν $\rightarrow$ kinematic viscosity of liquid

Dimension of Reynolds number:

$$\text{Re. } no. = \frac{vd}{\nu} = \frac{\frac{L}{T} \times L}{L^2/T} = \frac{\frac{L^2}{T}}{L^2/T} = 1 \left[\therefore \nu = \frac{Area}{Time} \right]$$

which shows that Reynolds number has no units but it is a dimensionless number.

Reynolds number has much importance and gives the information about the type of flow (i.e. laminar or turbulent).

Reynolds [102] after carrying out a series of experiments found that if the Reynolds number for a particular flow is less than 2000, the flow is laminar flow. But the Reynolds number is between 2000 and 2300, it neither laminar flow nor turbulent flow. But the Reynolds number, exceeds 2300, the flow is turbulent flow.

Critical Reynolds number:

Reynolds [102] investigated circumstances of the transition from laminar to turbulent flow. Based on his experimental results Reynolds [102] concluded that the transition from laminar to turbulent flow in pipes always occurred at nearly the same Reynolds number. This suggested that the Reynolds number at which the flow the change from laminar flows to turbulent flow is called critical Reynolds number. The approximate value the critical Reynolds number, Re_{cr} at which the laminar regime breaks down was established to be order of 2×10^3. Later with Reynolds apparatus, Ekman [32(a)] was able to maintain laminar flow up to a critical Reynolds number of 4×10^4 when the testing conditions were made extremely free from disturbances. There are two types of critical Reynolds number.

(i) Upper critical Reynolds number.

(ii) Lower critical Reynolds number.

Upper critical Reynolds number: The Reynolds number, which defines the upper limit of laminar flow, is called the upper critical Reynolds number or in other words the Reynolds number at which the flow enters from transition to turbulent flow is known as upper critical Reynolds number. However, several more recent investigators [39(a), 101(a)] have repeatedly demonstrated that there is no definite upper critical Reynolds number; rather the numerical value depends largely on the test conditions affecting the initial turbulence of flow.

Obviously, the upper critical Reynolds number is a function of initial disturbances; its numerical values always increase with a decrease in disturbances. For engineering purposes, high numerical values of the upper critical Reynolds number are of limited practical significance; the transition from to turbulent flow in a tube may be assumed to take placed at 2100-4000.

Lower critical Reynolds number: The Reynolds number which defines the below limit of laminar flow is called the lower critical Reynolds number or in other words the critical Reynolds number at which the flow enters from laminar to transition period is known as a lower critical Reynolds number. At lower critical Reynolds number the flow is always laminar. For flow through round tubes the lower critical Reynolds number is taken to be approximately 2000. This lower critical Reynolds number has considerable practical significance since it defines a definite limit below which all initial disturbances in the flow will eventually be damped out by fluid viscosity; therefore the flow is always laminar.

From the above discussion we conclude that if the Reynolds number is smaller than the critical Reynolds number i,e $R < Re_{cr}$, the flow is laminar. If the Reynolds number is greater than the critical Reynolds number i,e $R < Re_{cr}$, the flow is turbulent. Transition normally takes place at Reynolds number 2000-4000.

Other factors affecting transition from laminar to turbulent flow are

(a) The influence of the pressure gradient on transition from laminar flow to a turbulent flow is demonstrated by Schubaner and Shramstad's [123(a)] experimental results. For accelerated flow ($\frac{dp}{dx} < 0$ and $\frac{du}{dx} > 0$) the critical Reynolds number, Re_{cr} increases, where as for retarded flow ($\frac{dp}{dx} > 0$ and $\frac{du}{dx} < 0$) Re_{cr} decreases i.e transition to turbulent flow much more easily provoked.

(b) The effect of roughness of the wall on transition is to decrease the critical Reynolds number. This is clear because roughness would be equivalent to additional disturbances in the laminar strewing.

(c) Curvature has negligible effect on convex surface (the radious of curvature is negative), but the critical Reynolds number was found to decrease as the radious of curvature of a concave surface decreases.

(d) Suction of the boundary layer at the wall has a very powerful influence on stabilizing the laminar flow; but injection will tend to decrease the critical Reynolds number.

(e) For heated wall when $T_w > T_\infty$, the transfer of heat from the wall to the gas tends to destabilize the laminar boundary layer, where as the heat transfer from the gas to the wall $T_w < T_\infty$ increases the critical Reynolds number.

1.2 Historical Back Ground of Early Work of Turbulence:

The study of turbulence began with the works of Boussinesq [1] and Reynolds [102]. Boussinesq cast much light on the physics of turbulence. He pointed out that turbulent motion is chaotic in nature and can't be treated by deterministic laws, hence indicating the use of theory of probability. Reynolds, O. in 1883 [102] first made the systematic investigations and gave the experimental results to understanding the facts of turbulent flow. He made the remarkable difference between laminar and turbulent flows by proposing the Reynolds number and gave the Reynolds stresses to describe the turbulent phenomena. Reynolds averaged the Navier-Stockes equations for an incompressible fluid. Thus he established the so-called Reynolds equations for the mean values. His technique followed closely that used by Maxwell in 1850 when Maxwell deduced the Navier-Stockes equation from the Kinetic theory of gases. Therefore, the theory of turbulence was based on analogies with the discontinuous collisions between the discrete entities studied in Kinetic gas theory. Prandtl [93] developed His "mixing length" theory based on the problems of practical importance such as pipe flows over boundaries of specific shapes. Prandtl's theory was successfully applied to the turbulent flow of a liquid in a circular pipe and also to the meteorological problem of wind distribution in the layer of air adjacent to the ground. However, his theory had a serious weakness in the sense that it requires some adhoc assumption on the mixing length.

The origin of the idea of statistical approach to the problem of turbulence may be traced back to Taylor's paper of 1921 [125] in which he has advanced the concept of the Lagrangian correlation coefficient that provides a theoretical basis for turbulent diffusion. Tailor, G. I. [126,127] and Von Karman, T. [133,134] broke away from the concept, which described turbulence in terms of collisions between discrete entities and instead introduce the concept of velocity correlation at two or more points, as one of the parameters involved in describing turbulent motion. Taylor, G. I. introduced the so-called " energy spectrum" method to describe the probability density function for energy in the turbulent flow field. Von Karman proved that the correlation of velocities at two points is a tensorial character. He introduced the "correlation tensor" method. Taylor, G. I. [126] introduces the idea that the velocity of the fluid of turbulent motion is a random continuous function of position and time. To make the turbulent motion amenable to mathematical treatment, he assumes the turbulent fluid to be homogeneous and isotropic. In its supports, he describes the measurements showing that the turbulence generated downstream from a regular array to rods in a wind tunnel is approximately homogeneous and isotropic. In spite of the fact that the turbulence in nature is not always exactly homogeneous and isotropic, it is essential to study homogeneous and isotropic turbulence as a first step to understand the more complicated phenomenon of non-homogeneous turbulence.

Taylor, G. I. [130] in 1938 took into account the non-linearity of the dynamical equations and showed that it results in the skew ness of the probability distribution of the difference between the velocity components at two points. He showed that the non-linearity of the dynamical equation is also responsible for the existence of the interaction between components of the turbulent having different fluctuations.

Kolmogoroff's [67, 68] work contributed significantly to understanding the physics of turbulence. His outstanding works in the theory of local homogeneous and local isotropic turbulent flow resulted in the "2/3 Kolmogoroff law", the analog of which in the language of spectra is the 5/3 law.

Another significant contribution came from Hopf, E. [44,45] who applied the theory of the characteristic functional to turbulence, and Kampe de Feriet, J. [33], who used the theory of group transformation in order to mathematically certain characteristics of turbulent motion.

Kampe de Feriet, J. and Batchelor, G. K. introduced the three dimensional spectrum functions and by means of Fourier transformations, investigated many of its properties in connection with the energy spectrum.

Other significant contributors to the theory of turbulence were S. Chandrasekhar, R. Betchov, J. Laufer, A. A. Townsend, J. O. Hinze, S. Corrsin, O. Phillips, A. S. Monin, A. S. Obuklov, A. M. Yaglom and G. Yamamoto modified theory of incompressible turbulence to accommodate compressible turbulence, but without significant success.

Continuous through today, modern theories in turbulence are still statistical in nature, but are phenomena logically different from previous efforts. Among the most important recent developments is Kraichnan's [69] theory of direct interaction approximation. In fact Kraichnan's theory represents an effort to determine an average green's function of a nonlinear stochastic field.

In the following instead of giving a detailed account of the historical development of the subject, we shall confine to mere concepts and method of turbulence together with a few theories of turbulence, which have been used in subsequent chapters.

1.3 Averaging Procedure:

In mathematical description of turbulent flow, it is convenient to consider an instantaneous velocity such as u is the sum of the time average part $\overline{u}$ and momentary fluctuation (fluctuating velocity) u′ i,e

$$u = \overline{u} + u' \qquad \text{----------------} (1.3.1)$$

where $\overline{u} \rightarrow$ average value or mean value

$u' \rightarrow$ fluctuating velocity and

u $\rightarrow$ velocity of motion

In a steady flow $\overline{u}$ does not change with time. In talking the average of a turbulent quantity, the result depends not only on the scale used but also on the method of averaging. These are four different kinds of averaging procedure introduced by Pai [91] that are found to be useful for the study of turbulent flows. These are

(i) time average,

(ii) space average,

(iii) space-time average and

(iv) ensemble average or the statistical average.

If the turbulent flow field is quasi-steady, time average can be used. For a homogeneous turbulence flow field, space average can be considered. If the flow field is steady and homogeneous, space-time average is used. Lastly, if the flow field is neither steady nor homogeneous, we assume that averaging is taken over a large number of experiments that have initial and boundary conditions. This type of average is called ensemble average or statistical average. Ensemble average is more general than the time and space averages and very useful for the study of in homogeneous, non-stationary turbulent flow. This type of averaging can be applied to any flow. Most of the modern theories have used the ensemble averaging procedure for describing the statistical properties of turbulence. However, like the time and space averages, the physical interpretation of the ensemble average is not so simple. In general any turbulent field is completely determined by the hierarchy of correlations.

$\langle u_i(r,t)\rangle$, $\langle u_i(r,t)u_j(r',t)\rangle$, $\langle u_i(r,t)u_j(r',t)u_m(r'',t)\rangle$, where, $\langle \quad \rangle$ denote the ensemble average defined in Leslie's Book (79)

In homogeneous isotropic turbulence the first correlation represents the mean velocity, and is simply zero, the pair correlation $\langle u_i(r)u_j(r')\rangle$ is often considered to be a sufficient description of turbulent flow. The higher order correlations are assumed to give less and less information so that only a finite number of correlations are required to determine the statistical properties of turbulence. This is a possible method of reducing the infinite hierarchy of equations into a closed set.

The double correlation tensor $R_{ij}(\hat{r},\hat{x},t)$ for two points separated by the space vector $\hat{r}$ is defined by

$$R_{ij}(\hat{r},\hat{x},t)=\langle \ (\ \hat{x}-\frac{1}{2}(\hat{r},t\ )u_j(\ \hat{x}+\frac{1}{2}\hat{r},t\ )\rangle$$

Similarly, the triple correlation tensor T_{ijk} or higher correlation tensors can be introduced.

The Fourier transform of R_{ij} with respect to $\hat{r}$ defined by

$$\phi_{ij}\left(\hat{k},\hat{x},t\right)=\frac{1}{(2\pi)^3}\int\int\int_{-\infty}^{\infty}\bar{e}^{i}\left(\hat{k},\hat{r}\right)R_{ij}\left(\hat{r},\hat{x},t\right)d\hat{r}$$

represents the energy spectrum function $E\left(\hat{k},t\right)$ in the sense that it describes the distribution of kinetic energy over the various wave number component of turbulent flows. The Fourier transform defined above can be treated as generalized functions or distributions in the sense of Lighthill [71]. It follows from the inverse Fourier transform that

$$\frac{1}{2}\left\langle u^2\right\rangle=\frac{1}{2}\left\langle u_i(\hat{x})u_i(\hat{x})\right\rangle=\frac{1}{2}R_{ij}(o,\hat{x},t)=\int_0^{\infty}E\left(\hat{k},t\right)d\hat{k}$$

So that $E\left(\hat{k},t\right)$ represents the density of contributions to the kinetic energy in the wave numbers of space k, thus the investigation of the energy spectrum function $E\left(\hat{k},t\right)$ is the central problem of the dynamics of turbulence.

Expressed in mathematical form the four methods of averaging applied for instance.

(a) Time average for a stationary turbulence

$$\bar{u}^{t}(x,t)=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{+T}u(x,t)ds$$

In practice the scale used in the averaging process determines the value of the period 2T.

(b) Space average in which we take the average over all the space at given time, i.e

$$\bar{u}^{s}(x,t)=\lim_{v_b\to\infty}\frac{1}{v_b}\int_{v_b}u(s,t)ds$$

In practice the volume of space the scale used in the averaging process determines V_b.

(c) Space time average in which we take the average over a long period of time and over the space i.e.,

$$\overline{\overline{u}}^{s,t}(x,t) = \lim_{T\to\infty,\ V_b\to\infty} \frac{1}{2T.V_b} \int_{-TV_b}^{+T}\int^{t} u(s,y)\,ds\,dy$$

In practice the scale used determines both the values of T and of V_b.

(d) Statistical average in which we take the average over the whole collection of sample turbulent functions for a fixed time, i.e.

$$\overline{\overline{u}}^{\omega}(x,t,\omega) = \int_{\Omega} u(x,t,\omega)\,du(\omega)$$

over the whole Ω space of ω, the random parameter. The measure is

$$\int_{\Omega} d\mu(\omega) = 1$$

Some explanations are neglected for the statistical average. The essential characteristic of the turbulent motion is that the turbulent fluctuations are random in nature. A turbulent velocity field can be regarded as a random vector field of a set of vectors in space and time. Any random vector field can be regarded as a field consisting of three random scalar fields as its components. A random scalar function $u(x,t,\omega)$ is a function of the spatial co-ordinates x and time t, which depends on a parameter ω. The parameter ω is chosen at random according to some probability law in a space.

In the experimental investigation we use time averages almost exclusively, space averages seldom and never statistical averages. In the theory, we use almost exclusively the statistical averages.

For stationary homogeneous turbulence we may expect and assumed that the three averaging lead to the same result.

$$\overline{u}^{t} = \overline{u}^{s} = \overline{u}^{\omega}$$

This assumption is known as the ergodic hypothesis.

1.4 Reynolds Rules of Averages:

Reynolds [102] was the first to introduce elementary statistical motion into the consideration of turbulent flow. In the theoretical investigation of turbulence, he assumed that instantaneous fluid velocity satisfies the Navier-Stockes equations for a viscous fluid and that the instantaneous velocity may be separated into a mean velocity and a turbulent fluctuating velocity. u, P, T and ρ be respectively the instantaneous velocity, pressure time and density, then the process of averaging we write

$$u = \overline{u} + u', \quad P = \overline{P} + P', \quad \rho = \overline{\rho} + \rho', \quad T = \overline{T} + T' \text{ etc}$$

In these expressions the quantities with bars denote mean variables and the quantities with prime denote the fluctuating variables.

Further more $\overline{u}' = \overline{P}' = \overline{T}' = 0$

In the study of turbulence we often have to carry out an averaging procedure not only on single quantities but also on products of quantities. Here the over scores have the following properties.

Let $A = \overline{A} + A'$ and $B = \overline{B} + B'$

In any further averaging procedure we can show that

$$\overline{A} = \overline{\overline{A} + A'} = \overline{\overline{A}} + \overline{A}' = \overline{A} \qquad \text{whence} \quad \overline{A}' = 0 \qquad \text{------------- (1.4.1)}$$

$$\overline{B} = \overline{\overline{B} + B'} = \overline{\overline{B}} + \overline{B}' = \overline{B} \qquad \text{whence} \quad \overline{B}' = 0 \qquad \text{------------- (1.4.2)}$$

In the above relations we used the properties that the average of the sum is equal to the sum of the averages and the average of a constant times B is equal to the constant times the average of B.

Next

$$\overline{\overline{A}\,\overline{B}} = \overline{\overline{A}}\,\overline{\overline{B}} = \overline{A}\,\overline{B} \qquad \text{------------ (1.4.3)}$$

$$\overline{\overline{A}B'} = \overline{\overline{A}}\,\overline{B'} = \overline{A}\,\overline{B}' = 0 \qquad \therefore \overline{B}' = 0 \qquad \text{------------ (1.4.4)}$$

$$\overline{BA'} = \overline{\overline{B}A'} = \overline{B}\overline{A'} = 0 \qquad \therefore\ \overline{A'} = 0 \qquad \text{------------ (1.4.5)}$$

Similarly,

$$\overline{AB} = \overline{(\overline{A} + A')(\overline{B} + B')} = \overline{\overline{A}\,\overline{B}} + \overline{\overline{A}B'} + \overline{A'\overline{B}} + \overline{A'B'} = \overline{A}\,\overline{B} + \overline{A'B'} \qquad \text{------------ (1.4.6)}$$

Note that the average of a product is not equal to the product of the averages. Terms such as $\overline{A'B'}$ are called correlations.

1.5 Reynolds Equations and Reynolds Stresses:

In turbulent flow, we usually assume that instantaneous velocity components satisfy the Navier-Stockes equations,

$$\frac{\partial U}{\partial t} + (U.\nabla)U = F - \frac{1}{\rho}\nabla p + \nu\nabla^2 U \qquad \text{---------- (1.5.1)}$$

The tensor form the equation (1.5.1) can be written as

$$\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j} = F - \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu\frac{\partial^2 u_i}{\partial u_j \partial u_j} \qquad \text{---------- (1.5.2)}$$

Substituting the expression for the instantaneous velocity components $u_i = \overline{u}_i + u_i'$ into the Navier-Stockes equation (1.5.2) for an incompressible fluid after neglecting the body forces and taking the mean values of these equations according to Reynolds rule of averaging (1.4.1)-(1.4.6), we have the following Reynolds equation of motion for the turbulent flow of an incompressible fluid.

$$\rho\left(\frac{\partial \overline{u}_i}{\partial t} + \overline{u}_j\frac{\partial \overline{u}_i}{\partial x_j}\right) = -\frac{\partial \overline{P}}{\partial x_i} + \mu\frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} + \frac{\partial}{\partial x_j}\left(\rho\overline{u_i'u_j'}\right) \qquad \text{---------- (1.5.3)}$$

where i and j run from 1 to 3 and Einstein's summation convection is used. The bar represents the mean value and the prime represents the turbulent fluctuation. Additional terms over the Navier–Stockes equations are due to Reynolds stress are $-\overline{\rho u_i'^2}$ and the eddy stresses are $\overline{-\rho u_i'u_j'}(i \neq j)$, where ρ is the density of the fluid. These stresses represent the rate of transfer of momentum across the corresponding surfaces because of turbulent velocity fluctuations.

The solutions of Reynolds equation will be represent the turbulent flow, but as in the case of Navier-Stockes equations it is not at the present time possible to solve the Reynolds equations for many practical purposes. In general the Reynolds equations are not sufficient to determine the unknown variable u_i, u_j(i, j=1,2,3), p and Reynolds stresses. This is one of the main difficulties in theoretical investigation of turbulent flow. In similar way, Reynolds equation of motion for the turbulent flow of a compressible fluid may be obtained. But the expressions for the eddy stresses (Reynolds stresses) of compressible fluid are much more complicated because the fluctuations of density should be considered.

1.6 Correlation Functions:

Taylor, G. I. [126] introduced new notions into the study of the statistical theory of turbulence in a most important series of papers in 1935. He successfully developed a statistical theory of turbulence, which is applicable to continuous movements and satisfies the equation of motion.

The first important new notion was that of studying the correlation or coefficient of correlation between two fluctuating quantities in turbulent flow. In his theory, Taylor makes much use of the correlation between the components of the fluctuations neighbouring points.

The statistical property of a random variable may be described by the correlation function, which is defined as follows:

Consider the fluctuating variables u_i and u_j and assume that there exists certain correlation between them. The correlation function is defined as

$$P_{ij} = \overline{u_i u_j} \qquad \text{-----------} (1.6.1)$$

where the bar denotes the average by certain process. Some times it is convenient to use the correlation coefficient such as

$$R_{ij} = \frac{\overline{u_i u_j}}{\sqrt{\overline{u_i^2}}\sqrt{\overline{u_j^2}}} \qquad \text{-----------} (1.6.2)$$

By Cauchy inequality, we have

$$\overline{u_i u_j} - \sqrt{\overline{u_i^2}}\sqrt{\overline{u_j^2}} \le 0 \qquad \text{------------} (1.6.3)$$

hence $-1 \le R_{ij} \le 1$

If we consider u_i and u_j as the velocity components in a flow field, the correlation of Equation (1.6.1) as a tensor of second rank.

By a different process of averaging we obtain different kinds of correlation functions. If we consider u_i and u_j as the velocity components at a given point in space, u_i and u_j are functions of time; hence, we showed take the time average in equation (1.6.1) to get the correlation function P_{ij}.

If we consider u_i and u_j as the velocity components at a given time, u_i and u_j are functions of space co-ordinates $x(x_1, x_2, x_3)$; hence, we should take the space average in equation to get the correlation function.

More generally if we consider u_i and u_j as functions of both time t and spatial co-ordinates $x(x_1, x_2, x_3)$, we obtain take a space-time average in equation (1.6.1) to get the correlation function.

The correlation function between the components of the fluctuating velocity at the same time at two different points of the fluid, first introduce by Taylor, G. I. [126], has been investigated extensively in the isotropic turbulence.

The correlation function between two fluctuating velocity components at the same point and at the same time gives the Reynolds Stress. The correlation function between two fluctuating quantities may also be defined in a manner similar to above.

1.7 Spectral Representation of the Turbulence:

Theoretical treatment of the turbulence is merely related to the solution of the Navier-Stockes equations. These equations, however, contain more unknowns than number of equations and therefore additional assumptions must be made. This is known as " Closure problem". An alternative approach is based on the spectral form of the dynamical Navier-Stockes equation. The spectral form of the turbulence is still under-determined, but it has a simple physical interpretation and is more convenient. The spectral approach is, however, almost exclusively used for the description of homogeneous turbulence [85,86]. The principal concepts of spectral representation in the study of turbulence are described below:

If we neglect the body forces from the Navier-Stockes equation (1.5.2) and multiply the x_i-component of Navier-Stockes equation written for the point P by u'_j and multiply the x'_j component of the equation written for the point P′ by u'_i adding and taking ensemble averages we get.

$$\frac{\partial}{\partial t}\overline{u_i u'_j} + \overline{u'_j u_l \frac{\partial u_i}{\partial x_l}} + \overline{u_l u'_l \frac{\partial u'_j}{\partial x'_l}} = -\frac{1}{\rho}\left[\overline{u'_j \frac{\partial p}{\partial x_i}} + \overline{u_i \frac{\partial p'}{\partial x'_j}}\right] + v\left[\overline{u'_j \frac{\partial^2 u_i}{\partial x^2{}_l}} + \overline{u_i \frac{\partial^2 u'_j}{\partial x^2{}_l}}\right] \quad \text{------ (1.7.1)}$$

Since in homogeneous turbulence the statistical quantities are independent of position in space and considering the point P and P′. Separated by a distance vector $\bar{r}$ and applying the laws of spatial covariances, a simplified form of equation (1.7.1) is obtained as:

$$\frac{\partial}{\partial t}\overline{u_i u'_j} = -\frac{\partial}{\partial r_1}\left(\overline{u_i u'_j u_l} - \overline{u_i u'_j u'_l}\right) + \frac{1}{\rho}\left[\frac{\overline{\partial p u'_j}}{\partial r_i} - \frac{\overline{\partial p' u_i}}{\partial r_j}\right] + 2v\frac{\partial^2 \overline{u_i u'_j}}{\partial r_l^2} \quad \text{-------- (1.7.2)}$$

The covariance $\overline{u_i u'_j}$ is not suitable for direct analysis of quantitative estimate of the turbulent flows and it is better to use the three-dimensional Fourier transforms of $\overline{u_i u'_j}$ with respect to $\bar{r}$. The variable that corresponds to $\bar{r}$ in the three dimensional wave-number space is a vector $\bar{K} = (K_1, K_2, K_3)$. We define the wave number spectral density as:

$$\phi_{ij}\left(\vec{K}\right) = \frac{1}{(2\pi)^3}\int \overline{u_i u'_j}\exp\left(-i\bar{K}.\bar{r}\right)d\bar{r} = \frac{1}{(2\pi)^3}\iiint \overline{u_i u_j}\exp\{-i(K_1 r_1 + K_2 r_2 + K_3 r_3)\}dr_1 dr_2 dr_3$$

-------- (1.7.3)

It can be shown that if $\overline{u_i u'_j}$ has a continuous range of wavelength, $\phi_{ij}\left(\vec{K}\right)$has a continuous distribution in wave number space. We can rigorously regard $\phi_{ij}\left(\vec{K}\right)dK_1dK_2dK_3$ as the contribution of elementary volume $dK_1dK_2dK_3$ (centred at wave number $\hat{K}$ and therefore representing a wave number of length $\frac{2\pi}{|\vec{K}|}$, in the direction of vector $\vec{K}$) to the value of $\overline{u_i u'_j}$

hence the name "Spectral density". This is consistent with the behaviour of the inverse transform

$$\overline{u_i u_j'(r)} = \int_{-\infty}^{\infty} \phi_{ij}(\vec{K}) \exp(i\vec{K}.r)\, d\vec{K} \qquad \text{---------- (1.7.4)}$$

The one dimensional wave number spectrum of $\overline{u_i u_j'}$ for a wave number component in the x_l direction is

$$\phi_{ij}(K_l) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{u_i u_j'}(r_l) \exp(-i\vec{K}_l.r_l)\, dr_l \qquad \text{---------- (1.7.5)}$$

whose inverse is

$$\overline{u_i u_j'(r)} = \int_{-\infty}^{\infty} \phi_{ij}(K_l) \exp(ik_l, r_l).dK_l \qquad \text{---------- (1.7.6)}$$

The equation (1.7.2) for unstrained homogeneous turbulence becomes on Fourier transforming as:

$$\frac{\partial \phi_{ij}(\vec{K})}{\partial t} = \Gamma_{ij}(\vec{K}) + \Pi_{ij}(\vec{K}) - 2\nu K_l^2 .\phi_{ij}(\vec{K}) \qquad \text{------------ (1.7.7)}$$

where Γ and Π are the transforms of the triple product and pressure terms respectively.

1.8 Equation of Motion of Dust Particles:

Knowledge of the behaviour of discrete particles in a turbulent flow is of great interest to many branches of technology, particularly if there is a substantial difference between particles and the fluid. Saffman [106] derived an equation that described the motion of a fluid containing small dust particles, which is applicable to laminar flows as well as turbulent flow.

A more plausible explanation seems to be that the dust damps the turbulence. A dust particle in air, or in any other gas, has a much larger inertia than the equivalent volume of air and will not therefore participate readily in turbulent fluctuations. The relative motion of dust particles and the air will dissipate energy because of the drag between dust and air, and extract energy from turbulent fluctuations. If as certainly seems possible, the turbulent intensity is reduced than the Reynolds stresses will be decreased and the force required to maintain a given flow rate will likewise be reduced.

In order to formulate the problem in a reasonably simple manner and to bring out the essential features, we shall make simplifying assumption about the motion of dust particles. It will be supposed that their velocity and number density can be described by fields $u(\vec{x},t)$ and $N(\vec{x},t)$. We also assume that the bulk concentration (i.e. concentration of volume) of dust is very small so that the effect of dust particles on the gas is equivalent to an extra force $KN(\vec{v}-\vec{u})$ per unit volume, where $\vec{u}(\vec{x},t)$ is the velocity of the gas and K is constant. It is also supposed that the Reynolds number of the relative motion of dust and gas is small compared with unity, so that the force between the dust and gas is proportional to the relative velocity. Then with small bulk concentration and the neglect of the compressibility of the gas, the equations of motion and continuity of the gas are:

$$\rho\left[\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u}\right] = -\nabla p + \nu\nabla^2\vec{u} + KN(\vec{v}-\vec{u}) \qquad \text{---------- (1.8.1)}$$

$$\text{div } \vec{u} = 0 \qquad \text{---------- (1.8.2)}$$

where p, ρ and μ are the pressure, density and viscosity of the clean gas respectively. If dust particles are spheres of radius ε, then by Stocke's drag formula, $K = 6\pi\mu\varepsilon$.

As will be seen below, the effect of the dust is measured by the mass concentration, say f. The bulk concentration is $f\frac{\rho}{\rho_1}$ where ρ_1 is the density of the material in the dust particles. For common materials $\frac{\rho}{\rho_1}$ will be of the order of several thousand or more, so that the mass concentration may be significant fraction of unity, while the bulk concentration is small. it is to be noted that for suspension in liquids, the bulk and mass concentration will roughly be the same. So that the qualitative differences in the motion of dusty gases and the suspensions in the liquids may be expected. For spherical particles, the Einstein increase in the viscosity is $\frac{5}{2}\mu f\frac{\rho}{\rho_1}$, which is negligible for a dusty gas but may be significant for a liquid suspension. The force exerted on the dust by the gas is equal and opposite to the force exerted on the gas by dust, so that the equation of motion of the dust is:

$$mN\left[\frac{\partial\vec{v}}{\partial t}+(\vec{v}.\nabla)\vec{v}\right]=mN\vec{g}+KN(\vec{v}-\vec{u}) \qquad \text{------------- (1.8.3)}$$

where mN the mass of the dust per unit volume and $\vec{g}$ is the acceleration due to gravity. The buoyancy force is neglected since $\frac{\rho}{\rho_1}$ is small.

The equation of continuity of the dust is:

$$\frac{\partial N}{\partial t}+div(N\vec{v})=0 \qquad \text{------------- (1.8.4)}$$

Here, $\nu=\frac{\mu}{\rho}$ is kinetic viscosity of the clean gas and $\tau=\frac{M}{K}$ is called the relaxation time of the dust particles. It is measure of the time for the dust to adjust to changes in the gas velocity. For spherical particles of radius ε,

$$\tau=\frac{\frac{4}{3}\pi\varepsilon^3\rho_1}{6\pi\varepsilon\mu}, \text{ or } \qquad \tau=\frac{2}{9}\frac{\varepsilon^2}{\nu}\frac{\rho_1}{\rho} \qquad \text{------------- (1.8.5)}$$

where $\frac{4}{3}\pi\varepsilon^3\rho$, mass of single spherical dust particle of radius ε; ρ_1, density of the material in the dust particles.

The effect of dust is described in two parameters f and τ. The former describes how much dust is present and the latter is determined by the size of individual particles. Making the dust fine, will decrease τ, and making coarse, will increase τ in a manner proportional to the surface area of the particles.

1.9 Decay of Turbulence before the final period and in the final period:

In considering the dynamic equations for the velocity correlation and for the energy spectrum, it has been shown that these correlations and spectra change with time and the turbulence decays if no energy sources are present to sustain it. As in all fluid flows, an important parameter is the Reynolds number and the character of the turbulence may vary appreciably whether the Reynolds number of turbulence is large or small.

Batchelor and Townsend [2,3,4] have made many measurements of the decay of an isotropic turbulence produced by grids. From the results of these measurements Batchelor [3] arrives at the conclusion that different periods of decay may actually be distinguished; an initial period, a final period and a transition period. This is considered with respect to time, but for turbulence behind grids it applies to consecutive regions downstream of the grid.

How long the initial period will last is difficult to say, since the transition to other periods is very gradual, and since moreover, it would depend strongly on the initial value of the Reynolds number. The above-mentioned experimental data by Townsend seen to show that the turbulence may be considered to be in the initial period up to $x_1/M = 100$ to 150, but data obtained by other experimenters have shown from the initial period-decay law may occur.

Townsend's experiments have shown that the final period seems to apply to distances greater than 500M. Of course, this value too should depend on the initial Reynolds number of turbulence.

In Townsend's experiments the Reynolds number $\mathrm{Re}_M = \overline{U}_1 M/\nu$ was about 650. Where, $\mathrm{Re}_M \rightarrow$mesh Reynolds number; M, Mesh of a grid; $\overline{U}_1$, speed; ν, kinematic viscocity.

In the initial period the decay is determined predominantly by the decay of the energy containing eddies; in the final period the viscous effects predominate over inertial effects. Thus, in the final period, where the Reynolds number of turbulence is very small, the inertial terms in the dynamic equations may be neglected.

According to Deissler [27], in the final period of decay the inertia terms (triple correlations) in the two point correlation equation obtained from the momentum and continuity equations can be neglected because the Reynolds number of the eddies is small, and a solution can be obtained. However, at earlier times the inertia terms in the two-point correlation equation can't be neglected. So that in order to obtain a solution, an intuitive assumption is generally introduced to relate the triple correlations to the double correlations. The situation in homogeneous turbulence is therefore analogous to that in turbulent shear flow where intuitive assumptions have been introduced to relate the Reynolds stress or the eddy diffusivity to the mean flow; although one case of homogeneous turbulence, the turbulence in the final period, has been solved without introducing intuitive hypothesis. Where as those analyses aided greatly in unifying much of the information on turbulent flow and in clarifying some of the physical aspects of turbulence, they do not of course, constitute deductive theories based on the momentum and continuity equations.

It should be possible to predict the turbulent decay at times before the final period from the momentum and continuity equations. If the initial distribution of velocities and pressure is known, the momentum and continuity equations could be used numerically to predict the distributions a short time later. However it appears that because of the small size of some of the eddies, the step sizes in the calculations would have to be extremely small.

A better plan may be to construct, from the momentum and continuity equations, equations involving correlations between velocities and pressures at more than two points. Then, for instance, in the three-point correlations equation, one neglects the quadruple correlations, which should be applicable before the final period. In the final period the triple correlations are of course negligible. Using the expression for the triple correlations so obtained, the two-point equation can be solved and the various quantities describing the turbulence at times the final period can be obtained. Higher order approximations, valid at still earlier times can be obtained in the same way by constructing four- or five-point correlation equations. Each time the set of equations is made determinate by neglecting the highest order correlation.

1.10 Distribution Functions in Turbulence and it's Properties:

The distribution function in the statistical theory are discussed by several authors in the past, but the dynamical equations describing the time evolution of the finite dimensional probability distributions in turbulence were first proposed by Lundgren [75] and Monin [84,85]. Lundgren [75] considered a large ensemble of identical fluid system in turbulent state. In his consideration each number of the ensemble is an incompressible fluid in an infinite space with velocity $\hat{u}(\hat{r},t)$ satisfying the continuity and Navier-Stockes equations. The only difference in the members of ensemble is the initial conditions that vary from member to member. He considered a function $F(\hat{u}(\hat{r}_1,t),\hat{u}(\hat{r}_2,t)---)$ whose ensemble is given as $\langle F(\hat{u}(r_1,t),\hat{u}(r_2,t)---)\rangle$ and defined one point distribution function $f_1(\hat{r}_1.\hat{v}_1,t)$ such that

$\int f_1(\hat{r}_1.\hat{v}_1,t)\,d\hat{v}_1$ is the probability that the velocity at a point $\hat{r}_1$ at time t is in element $d\hat{v}_1$ about $\hat{v}_1$ and is given by $f_1(\hat{r}_1,\hat{v}_1,t) =< \delta(\hat{u}(\hat{r}_1,t)-\hat{v}_1)\ \rangle$

and two points distribution function is given by

$$f_2(\hat{r}_1,\hat{v}_1,\hat{r}_2,\hat{v}_2,t) = \langle \delta(\hat{u}(r_1,t)-\hat{v}_1)\delta(\hat{u}(r_2,t)-\hat{v}_2)\rangle$$

In short one and two point distribution functions are denoted as $f_1^{(1)}$ and $f_2^{(1,2)}$. Here δ is the dirac-delta function, which is defined as

$$\int \delta(\overline{u}-\overline{v})d\overline{v} = \begin{cases} 1 & \text{at the point } \overline{u}=\overline{v} \\ 0 & \text{elsewhere} \end{cases}.$$

and $\langle\quad\rangle$ denote the ensemble average.

1.11 Fourier Transformation of the Navier-Stockes Equation:

The principal reason for using Fourier transformation is that they convert differential operators into multipliers. The equations are so complicated in configuration (or coordinate) space that very little can be done with them, and the transformation to wave number (or Fourier) space simplifies them very considerably.

Another and more mathematical argument shows that these transforms are right method of treating a homogeneous problem. Associated with any correlation function, $\phi(\vec{x},\vec{x}')$ is a sequence of eigen functions $\phi(\vec{n},\vec{x}')$ and their associated eigen-values $\lambda(\vec{n})$. These quantities satisfy the value equation.

$$\int \phi(\vec{x},\vec{x}')\Psi(\vec{n},\vec{x})d^3\vec{x}' = \lambda(\vec{n})\Psi(\vec{n},\vec{x}) \qquad \text{-------------- (1.11.1)}$$

and the orthonormalization relation

$$\int \Psi(\vec{n},\vec{x})\Psi^*(\vec{m},\vec{n})d^3\vec{x} = 1, \quad \text{if m=n} \qquad \text{-------------- (1.11.2)}$$
$$=0 \quad \text{otherwise}$$

These equations imply that ϕ is a scalar. Actually it is a tensor of order two, but this complicates the argument without introducing anything essentially new. The index $\vec{n}$ is in general a complex variable and ψ* denotes the complex conjugate of ψ (strictly, ψ* is the adjoint of ψ, but since ϕ is real and symmetric the adjoint is simply the complex conjugate). The integrations in equations (1.11.1) and (1.11.2) are over all space, which may be finite or infinite. If the space is finite $\vec{n}$ is usually an infinite but countable sequence, while if space is infinite, $\vec{n}$ will be a continuous variable. Here the eigen functions all have real eigen-values. If follows from (1.11.1) and (1.11.2) that.

$$\phi(\vec{x}.\vec{x}) = \sum_{all\ n} \lambda(\vec{n}).\psi(\vec{n}.\vec{x})\psi^*(\vec{n},\vec{x}') \qquad \text{------------- (1.11.3)}$$

and this is the diagonal representation of the correlation function in terms of its eigen functions. Evidently these functions are only defined "within a phase" that is, a factor exp(iγ) can be added to $\psi(\vec{n},\vec{x})$ without altering $\phi(\vec{x},\vec{x}')$ provided γ is real and independent of x. For a homogeneous field, ϕ is a function of $\vec{x},\vec{x}'$ only and the problem is to find the eigen functions which are also homogeneous within a phase in the sense that

$$\psi(\vec{n},\vec{x}') = \exp(i\gamma)\psi(\vec{n},\vec{x}+\vec{a})$$

This equation is satisfied by the Fourier equation

$$\psi(\vec{n},\vec{x}) = \exp(i\vec{n}.\vec{x}) = \exp(i\vec{n}_j\vec{x}_j)$$

with $\gamma = -\vec{n}.\vec{a}$. In this situation (instance), therefore, "the index", $\vec{n}$ is a wave number. Equation (1.11.3) becomes.

$$\phi(\vec{x},\vec{x}') = \sum \lambda(\vec{n})\exp\{in(\vec{x}-\vec{x}')\}$$

so that $\lambda(\vec{n})$ may be identified with $\phi(\vec{n})$, the Fourier transform of the correlation function.

Since we are considering homogeneous isotropic turbulence, the turbulent field must be infinite in extent. This produces, mathematical difficulties, which can only be resolved by using functional calculus. This difficulty is avoided by supposing that the turbulence is confined to the inside of a large box with sides (a_1, a_2, a_3) and that it obeys cyclic boundary conditions on the sides of this box. The a_i is allowed to tend to infinity at an appropriate point in the analysis. Thus the Fourier transform is defined by

$$U_i(\vec{x}) = (2\pi)^3(a_1,a_2,a_3)^{-1}\sum_K u_i(\vec{K})\exp(i\vec{K}.\vec{x}) \qquad \text{--------------(1.11.4)}$$

Here $\vec{K}$ is limited to wave vectors of the from

$$\frac{2n_1\pi}{a_1}, \frac{2n_2\pi}{a_2}, \frac{2n_3\pi}{a_3}$$

where n_i are integers while the a_i are, as before the sides of the elementary box. As these sides become infinitely large, equation (1.11.4) goes over into standard form,

$$U_i(\vec{x}) = \int u_i(\vec{K})\exp(i\vec{K}.\vec{x})d^3\vec{K} \qquad \text{------------- (1.11.5)}$$

The inverse of (1.11.5) is,

$$u_i(\vec{K}) = (2\pi)^{-3} \int_{box} u_i(\vec{x})\exp(-i\vec{K}\vec{x})d^3x \qquad \text{-------------- (1.11.6)}$$

The Fourier transform of Navier-Stockes equation may be written as

$$\left[\frac{d}{dt} + vK^2\right]u_i(\vec{K}) = M_{ijm}(\vec{K})\overset{\Delta}{\sum} u_j(\vec{P})U_m(\vec{r}) \qquad \text{-------------- (1.11.7)}$$

where $\overset{\Delta}{\sum}$ is a short notation for the integral operator in

$$\iint U_j(\vec{K})U_m(\vec{r}).\delta(\vec{K} - \vec{P} - \vec{r})(d^3\vec{p})(d^3\vec{r}) \qquad \text{-------------- (1.11.8)}$$

where δ_K, p+r is the Kronecker delta symbol which is zero unless

$$\vec{K} = \vec{p} + \vec{r}$$

Here, $M_{ijm}(\vec{K})$ is a simple algebraic multiplier and not a differential operator. We have

$$M_{ijm}(\vec{K}) = -\frac{1}{2}i.P_{ijm}(\vec{K}) \qquad \text{--------------- (1.11.9)}$$

where, $P_{ijm}(\vec{K}) = K_m P_{ij}(\vec{K}) + K_j P_{im}(\vec{K})$

and $P_{ij} = \delta_{ij} - \dfrac{K_i K_j}{K^2}$

$P_{ij}(\vec{K})$ is the Fourier transform of $P_{ij}(\nabla)$ but $P_{ijm}(\vec{K})$ is not the transform of $P_{ijm}(\nabla)$.

As it is stands, equation (1.11.7) can't describe stationary turbulence since it contains no input of energy to balance the dissipative effect of viscosity. In real life this input is provided by effects, such as the interaction of mean velocity gradient with the Reynolds stress, which are incompatible with the ideas of homogeneity and isotropy. To avoid this difficulty, we introduce in to the right hand side of equation (1.11.7) a hypothetical homogeneous isotropic stirring force f_i. The equation then reads.

$$\left[\frac{d}{dt}+\nu K^2\right]u_i(\vec{K}) = M_{ijm}(\vec{K})\sum^{\Delta} u_j(\vec{P})u_m(\vec{r}) + \partial_i(\vec{K}) \qquad \text{---------------- (1.11.10)}$$

1.12 Magneto-hydrodynamic (MHD) Turbulence:

Magneto-hydrodynamic (MHD) is an important branch of fluid dynamics. MHD is the science, which deals with the motion of highly conducting fluids in the presence of a magnetic field. The motion of the conducting fluid across the magnetic field generates electric current, which changes the magnetic field and the action of the magnetic field on these currents gives rise to mechanical force, which modifies the flow of the field. From historical point of view it seems that the first attempt to study the problem of MHD is due to Faraday. Later on in 1937 Hartmann took up Faraday's idea in understood conditions. Hartmann carried out experiments, which demonstrated the influence of a very intense magnetic field on the motion of mercury.

There are two basic approaches to the problem, the macroscopic fluid continuum model known as MHD, and the microscopic statistical model known as plasma dynamics. We shall be concerned here only with the MHD, that is electrically conducting fluids, and study the problems of MHD turbulent flow.

The magneto-hydrodynamic turbulence is the study of the interaction between a magnetic field and the turbulent motions of an electrically conducting fluid. The interaction between the velocity and the magnetic fields results in a transfer of energy between the Kinetic and magnetic spectra, and it is thought that the interstellar magnetic field is maintained by a "dynamo" action from turbulence in the interstellar gas.

Modern applications of magneto-hydrodynamics in the fields of propulsion, nuclear fission and electrical power generation make the problem of magneto-hydrodynamic turbulence one of considerable interest to engineers, since turbulent phenomena seen to be inherent in almost all type of flow problems.

The theory of turbulence in an incompressible viscous and electrically conducting fluid is formulated probabilistically through the use of the joint characteristic functional and the functional calculus. The use of the joint characteristic functional approach relies upon the fact that the velocity and magnetic fields are both solenoid, and hence, in the probabilistic sense, are jointly distributed over the phase space consisting of the set of all solenoid vector fields. The formulation of the problem in phase space is completely carried out. The full space-time functional formulation of the problem as developed by Lewis and Kraichnan [73] for "ordinary turbulence" is extended to magneto-hydrodynamic turbulence. This approach enables us to generate space-time correlation between the velocity and magnetic field components rather than merely spatial correlations as were used in the original [43] Hopf presentation. Dynamical equation for various order space-time correlations between velocity and magnetic field components are derived from the joint characteristic functional by its expansion in a Taylor series.

The concept of Kolomogoroff's [68] equilibrium hypothesis for ordinary turbulence is extended to magneto-hydrodynamic turbulence. The problem of predicting the form of the energy spectra in the equilibrium range is taken up.

The fundamental equations of magneto-hydrodynamics for an incompressible fluid are

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} = -\frac{1}{\rho}\nabla p + \frac{\rho_e}{\rho}\vec{E} + \frac{\mu}{\rho}\vec{j} \times \vec{H} + \nu\nabla^2\vec{u} + \vec{F} \qquad \text{------------ (1.12.1)}$$

$$\nabla.\vec{u} = 0 \qquad \text{------------ (1.12.2)}$$

$$\frac{K}{c}\frac{\partial \vec{E}}{\partial t} = curl\vec{H} - 4\pi\vec{J} \qquad \text{----------- (1.12.3)}$$

$$\frac{\mu_e}{c}\frac{\partial \vec{H}}{\partial t} = -curl\vec{E} \qquad \text{----------- (1.12.4)}$$

$$\nabla.\vec{H} = 0 \qquad \text{----------- (1.12.5)}$$

$$J = \sigma\left(c\vec{E} + \mu_e \vec{u} \times \vec{H}\right) + \rho_e \frac{\vec{u}}{c} \qquad \text{----------- (1.12.6)}$$

where $\vec{u}$, the velocity vector; $\vec{F}$, the body force; P, the pressure; ρ, the density of the fluid which is constant; ρ_e, the excess electric charge; $\vec{E}$, the electric field strength; μ_e, the magnetic permeability; $\vec{J}$, the electric current density; $\hat{H}$, the magnetic field strength; ν, the coefficient of kinematic viscosity; k, the dielectric constant; c, the speed of light; σ, the electrical conductivity; ∇, the gradient operator, $\nabla.\nabla = \nabla^2$ and t is the time.

When conductivity σ of the fluid tends to infinity the electric field strength $\vec{E}$, at each point must tend to the value $\frac{\mu_e \vec{u} \times \vec{H}}{c}$, otherwise the current $\vec{J}$ given by equation (1.12.6) becomes very large even when very slightest mass motion is present. Hence when σ is large we may assume that

$$\vec{E} = -\mu_e \frac{\vec{u} \times \vec{H}}{c} \qquad \text{------------ (1.12.7)}$$

a relation which is increasingly valid as $\sigma \to \infty$

An important consequence of relation (1.12.7) is that under the circumstances in which this is a good approximation the energy in the electric field is of the order of $|u|^2/c^2$ of the energy in the magnetic field and can, therefore, be neglected.

This approximation is known as the approximation of Magneto-hydrodynamics. We have to consider only the interaction between the two fields $\vec{u}$ and $\vec{H}$.

In the MHD approximation, Maxwell equation (1.12.3) becomes,

$$\vec{J} = \frac{1}{4\pi} curl\vec{H} \qquad \text{---------- (1.12.8)}$$

In the framework of approximations (1.12.7) and (1.12.8) the Navier-Stockes equation are modified to take into account the electromagnetic body force (assuming that there is no body force $\vec{F}$) and equation (1.12.1) becomes

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} = \frac{\mu_e}{4\pi\rho} curl\vec{H} \times \vec{H} - \frac{1}{\rho}\nabla P + \nu\nabla^2\vec{u} \qquad \text{---------- (1.12.9)}$$

Again, in the approximation (1.12.7), Maxwell equation (1.12.4) becomes

$$\frac{\partial \vec{H}}{\partial t} = curl(\vec{u} \times \vec{H}) \qquad \text{-------- (1.12.10)}$$

In a higher approximation in which the loss of energy by Joule heat is allowed for the equation (1.12.10) is modified to [6]

$$\frac{\partial \vec{H}}{\partial t} - curl(\vec{u} \times \vec{H}) = \lambda\nabla^2\vec{H} \qquad \text{--------- (1.12.11)}$$

where $\lambda = (4\pi\mu_e\sigma)^{-1}$ is the magnetic diffusivity

Now the magnetic field intensity $\vec{H}$ is a solenoidal vector, and in an incompressible fluid the velocity $\vec{u}$ is also a solenoidal vector. When we use this property of $\vec{u}$ and $\vec{H}$ equations (1.12.9) and (1.12.11) can be written in the form [5] as

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i \partial u_k}{\partial x_k} - \frac{\mu_e}{4\pi\rho}\frac{\partial}{\partial x_k}(H_i H_k) = -\frac{1}{\rho}\frac{\partial}{\partial x_k}(P + \mu_e\frac{|\vec{H}|^2}{8\pi}) + \nu\nabla^2 u_i \qquad \text{---------- (1.12.12)}$$

and

$$\frac{\partial H_i}{\partial t} + \frac{\partial}{\partial x_k}(H_i u_k - u_i H_k) = \lambda\nabla^2 H_i \qquad \text{----------- (1.12.13)}$$

where, here and in the sequel, summation over the repeated indices is to be understood. Equations (1.12.12) and (1.12.13) form the basis of Batchelor's [6] discussion. Chandrasekhar [16] extended the invariant theory of turbulence to the case of magneto-hydrodynamics. He introduced the new variable as

$$\bar{h} = \sqrt{\frac{\mu_e}{4\pi\rho}}.\vec{H} \qquad \text{----------- (1.12.14)}$$

for H. It has the dimension of velocity (known as Alfven's velocity) but be haves as vorticity.

In terms of $\bar{h}$ the equations (1.12.12) and (1.12.13) can be expressed as

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial P_n}{\partial x_i}+\nu\nabla^2 u_i \qquad \text{---------- (1.12.15)}$$

and $$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \lambda\nabla^2 h_i \qquad \text{---------- (1.12.16)}$$

where $P_n = \frac{P}{\rho}+\frac{1}{2}|\vec{h}|^2$, is the total MHD pressure and $\lambda = (4\pi\mu_e\sigma)^{-1}$ is the magnetic diffusivity. Chandrasekhar [21, 22] in his theory, considered the correlation's between $\vec{u}$ and $\vec{h}$ at two points p and p′ in the field of isotropic turbulence in the same manner as in the ordinary turbulence. Here, we have the double correlation, $\overline{u_i u_j'}$, $\overline{h_i h_j'}$ and $\overline{u_i h_j'}$, and the triple correlation,

$\overline{u_i u_j u_k'}$, $\overline{h_i h_j u_k'}$, $\overline{u_i u_j h_k'}$, $\overline{h_i h_j h_k'}$, $\overline{(h_i u_j - u_i h_j)h_k'}$, and $\overline{(h_j' u_k' - h_k' u_j')u_i}$,

where the subscripts refer to the components of the vectors i,j,k=1,2,3. Each of these double and triple correlations depends on one scalar function in the case of isotropic turbulence because the divergence of both $\vec{u}$ and $\vec{h}$ is zero.

Equations (1.12.15) and (1.12.16) are derived by coupling Maxwell's equations for the electromagnetic field and Navier-Stockes equations for the velocity field. The Maxwell equations are modified to include the induced electric field due to the fluid motion and the Navier-Stockes equations are modified to include the Lorentz force on fluid elements due to the magnetic field. The so-called "Magneto-hydrodynamic approximation" is made, in which displacement currents are neglected in Maxwell's equations. This approximation is well founded provided we are not dealing with very rapid oscillations of the electromagnetic field quantities, as is the case in the propagation of electromagnetic waves. Under this approximation, the energy in the electric field is of the order of $\frac{1}{c^2}$ times the energy in the magnetic field, where c is the speed of light and hence may be neglected. Therefore, we have only to consider the interaction between the velocity field $\vec{u}$ and the magnetic field $\vec{h}$.

1.13 A brief Description of Past Researches Relevant to this Thesis work:

In geophysical flows, the system is usually rotating with a constant angular velocity. Such large-scale flows are generally turbulent. When the motion is referred to axes, which rotate steadily with the bulk of the fluid, the coriolis and centrifugal force must be supposed to act on the fluid. The coriolis force due to rotation plays an important role in a rotating system of turbulent flow, while the centrifugal force with the potential is incorporated into the pressure.

Kishore and Dixit [52], Kishore and Singh [54], Dixit and Upadhyay [30], Kishore and Golsefied [57] discussed the effect of coriolis force on acceleration covariance in ordinary and MHD turbulent flow. Funada, Tuitiya and Ohji [37] considered the effect of coriolis force on turbulent motion in presence of strong magnetic field. Kishore and Sarker [62] studied the rate of change of vorticity covariance in MHD turbulence in a rotating system. Sarkar [109] discussed the vorticity covariance of dusty fluid turbulence in a rotating frame. Shimomura and Yoshizawa [119], Shimomura [120] and [121] also discussed the statistical analysis of turbulent viscosity, turbulent scalar flux and turbulent shear flows respectively in a rotating system by two-scale direct interaction approach. Sarker [111] studied the Thermal decay process of MHD turbulence in a rotating system.

Saffman [106] derived an equation that described the motion of a fluid containing small dust particles, which is applicable to laminar flows as well as turbulent flow. Using the Saffman's equation Michael and Miller [83] discussed the motion of dusty gas occupying the semi-infinite space above a rigid plane boundary. Sinha [122], Sarker [110], Sarker and Rahman [112] considered dust particles on their won works.

The essential characteristic of turbulent flows is that turbulent fluctuations are random in nature and therefore, by the application of statistical laws, it has been possible to give the idea of turbulent fluctuations. The turbulent flows, in the absence of external agencies always decay. Millionshtchikov [81], Batchelor and Townsend [4], Proudman and Reid [101], Tatsumi [124], Deissler [27,28], Ghosh [38,39] had given various analytical theories for the decay process of turbulence so far. Further Monin and Yaglom [85] gave the spectral approach for the decay process of turbulence. Although, MHD turbulent fluctuations are random in nature but exhibit the characteristic structure likewise the hydrodynamic turbulence, hence the statistical laws can also be applied in MHD turbulence. Mazumdar [87] derived an early period decay equation for general type of turbulence for an incompressible fluid. Also Sinha [122] discussed the decay process of MHD turbulence and derived an early period decay equation. Sarker and Kishore [108] discussed the decay of MHD turbulence before the final period. The approach is phenomenological in the sense that they considered the region where the variations of the mean temperature and mean velocity may be neglected because of the transportation of the thermal energy from place to place is very rapid.

Deissler [27,28] developed a theory for homogeneous turbulence, which was valid for times before the final period. Using Deissler's theory Loeffler and Deissler [72] studied the temperature fluctuations in homogeneous turbulence before the final period. Following Deissler's approach Sarker and Islam [116] also studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time. Sarker and Rahman [113] studied the decay of temperature fluctuations in MHD turbulence before the final period, Sarker and Islam [117] considered the decay of dusty fluid turbulence before the final in a rotating system.

Sarker and Rahman [112] discussed the decay of turbulence before the final period in presence of dust particles. Sarker and Islam [118] studied the effect of very strong magnetic field on acceleration covariance in MHD turbulence of dusty fluid turbulence in a rotating system. Further Rahman and Sarker [105] studied the decay of dusty fluid MHD turbulence before the final period. In their approach they considered the two and three point correlation equations and solved these equations after neglecting the fourth and higher order correlation terms compared to the second and third order correlation terms.

Using Deissler's theory Kumar and Patel [64] studied the first order reactants in homogeneous turbulence before the final period for the case of multi-point and single time. The problem [64] also extended to the case of multi-point and multi-time concentration correlation in homogeneous turbulence by Kumar and Patel [65]. The numerical result of [65] carried out by Patel [97].

Following Deissler's approach Sarker and Kishore [108] studied the decay of MHD turbulence before the final period. Sarker and Islam [115] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Islam and Sarker [46] discussed the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Sarker and Islam [117] also studied the decay of dusty fluid turbulence before the final period in a rotating system.

By analyzing the above all theories, we have studied the chapter II, chapter IV and chapter V.

Hopf [43], Kraichnan [69], Edward [32] and Hering [40] have given various analytical theories in the statistical theory of turbulence. But at first Lundgren [74] derived the dynamical equations, which are describing the time evolution of the finite dimensional probability distribution of turbulent quantities. Lundgren [74] derived a hierarchy of coupled equations for multi-point turbulence velocity distribution functions, which resemble with BBGKY hierarchy of equations of Ta-Yu-Wu [131] in the kinetic theory of gases. Further Lundgren [75] considered a similar problem for non-homogeneous turbulence. The basic difficulty is that the above theories faced to closure problem. Some general approaches to closure problem for multi dimensional probability density equations those were made by Lyubimov and Ulinch [77,78]. Two other closure hypotheses for the probability distribution equation of single time values were investigated by Fox [35], Lundgren [76], Bray and Moss [14] considered the probability density function of a progress variable in a idealized premixed turbulent flow. Bigler [13] gave the hypothesis that in turbulent flow, the thermo-chemical quantities can be related locally a few scalars.

Further Janicka, Kolbe and Kollmann [50] and Pope [98] gave a more suitable model for the probability density functions of scalars in turbulent reacting flows.

Also Kishore [51] studied the distributions functions in the statistical theory of MHD turbulence of an incompressible fluid. Pope [100] derived the transport equation for the joint probability density function of velocity and scalars in turbulent flow. Kishore and Singh [53] derived the transport equation for the bivariate joint distribution function of velocity and temperature in turbulent flow. Kishore and Singh [55] have been derived the transport equation for the joint distribution function of velocity, temperature and concentration in convective turbulent flow. Dixit and Upadhyay [31] considered the distribution functions in the statistical theory of MHD turbulence of an incompressible fluid in the presence of the coriolis force. Kollman and Janicka [66] derived the transport equation for the probability density function of a scalar in turbulent shear flow and considered a closure model based on gradient – flux model

But at this stage, one is met with the difficulty that the N-point distribution function depends upon the N+1-point distribution function and thus result is an unclosed system. This so-called "closer problem" is encountered in turbulence, kinetic theory and other non-linear system. Sarker and Kishore [107] discussed the distribution functions in the statistical theory of convective MHD turbulence of an incompressible fluid. Further Sarker and Kishore [114] discussed the distribution functions in the statistical theory of convective MHD turbulence of mixture of a miscible incompressible fluid.

Following the above theories, in chapter III, an attempt is made to define the distribution function for simultaneous velocity, magnetic, temperature and concentration fields in MHD turbulence in a rotating system in presence of dust particles. Finally, the transport equations for evolution of distribution functions have been derived and various properties of the distribution function have also been discussed. The resulting one-point equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed in the case of ordinary turbulence.

In chapter II-A, we have considered the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in a rotating system.

In chapter II-B, we have studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in presence of dust particles.

In chapter II-C, we have studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in a rotating system in presence of dust particles and we have derived the decay law of temperature energy fluctuations in homogeneous turbulence before the final period for the case multi-point and multi-time in a rotating system in presence of dust particles. In all cases two- and three-point correlations equations are made to determinate by neglecting the quadruple correlation in comparison with lower order correlation applicable at times before the final period.

In chapter-III, we have studied the statistical theory of certain distribution functions in MHD turbulent flow in a rotating system in presence of dust particles.

In chapter IV-A, we have studied the decay of temperature fluctuations in MHD turbulence before the final period in a rotating system.

In chapter IV-B, we have considered the decay of temperature fluctuations in MHD dusty fluid turbulence before the final period in a rotating system and we have obtained the decay law of temperature fluctuations in dusty fluid MHD turbulence before the final period in a rotating system.

In chapter V-A, we have studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time in a rotating system.

In chapter V-B, we have considered the decay of MHD turbulence before the final period for the case of multi-point and multi-time in presence of dust particles.

In chapter V-C, we have discussed the decay of dusty fluid MHD turbulence before the final period in a rotating system for the case of multi-point and multi-time. We have derived the decay law of magnetic energy fluctuations of dusty fluid MHD turbulence in a rotating system before the final period for the case of multi-point and multi-time.

CHAPTER-II

PART-A

DECAY OF TEMPERATURE FLUCTUATIONS IN HOMOGENEOUS TURBULENCE BEFORE THE FINAL PERIOD FOR THE CASE OF MULTI-POINT AND MULTI-TIME IN A ROTATING SYSTEM

2.1 Introduction:

The essential characteristic of turbulent flows is that turbulent fluctuations are random in nature and therefore, by the application of statistical laws, it has been possible to give the idea of turbulent fluctuations. The turbulent flows, in the absence of external agencies always decay. Millionshtchikov [81], Batchelor and Townsend [4], Proudman and Ried [101], Tatsumi [124], Deissler [27,28], Ghosh [38,39] had given various analytical theories for the decay process of turbulence so far. Further Monin and Yaglom [85] gave the spectral approach for the decay process of turbulence.

In geophysical flows, the system is usually rotating with a constant angular velocity. Such large-scale flows are generally turbulent. When the motion is referred to axes, which rotate steadily with the bulk of the fluid, the coriolis and centrifugal force must be supposed to act on the fluid. The coriolis force due to rotation plays an important role in a rotating system of turbulent flow, while the centrifugal force with the potential is incorporated into the pressure. Kishore and Dixit [52], Kishore and Singh [54], Dixit and Upadhyay [30], Kishore and Golsefied [57] discussed the effect of coriolis force on acceleration covariance in ordinary and MHD turbulent flow. Funada, Tuitiya and Ohji [37] considered the effect of coriolis force on turbulent motion in presence of strong magnetic field. Kishore and Sarker [62] studied the rate of change of vorticity covariance in MHD turbulence in a rotating system. Sarker [109] discussed the vorticity covariance of dusty fluid turbulence in a rotating frame. Shimomura and Yoshizawa [119], Shimomura [120] and [121] also discussed the statistical analysis of turbulent viscosity, turbulent scalar flux and turbulent flows respectively in a rotating system by two-scale direct interaction approach. Sarker [111] studied the thermal decay process of MHD turbulence in a rotating system. Deissler [27,28] developed a theory for homogeneous turbulence, which was valid for times before the final period. Following Deissler's theory Loeffler and Deissler [72] studied the decay of temperature fluctuations in homogeneous turbulence before the final period. In their study, they presented the theory, which is valid during the period for which the quadruple correlation terms are neglected compared to the second and third–order correlation terms. Using Deissler's same theory Kumar and Patel [64] studied the first-order reactants in homogeneous turbulence before the final period for the case of multi-point and single-time. The problem [64], which is extended to the case of multi-point and multi-time concentration correlation by Kumar and Patel [65] and also the numerical result of [65] carried out by Patel [97]. Following Deisssler's approach Sarker and Islam [115] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Islam and Sarker [46] also studied the first–order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Sarker and Rahman [113] studied the decay of temperature fluctuations in MHD turbulence before the final period. Sarker and Islam [116] also studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time. In their approach, they considered two and three point correlations and neglecting fourth- and higher-order correlation terms compared to the second-and third-order correlation terms.

In this chapter the method of [27,28] is used and we have studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in a rotating system and have considered correlations between fluctuating quantities at two and three point. Two and three point correlation equation in a rotating system is obtained and the set of equations is made to determinate by neglecting the quadruple correlations in comparison to the second- and third-order correlations. The triple correlation equations should be applicable before the final period. In the final period the triple correlations are of course negligible. Using the expressions for the triple correlations so obtained, the two-point equation can be solved and the various quantities describing the turbulence at times before the final period can be obtained. For solving, the correlation equations are converted to spectral form by taking their Fourier transforms. Finally integrating the energy spectrum over all wave numbers, the energy decay law of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in a rotating system is obtained.

2.2 Correlation and spectral equations :

For an incompressible fluid with constant properties and for negligible frictional heating, the energy equation may be written as

$$\frac{\partial \tilde{T}}{\partial t}+\tilde{u}_i\frac{\partial \tilde{T}}{\partial x_i}=\frac{k}{\rho C_p}\frac{\partial^2 \tilde{T}}{\partial x_i \partial x_i} \qquad \text{------------ (2.2.1)}$$

where $\tilde{T}$ and $\tilde{u}_i$ are instantaneous values of temperature and velocity; k, thermal conductivity; ρ, fluid density; C_p heat capacity at constant pressure; x_i, space co-ordinate; t, time and the repeated subscripts are summed from 1 to 3.

Breaking these instantaneous values into time average and fluctuating components as $\tilde{T}=\overline{T}+T$ and $\tilde{u}_i=\overline{u}_i+u_i$ allows equation (2.2.1) to be written as

$$\frac{\partial}{\partial t}(\overline{T}+T)+(\overline{u}_i+u_i)\frac{\partial}{\partial x_i}(\overline{T}+T)=\gamma\frac{\partial^2(\overline{T}+T)}{\partial x_i\partial x_i}$$

$$\text{or, } \frac{\partial \overline{T}}{\partial t}+\frac{\partial T}{\partial t}+\overline{u}_i\frac{\partial \overline{T}}{\partial x_i}+\overline{u}_i\frac{\partial T}{\partial x_i}+u_i\frac{\partial \overline{T}}{\partial x_i}+u_i\frac{\partial T}{\partial x_i}=\gamma[\frac{\partial^2 \overline{T}}{\partial x_i}+\frac{\partial^2 T)}{\partial x_i\partial x_i}], \qquad \text{----------- (2.2.2)}$$

where $\gamma=\frac{k}{\rho C_p}$. From the condition of homogeneity it follows that $\frac{\partial \overline{T}}{\partial x_i}=0$, and in addition the usual assumption is made that $\overline{T}$ is independent of time and that $u = 0$. Thus equation (2.2.2) becomes

$$\frac{\partial T}{\partial t}+u_i\frac{\partial T}{\partial x_i}=\left(\frac{v}{p_r}\right)\frac{\partial^2 T}{\partial x_i.\partial x_i} \qquad \text{------------ (2.2.3)}$$

where $p_r=v/\gamma$, Prandtl number; ν, kinematic viscosity.

Equation (2.2.3) is assumed to hold at the arbitrary point p. For the point p' the corresponding equation can be written

$$\frac{\partial T'}{\partial t'}+u_i'\frac{\partial T'}{\partial x_i'}=\left(\frac{\nu}{p_r}\right)\frac{\partial^2 T'}{\partial x_i'\partial x_i'}. \qquad \text{---------- (2.2.4)}$$

Multiplying equation (2.2.3) by T', equation (2.2.4) by T and taking ensemble average, result in

$$\frac{\partial\langle TT'\rangle}{\partial t}+\frac{\partial\langle TT'u_i\rangle}{\partial x_i}=\left(\frac{\nu}{p_r}\right)\frac{\partial^2\langle TT'\rangle}{\partial x_i\partial x_i}, \qquad \text{---------- (2.2.5)}$$

$$\frac{\partial\langle TT'\rangle}{\partial t'}+\frac{\partial\langle TT'u_i'\rangle}{\partial x_i'}=\left(\frac{\nu}{p_r}\right)\frac{\partial^2\langle TT'\rangle}{\partial x_i'\partial x_i'} \qquad \text{---------- (2.2.6)}$$

with the continuity equation,

$$\frac{\partial u_i}{\partial x}=\frac{\partial u_i'}{\partial x_i'}=0. \qquad \text{---------- (2.2.7)}$$

Angular bracket $\langle ---\rangle$ is used to denote an ensemble average. Using the transformations

$$\frac{\partial}{\partial x_i}=-\frac{\partial}{\partial r_i},\frac{\partial}{\partial x_i'}=\frac{\partial}{\partial r_i},\left(\frac{\partial}{\partial t}\right)_{t'}=\left(\frac{\partial}{\partial t}\right)_{\Delta t}-\frac{\partial}{\partial \Delta t},\frac{\partial}{\partial t'}=\frac{\partial}{\partial \Delta t}$$

into equation (2.2.5) and (2.2.6), one obtains

$$\frac{\partial\langle TT'\rangle}{\partial t}-\frac{\partial\langle u_iTT'\rangle}{\partial r_i}(-r_i-\Delta t,t+\Delta t)+\frac{\partial\langle TT'u_i'\rangle}{\partial r_i}(\hat{r},\Delta t,t)=2\left(\frac{\nu}{p_r}\right)\frac{\partial^2\langle TT'\rangle}{\partial r_i\partial r_i}, \qquad \text{---------- (2.2.8)}$$

$$\frac{\partial\langle TT'\rangle}{\partial \Delta t}+\frac{\partial\langle u_iTT'\rangle}{\partial r_i}(-\hat{r},-\Delta t,t+\Delta t)=\left(\frac{\nu}{p_r}\right)\frac{\partial^2\langle TT'\rangle}{\partial r_i\partial r_i}. \qquad \text{---------- (2.2.9)}$$

It is convenient to write this equation in spectral form by use of the following three-dimensional Fourier transforms.

$$\langle TT'(\hat{r},\Delta t,t)\rangle=\int_{-\infty}^{\infty}\langle\tau\tau'(\hat{K},\Delta t,t)\rangle\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K}, \qquad \text{---------- (2.2.10)}$$

$$\langle u_i TT'(\hat{r},\Delta t,t)\rangle = \int_{-\infty}^{\infty}\langle \phi_i\ \tau\tau'(\hat{K},\Delta t,t)\rangle \exp[\hat{i}(\hat{K}.\hat{r})]d\hat{K} \qquad \text{---------- (2.2.11)}$$

and $\langle u_i' TT'(\hat{r},\Delta t,t)\rangle = \langle u_i TT'(-\hat{r},-\Delta t,t+\Delta t)\rangle$

$$= \int_{-\infty}^{\infty}\langle \phi_i \tau\tau'(-\hat{K},-\Delta t,t+\Delta t)\rangle \exp[\hat{i}(\hat{K}.\hat{r})]d\hat{K} \qquad \text{---------- (2.2.12)}$$

(Interchange are made between the points p and p')

where $\hat{K}$ is known as a wave number vector and magnitude of $\hat{K}$ has the dimension 1/length and can be considered to be the reciprocal of an eddy size.

Substitution of equations (2.2.10)-(2.2.12) into equations (2.2.8) and (2.2.9) we get the following spectral equations.

$$\frac{\partial\langle\tau\tau\rangle}{\partial t} + 2\left(\frac{\nu}{p_r}\right)k^2\langle\tau\tau'\rangle = iK_i\left[\langle\phi_i\tau\tau'(\hat{K},\Delta t,t)\rangle - \langle\phi_i\tau\tau'(-\hat{K},-\Delta t,t+\Delta t)\rangle\right], \qquad \text{-------- (2.2.13)}$$

$$\frac{\partial\langle\tau\tau'\rangle}{\partial\Delta t} + 2\left(\frac{\nu}{p_r}\right)k^2 = -iK_i\langle\phi_i\tau\tau'\rangle(-\hat{K},-\Delta t,t+\Delta t). \qquad \text{-------- (2.2.14)}$$

In equations (2.2.13)-(2.2.14) the quantity $\tau\tau'(\hat{K})$ may be interpreted as a temperature fluctuation "energy" contribution of thermal eddies of size 1/k. The time derivative of this "energy' as a function of the convective transfer to the wave numbers and the "dissipation" due to the action of the thermal conductivity. The term on the right hand side of equation (2.2.13) is also called transfer term while the 2nd term on the left hand side is the "dissipation" term.

2.3 Three-Point, Three-Time Correlation and Spectral Equations :

In order to obtain the three-point, three-time correlation and spectral equations, we write the Navier-Stockes equation for turbulent flow of incompressible fluid in a rotating system at the point P, energy equations at the points p' and p'' separated by the vectors $\hat{r}$ and $\hat{r}'$

$$\frac{\partial u_j}{\partial t}+\frac{\partial}{\partial x_i}\left(u_j u_i\right)=-\frac{1}{\rho}\frac{\partial p}{\partial x_j}+\nu\frac{\partial^2 u_j}{\partial x_i \partial x_i}-2\in_{mij}\Omega_m u_j, \quad \text{----------- (2.3.1)}$$

$$\frac{\partial T'}{\partial t}+u_i'\frac{\partial T'}{\partial x_i'}=\left(\frac{\nu}{P_r}\right)\frac{\partial^2 T'}{\partial x_i' \partial x_i'} \quad \text{----------- (2.3.2)}$$

and $$\frac{\partial T''}{\partial t}+u_i''\frac{\partial T''}{\partial x_i''}=\left(\frac{\nu}{P_r}\right)\frac{\partial^2 T''}{\partial x_i'' \partial x_i''}. \quad \text{---------- (2.3.3)}$$

Multiplying equations (2.3.1) – (2.3.3) by $T'T''$, u_jT'' and u_jT' respectively and then taking ensemble average, we obtained

$$\frac{\partial\langle u_jT'T''\rangle}{\partial t}+\frac{\partial\langle u_jT'T''u_i\rangle}{\partial x_i}=-\frac{1}{\rho}\frac{\partial\langle PT'T''\rangle}{\partial x_j}+\nu\frac{\partial^2\langle u_jT'T''\rangle}{\partial x_i\partial x_i}-2\in_{mij}\Omega_m\langle u_jT'T''\rangle, \text{ - (2.3.4)}$$

$$\frac{\partial\langle T'u_jT''\rangle}{\partial t'}+\frac{\partial\langle u_i'T'u_jT''\rangle}{\partial x_i'}=\left(\frac{\nu}{P_r}\right)\frac{\partial^2\langle T'u_jT''\rangle}{\partial x_i'\partial x_i'} \quad \text{------- (2.3.5)}$$

and $$\frac{\partial\langle T''u_jT'\rangle}{\partial t''}+\frac{\partial\langle u_i''T''u_jT'\rangle}{\partial x_i''}=\left(\frac{\nu}{P_r}\right)\frac{\partial^2\langle T''u_jT'\rangle}{\partial x_i''\partial x_i''}. \quad \text{------- (2.3.6)}$$

Using the transformations

$$\frac{\partial}{\partial x_i}=-\left(\frac{\partial}{\partial r_i}+\frac{\partial}{\partial r_i'}\right),\frac{\partial}{\partial x_i'}=\frac{\partial}{\partial r_i},\frac{\partial}{\partial x_i''}=\frac{\partial}{\partial r_i'},$$

$$\left(\frac{\partial}{\partial t}\right)_{t',t''}=\left(\frac{\partial}{\partial t}\right)_{\Delta t,\Delta t'}-\frac{\partial}{\partial \Delta t}-\frac{\partial}{\partial \Delta t'},\frac{\partial}{\partial \Delta t'}=\frac{\partial}{\partial \Delta t},\frac{\partial}{\partial \Delta t''}=\frac{\partial}{\partial \Delta t'},$$

into equations (2.3.4)–(2.3.6), we have

$$\frac{\partial\langle u_jT'T''\rangle}{\partial t}-\left(\frac{\partial}{\partial r_i}+\frac{\partial}{\partial r_i'}\right)\langle u_jT'T''u_i\rangle+\frac{\partial\langle u_i'T'u_jT''\rangle}{\partial r_i}+\frac{\partial\langle u_i'T''u_jT'\rangle}{\partial r_i'}$$

$$=\frac{1}{\rho}\left(\frac{\partial}{\partial r_j}+\frac{\partial}{\partial r_j'}\right)\langle PT'T''\rangle+\nu\left(\frac{\partial}{\partial r_i}+\frac{\partial}{\partial r_i'}\right)^2\langle u_jT'T''\rangle+\left(\frac{\nu}{P_r}\right)\left(\frac{\partial^2}{\partial r_i\partial r_i}+\frac{\partial^2}{\partial r_i'\partial r_i'}\right)\langle u_jT'T''\rangle$$

$$-2\in_{mij}\Omega_m\left\langle u_j T'T''\right\rangle, \qquad \text{---------- (2.3.7)}$$

$$\frac{\partial\left\langle u_j T'T''\right\rangle}{\partial \Delta t}+\frac{\partial\left\langle u_i' T' u_j T''\right\rangle}{\partial r_i}=\left(\frac{\nu}{P_r}\right)\frac{\partial^2\left\langle T' u_j T''\right\rangle}{\partial r_i \partial r_i}, \qquad \text{---------- (2.3.8)}$$

$$\frac{\partial\left\langle T' u_j T''\right\rangle}{\partial \Delta t'}+\frac{\partial\left\langle u_i' T'' u_j T'\right\rangle}{\partial r_i'}=\left(\frac{\nu}{P_r}\right)\frac{\partial^2\left\langle u_j T'T''\right\rangle}{\partial r_i' \partial r_i'}. \qquad \text{---------- (2.3.9)}$$

In order to convert equations (2.3.7)-(2.3.9) to spectral form, we can define the following six-dimensional Fourier transforms :

$$\left\langle u_j T'T''\right\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \beta_j \theta'\theta''\right\rangle \exp[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)]d\hat{K}dK', \qquad \text{--------- (2.3.10)}$$

$$\left\langle u_i u_j T'T''\right\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \beta_i\beta_j \theta'\theta''\right\rangle \exp[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)]d\hat{K}d\hat{K}', \qquad \text{--------- (2.3.11)}$$

$$\left\langle pT'T''\right\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\left\langle \alpha \theta'\theta''\right\rangle \exp[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)]d\hat{K}d\hat{K}'. \qquad \text{--------- (2.3.12)}$$

Interchanging the points p' and p'' shows that

$$\left\langle u_j u_i'' T'T''\right\rangle=\left\langle u_j u_i' T'T''\right\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \beta_j\beta_i' \theta'\theta''\right\rangle \exp[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)]d\hat{K}d\hat{K}'. \qquad \text{--------- (2.3.13)}$$

In these equations $\hat{K}$ and $\hat{K}'$ are known as wave number vectors and $d\hat{K}=dK_1dK_2dK_3$. The quantities $\beta_j\theta'\theta''$ etc are spectral tensors in wave number space corresponding to the correlation tensors in physical space.

Substituting the equations (2.3.10)-(2.3.13) in equations (2.3.7)-(2.3.9), we have

$$\frac{\partial\left\langle \beta_j\theta'\theta''\right\rangle}{\partial t}\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)+\frac{\nu}{p_r}[(1+p_r)k^2+2p_r k_i k_i'+(1+p_r)k'^2+\frac{2p_r}{\nu}\in_{mij}\Omega_m]\left\langle \beta_j\theta'\theta''\right\rangle$$

$$\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)=\frac{1}{\rho}i\left(k_j+k_j'\right)\left\langle \alpha\theta'\theta''\right\rangle+i(k_i+k_i')\left[\left\langle \beta_i\beta_j\theta'\theta''\right\rangle-\left\langle \beta_i'\beta_j\theta'\theta''\right\rangle\right]\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right),$$

$$\text{--------- (2.3.14)}$$

$$\frac{\partial\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)}{\partial\Delta t}+\left(\frac{\nu}{p_r}\right)k^2\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=-ik_i\langle\beta_i'\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t),$$

--------- (2.3.15)

$$\frac{\partial\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)}{\partial\Delta t'}+\left(\frac{\nu}{p_r}\right)k'^2\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=-ik_i'\langle\beta_i'\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t).$$

--------- (2.3.16)

If the derivative with respect to x_j is taken of the momentum equation (2.3.1) for the point p, the equation multiplied by $T'T''$ and taken the ensemble average, the resulting equation is

$$\frac{\partial^2\langle u_i u_j T'T''\rangle}{\partial x_j\partial x_i}=-\frac{1}{\rho}\frac{\partial^2\langle pT'T''\rangle}{\partial x_j\partial x_j}.$$ ----------- (2.3.17)

Writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$[\frac{\partial^2}{\partial r_i\partial r_i}+\frac{\partial^2}{\partial r_i\partial r_i'}+\frac{\partial^2}{\partial r_i'\partial r_j}+\frac{\partial^2}{\partial r_i'\partial r_j'}]\langle u_i u_j T'T''\rangle=-\frac{1}{\rho}[\frac{\partial^2}{\partial r_j\partial r_j}+2\frac{\partial^2}{\partial r_j\partial r_j'}+\frac{\partial^2}{\partial r_j'}]\langle pT'T''\rangle.$$ -(2.3.18)

Taking the Fourier transforms of equation (2.3.18)

$$\langle\alpha\theta'\theta''\rangle=\frac{-\rho(k_ik_j+k_ik_j'+k_i'k_j+k_i'k_j')\langle\beta_i\beta_j\theta'\theta''\rangle}{k_jk_j+2k_jk_j'+k_j'k_j'}.$$ -------- (2.3.19)

Equation (2.3.19) can be used to eliminate $\langle\alpha\theta'\theta''\rangle$ from equation (2.3.14).

2.4 Solution for times before the final period :

To obtain the equation for times before the final period of decay, the three point correlations are considered and the quadruple correlation terms are neglected in comparison with the 3rd order correlation terms. If this assumption is made the equation (2.3.19) shows that the term $\langle\alpha\theta'\theta''\rangle$ associated with the pressure fluctuations should also be neglected. Thus neglecting all the terms on the right hand side of equations (2.3.14)-(2.3.16), we have

$$\frac{\partial\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)}{\partial t}+\frac{\nu}{p_r}\left[(1+p_r)k^2+2p_rk_ik_i'+\frac{2p_r}{\nu}\in_{mij}\Omega_m\right]\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=0,$$

--------- (2.4.1)

$$\frac{\partial\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)}{\partial\Delta t}+\left(\frac{\nu}{p_r}\right)k^2\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=0, \quad \text{--------- (2.4.2)}$$

$$\frac{\partial\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)}{\partial\Delta t'}+\left(\frac{\nu}{p_r}\right)k'^2\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=0. \quad \text{--------- (2.4.3)}$$

Inner multiplication of equations (2.4.1), (2.4.2) and (2.4.3) by k_j and integrating between t_o and t we obtain

$$k_j\langle\beta_j\theta'\theta''\rangle=f_j\exp\left\{-\frac{\nu}{p_r}\left[(1+p_r)(k^2+k'^2)+2p_rkk'\cos\theta+\frac{2p_r}{\nu}\in_{mij}\Omega_m\right](t-t_0)\right\}, \text{ - (2.4.4)}$$

$$k_j\langle\beta_j\theta'\theta''\rangle=g_j\exp\left(-\frac{\nu}{p_r}k^2\Delta t\right) \quad \text{------------ (2.4.5)}$$

and $k_j\langle\beta_j\theta'\theta''\rangle=q_j\exp\left(-\frac{\nu}{p_r}k'^2\Delta t'\right)$ ------------ (2.4.6)

For these relations to be consistent, we have

$$k_j\langle\beta_j\theta'\theta''\rangle=k_j\langle\beta_j\theta'\theta''\rangle_0\exp\left[-\frac{\nu}{p_r}\{(1+p_r)(k^2+k'^2)(t-t_0)+k^2\Delta t+k'^2\Delta t'+2p_rkk'\cos\theta(t-t_0)\right.$$

$$\left.+\frac{2p_r}{\nu}\in_{mij}\Omega_m(t-t_0)\}\right] \quad \text{------------- (2.4.7)}$$

where θ is the angle between k and k' and $\langle\beta_j\theta'\theta''\rangle_0$ is the value of $\langle\beta_j\theta'\theta''\rangle$ at $t=t_o$, $\Delta t=\Delta t'=0$. Letting $\hat{r}'=0,\Delta t'=0$ in the equation (2.3.10) and comparing the result with the equation (2.2.11) shows that

$$\langle k_j\phi_j\tau\tau'(\hat{K},\Delta t,t)\rangle=\int_{-\infty}^{\infty}\langle k_j\beta_j\theta'\theta''(\hat{K},\hat{K}',\Delta t,0,t)\rangle d\hat{K}. \quad \text{---------- (2.4.8)}$$

Substituting the equation (2.4.7) and (2.4.8) into the equation (2.2.13) we obtain

$$\frac{\partial\langle\tau\tau'\rangle(\hat{K},\Delta t,t)}{\partial t}+2\frac{\nu}{p_r}k^2\langle\tau\tau'\rangle(\hat{K},\Delta t,t)=\int_{-\infty}^{\infty}ik_j\left[\langle\beta_j\theta'\theta''\rangle(\hat{K},K',\Delta t,0,t)-\langle\beta_j\theta'\theta''\rangle(-\hat{K},-\hat{K}',\Delta t,0,t)\right]_0$$

$$\exp\left[-\frac{\nu}{p_r}\{(1+p_r)(k^2+K'^2)(t-t_0)+k^2\Delta t+k'^2\Delta t'+2p_r kk'(t-t_0)\cos\theta\right.$$

$$\left.+\frac{2p_r}{\nu}\in_{mij}\Omega_m(t-t_0)\ \}\right]d\hat{K}'. \qquad \text{------------- (2.4.9)}$$

Now, $d\hat{K}'$ $(\equiv dK_1'dK_2'dK_3')$ can be expressed in terms of k' and θ as $-2\pi k'^2 d(\cos\theta)d\hat{k}'$ (cf. Deissler [28]).

i.e., $d\hat{K}'=-2\pi k'^2 d(\cos\theta)d\hat{k}'$ --------- (2.4.9a)

Substitution equation (2.4.9a) in equation (2.4.9) yields

$$\frac{\partial\langle\tau\tau'\rangle(\hat{K},\Delta t,t)}{\partial t}+2\frac{\nu}{p_r}k^2\langle\tau\tau'\rangle(\hat{K},\Delta t,t)$$

$$=2\int_{\infty}^{\infty}2\pi ik_j\left[\langle\beta_j\theta'\theta''\rangle(k,k')-\langle\beta_j\theta'\theta''\rangle(-\hat{K},\hat{K}')\right]_0 k'^2\left[\int_{-1}^{1}\exp\left\{-\frac{\nu}{p_r}\left[(1+p_r)(k^2+k'^2)(t-t_0)\right.\right.\right.$$

$$\left.\left.\left.+k^2\Delta t+k'^2\Delta t'+2p_r kk'(t-t_0)\cos\theta+\frac{2p_r}{\nu}\in_{mij}\Omega_m(t-t_0)\ \right]\right\}d(\cos\theta)\ \right]d\hat{k}'. \text{ -- (2.4.10)}$$

In order to find the solution completely and following Loeffler and Deissler [72] we assume that

$$ik_j\left[\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}')-\langle\beta_j\theta'\theta''\rangle(-\hat{K},-\hat{K}')\right]_0=-\frac{\delta_0}{(2\pi)^2}\left(k^2k'^4-k^4k'^2\right) \qquad \text{-------- (2.4.11)}$$

where δ_0 is a constant depending on the initial condition. The negative sign is placed in front of δ_0 in order to make the transfer of energy from small to large wave numbers for positive value of δ_0. The quantity $\left[\langle\beta_j\theta'\theta''\rangle(\hat{K},k')-\langle\beta_i'\theta'\theta''\rangle(-\hat{K},-\hat{K}')\right]_0$ depends on the initial conditions of the turbulence.

Substituting equation (2.4.11) into equation (2.4.10), we get

$$\frac{\partial}{\partial t}\langle\tau\tau'\rangle(\hat{K},\Delta t,t)2\pi+2\frac{\nu}{p_r}2\pi k^2\langle\tau\tau'\rangle(\hat{K},\Delta t,t)=-2\delta_o\int_0^{\infty}\left(k^2k'^4-k^4k'^2\right)k'^2$$

$$\times\left[\int_{-1}^{1}\exp\left\{-\frac{\nu}{p_r}\left[(1+p_r)(k^2+k'^2)(t-t_o)+k^2\Delta t+k'^2\Delta t'+2p_r kk'(t-t_o)\cos\theta\right.\right.\right.$$

$$\left.\left.\left.+\frac{2p_r}{\nu}\in_{mij}\Omega_m(t-t_o)\right]\right\}d(\cos\theta)\right]d\hat{k}'. \qquad \text{----------- (2.4.12)}$$

Multiplying both side of equation (2.4.12) by k^2, we get

$$\frac{\partial E}{\partial t}+2\frac{\nu}{p_r}k^2E=w, \qquad \text{---------- (2.4.13)}$$

where $E=2\pi k^2\langle\tau\tau'\rangle$, the energy spectrum function and w is given by

$$w=-2\delta_o\int_0^\infty\left(k^2k'^4-k^4k'^2\right)k^2k'^2\left[\int_{-1}^{1}\exp\left\{-\frac{\nu}{p_r}\left[(1+p_r)(k^2+k'^2)(t-t_o)+k^2\Delta t\right.\right.\right.$$

$$\left.\left.\left.+2p_r kk'(t-t_o)\cos\theta+\frac{2p_r}{\nu}\in_{mij}\Omega_m(t-t_o)\right]\right\}d(\cos\theta)\right]d\hat{k}'. \qquad \text{---------- (2.4.14)}$$

Integrating equation (2.4.12) with respect to θ, we have

$$w=-\frac{\delta_o}{\nu(t-t_o)}\int_0^\infty\left(k^3k'^5-k^5k'^3\right)\left[\exp\{-\frac{\nu}{p_r}[(1+p_r)(k^2+k'^2)(t-t_o)+k^2\Delta t\right.$$

$$\left.-2p_r kk'(t-t_o)+\frac{2p_r}{\nu}\in_{mij}\Omega_m(t-t_o)]]\}\right]dk'+\frac{\delta_o}{\nu(t-t_o)}\int_0^\infty\left(k^3k'^5-k^5k'^3\right)$$

$$\left[\exp\left\{-\frac{\nu}{p_r}\left[(1+p_r)(k^2+k'^2)(t-t_o)+k^2\Delta t+2p_r kk'(t-t_o)+\frac{2p_r}{\nu}\in_{mij}\Omega_m(t-t_o)\right]\right\}\right]dk'.$$

------- (2.4.15)

Again integrating equation (2.4.15) with respect to k', we have

$$w=-\frac{\delta_o\sqrt{\pi}\,p_r^{5/2}}{4\nu^{3/2}(t-t_o)^{3/2}(1+p_r)^{5/2}}\exp\{-\frac{2p_r}{\nu}\in_{mij}\Omega_m(t-t_o)\}\times\exp[\frac{-k^2\nu(1+2p_r)}{p_r(1+p_r)}$$

$$(t-t_o+\frac{1+p_r}{1+2p_r}\Delta t]\times[\frac{15p_rk^4}{4\nu^2(t-t_o)^2(1+p_r)}+\{\frac{5p_r^2}{(1+p_r)^2}-\frac{3}{2}\}\frac{k^6}{\nu(t-t_o)}$$

$$\times\exp[\frac{-k^2\nu(1+2p_r)}{p_r(1+p_r)}(t-t_o+\frac{p_r}{1+2p_r}\Delta t)]+\{\frac{p_r^3}{(1+p_r)^3}-\frac{p_r}{1+p_r}\}k^8]$$

$$-\frac{\delta_o\sqrt{\pi}\,p_r^{5/2}}{4\nu^{3/2}(t-t_o+\Delta t)^{3/2}(1+p_r)^{5/2}}\exp\{-\frac{2p_r}{\nu}\in_{mij}\Omega_m(t-t_o)\}\times[\frac{15p_rk^4}{4\nu^2(t-t_o+\Delta t)^2(1+p_r)}$$

$$+\{\frac{5p_r^2}{(1+p_r)^2}-\frac{3}{2}\}\frac{k^6}{\nu(t-t_o+\Delta t)}+\{\frac{p_r^3}{(1+p_r)^3}-\frac{p_r}{1+p_r}\}k^8]. \qquad \text{-------- (2.4.16)}$$

The series of equation (2.4.14) contains only even power of k and start with k^4. The quantity w is the contribution to the energy transfer arising from consideration of the three-point equation.

If we integrate equation (2.4.16) for $\Delta t=0$ over all wave numbers, we find that

$$\int_0^\infty w\,dk=0 \qquad \text{--------- (2.4.17)}$$

which is indicating that the expression for w satisfies the condition of continuity and homogeneity. Physically it was to be expected as w is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero.

The linear equation (2.4.13) can be solved to give

$$E=\exp[-\frac{2\nu}{p_r}k^2(t-t_o+\frac{\Delta t}{2})]\int w\exp[2\frac{\nu}{p_r}k^2(t-t_o+\frac{\Delta t}{2})]dt+J(k)\exp[-\frac{2\nu}{p_r}k^2(t-t_o+\frac{\Delta t}{2})] \text{-(2.4.18)}$$

where $J(k)=\frac{N_ok^2}{\pi}$ is a constant of integration and can be obtained as by Corrsin [24].

Substituting the values of w from (2.4.16) and J(k) into the equation (2.4.18) and integrating with respect to t_0 we get

$$E=\frac{N_ok^2}{\pi}\exp[-2\frac{\nu}{p_r}k^2(t-t_o+\frac{\Delta t}{2})]+\frac{\delta_o\sqrt{\pi}\,p_r^{5/2}}{4\nu^{3/2}(1+p_r)^{7/2}}\exp[-2\in_{mij}\Omega_m(t-t_o)]$$

$$\times\exp[\frac{-k^2\nu(1+2p_r)}{p_r(1+p_r)}(t-t_o+\frac{1+p_r}{1+2p_r}\Delta t)]\times[\frac{3p_rk^4}{2\nu^2(t-t_o)^{5/2}}+\frac{p_r(7p_r-6)k^6}{3\nu(1+p_r)(t-t_o)^{3/2}}$$

$$-\frac{4(3p_r^2-2p_r+3)k^8}{3(1+p_r)^2(t-t_o)^{1/2}}+\frac{8\sqrt{\nu}(3p_r^2-2p_r+3)k^9}{3(1+p_r)^{5/2}p_r^{1/2}}F(\eta)+\frac{\delta_o\sqrt{\pi}\,p_r^{5/2}}{4\nu^{3/2}(1+p_r)^{7/2}}$$

$$\times \exp[-2 \in_{mij} \Omega_m (t-t_o)] \times \exp[\frac{-\nu k^2(1+2p_r)}{p_r(1+p_r)}(t-t_o+\frac{p_r}{1+2p_r}\Delta t) \times [\frac{3p_r k^4}{2\nu^2(t-t_o+\Delta t)^{5/2}}]$$

$$+\frac{p_r(7p_r-6)k^6}{3\nu(1+p_r)(t-t_o+\Delta t)^{3/2}}-\frac{4}{3}\frac{(3p^2{}_r-2p_r+3)k^8}{(1+p_r)^2(t-t_o+\Delta t)^{1/2}}+\frac{8\sqrt{\nu}(3p_r^2-2p_r+3)k^9}{3(1+p_r)^{5/2}p_r^{1/2}}F(\eta), \text{-(2.4.19)}$$

where $F(\eta)=\bar{e}^{\eta^2}\int_0^{\eta} e^{x^2}dx$,

$$\eta = k\sqrt{\frac{\nu(t-t_o)}{p_r(1+p_r)}} \quad \text{or} \quad \eta = k\sqrt{\frac{\nu(t-t_o+\Delta t)}{p_r(1+p_r)}}.$$

By setting $\hat{r}=0$ in equation (2.2.10) and use is made of the definition of E, the result is

$$\frac{\langle TT' \rangle}{2}=\frac{\langle T^2 \rangle}{2}=\int_0^{\infty} Edk. \qquad \text{--------- (2.4.20)}$$

Substituting equation (2.4.19) into equation (2.4.20) and integrating with respect to k, gives

$$\frac{\langle T^2 \rangle}{2}=\frac{N_o p_r^{3/2}(T_0+\frac{\Delta T_0}{2})^{-3/2}}{8\sqrt{2\pi}\nu^{3/2}}+\frac{\pi\delta_o p_r^6}{4\nu^6(1+p_r)(1+2p_r)^{5/2}}\exp[-2\in_{mij}\Omega_m]$$

$$[\frac{9}{16T_0^{5/2}(T_0+\frac{1+p_r}{1+2p_r}\Delta T_0)^{5/2}}+\frac{9}{16(T_0+\Delta T_0)^{5/2}(T_0+\frac{p_r}{1+2p_r}\Delta T_0)^{5/2}}+\frac{5p_r(7p_r-6)}{16(1+2p_r)T_0^{3/2}(T_0+\frac{1+p_r}{1+2p_r}\Delta T_0)^{7/2}}$$

$$+\frac{5p_r(7p_r-6)}{16(1+2p_r)(T_0+\Delta T_0)^{3/2}(T_0+\frac{p_r}{1+2p_r}\Delta T_0)^{7/2}}+\frac{35p_r(3p_r^2-2p_r+3)}{8(1+2p_r)T_0^{1/2}(T_0+\frac{1+p_r}{1+2p_r}\Delta T_0)^{9/2}}$$

$$+\frac{35p_r(3p_r^2-2p_r+3)}{8(1+2p_r)(T_0+\Delta T_0)^{1/2}(T_0+\frac{p_r}{1+2p_r}\Delta T_0)^{9/2}}+\frac{8p_r(3p_r^2-2p_r+3)(1+2p_r)^{5/2}}{3.2^{23/2}(1+p_r)^{11/2}}$$

$$\sum_{n=0}^{\infty}\frac{1.3.5..........(2n+9)}{n!(2n+1)2^{2n}(1+p_r)^n}\times\{\frac{T_0^{(2n+1)/2}}{(T_0+\Delta T_0/2)^{(2n+1)/2}}+\frac{(T_0+\Delta T_0)^{(2n+1)/2}}{(T_0+\Delta T_0/2)^{(2n+1)/2}}\}], \text{ ---- (2.4.21)}$$

where $T_0=t-t_0$.

Equation (2.4.21) is the decay law of temperature energy fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in a rotating system.

2.5 Concluding Remarks :

In equation (2.4.21) we obtained the decay law of temperature fluctuation in homogeneous turbulence before the final period in a rotating system and the quadruple correlation terms are neglected in comparison with the third –order terms for the case of multi-point and multi-time. If the system is non rotating then $\Omega_m = 0$ the equation (2.4.21) becomes

$$\frac{\langle T^2\rangle}{2}=\frac{N_o p_r^{3/2}(T_0+\frac{\Delta T_0}{2})^{-3/2}}{8\nu^{3/2}\sqrt{2\pi}}+\frac{\pi\delta_o p_r^6}{4\nu^6(1+p_r)(1+2p_r)^{5/2}}\times[\frac{9}{16T_0^{5/2}(T_0+\frac{1+p_r}{1+2p_r}\Delta T_0)^{5/2}}$$

$$+\frac{9}{16(T_0+\Delta T_0)^{5/2}(T_0+\frac{p_r}{1+2p_r}\Delta T_0)^{5/2}}+\frac{5p_r(7p_r-6)}{16(1+2p_r)T_0^{3/2}(T_0+\frac{1+p_r}{1+2p_r}\Delta T_0)^{7/2}}$$

$$+\frac{5p_r(7p_r-6)}{16(1+2p_r)(T_0+\Delta T_0)^{3/2}(T_0+\frac{p_r}{1+2p_r}\Delta T_0)^{7/2}}+\frac{35p_r(3p_r^2-2p_r+3)}{8(1+2p_r)T_0^{1/2}(T_0+\frac{1+p_r}{1+2p_r}\Delta T_0)^{9/2}}$$

$$+\frac{35p_r(3p_r^2-2p_r+3)}{8(1+2p_r)(T_0+\Delta T_0)^{1/2}(T_0+\frac{p_r}{1+2p_r}\Delta T_0)^{9/2}}+\frac{8p_r(3p_r^2-2p_r+3)(1+2p_r)^{5/2}}{3.2^{23/2}(1+p_r)^{11/2}}$$

$$\sum_{n=0}^{\infty}\frac{1.3.5..........(2n+9)}{n!(2n+1)2^{2n}(1+p_r)^n}\times\{\frac{T_0^{(2n+1)/2}}{(T_0+\Delta T_0/2)^{(2n+1)/2}}+\frac{(T_0+\Delta T_0)^{(2n+1)/2}}{(T_0+\Delta T_0/2)^{(2n+1)/2}}\}], \quad \text{-------- (2.5.1)}$$

which is obtained earlier by Sarker and Islam [116].

If we put $\Delta T_0 = 0$ in equation (2.5.1), we can easily find out

$$\frac{\langle T^2 \rangle}{2} = \frac{N_o p_r^{3/2} T_0^{\ 3/2}}{8\sqrt{2\pi} v^{3/2}} + \frac{\pi \delta_o p_r^6 T_0^{\ 5}}{4v^6(1+p_r)(1+2p_r)^{5/2}} \times [\frac{9}{16} + \frac{5}{16} \cdot \frac{p_r(7p_r-6)}{1+2p_r} + \frac{35}{8} \frac{p_r(3p_r^2-2p_r+3)}{(1+2p_r)^2} + ---]$$

$$= AT_0^{-3/2} + BT_0^{-5} = A(t-t_0)^{-3/2} + B(t-t_0)^{-5}, \qquad \text{---------} \ (2.5.2)$$

where $A = \frac{N_o p_r^{3/2}}{8\sqrt{2\pi} v^{3/2}}$

and

$$B = \frac{\pi \delta_o p_r^6}{2v^6(1+p_r)(1+2p_r)^{5/2}} \times [\frac{9}{16} + \frac{5}{16} \cdot \frac{p_r(7p_r-6)}{1+2p_r} + \frac{35}{8} \cdot \frac{p_r(3p_r^2-2p_r+3)}{(1+2p_r)^2} + \ldots\ldots\ldots.].$$

which was obtained earlier by Loeffler and Deissler [72].

Here due to the effect of rotation in homogeneous turbulence, the temperature energy fluctuation decays more rapidly than the energy for non-rotating fluid for times before the final period.

If higher order correlation equations were considered in the analysis it appears that more terms of higher power of time would be added to the equation (2.4.21). For large times, the second term in the equation becomes negligible leaving the -3/2 power decay law for the final period.

CHAPTER-II

PART-B

DECAY OF TEMPERATURE FLUCTUATIONS IN HOMOGENEOUS TURBULENCE BEFORE THE FINAL PERIOD FOR THE CASE OF MULTI-POINT AND MULTI-TIME IN PRESENCE OF DUST PARTICLES

2.6 Introduction :

Knowledge of behavior of discrete particles in a turbulent flow is of great interest to many branches of technology, particularly if there is a substantial difference between the particles and the fluid. A dust particle in air, or in any other gas, has a much larger inertia than the equivalent volume of air and will not therefore participate readily in turbulent fluctuations. The relative motion of dust participate and the air will dissipate energy because of the drag between dust and air, and extract energy from turbulent intensity is reduced than the Reynolds stresses will be decreased and the force required to maintain a given flow rate will likewise be reduced.

Taylor [126] has been pointed out that the equation of motion of turbulence relates the pressure gradient and the acceleration of the fluid particles and the mean–square acceleration can be determined from the observation of the diffusion of marked fluid particles. The behavior of dust particles in a turbulent flow depends on the concentration of the particles and the size of the particles with respect to the scale of turbulent fluid. Saffman [106] derived and equation that describe the motion of a fluid containing small dust particle, which is applicable to laminar flows as well as turbulent flow. Using the Saffman's equations Michael and Miller [83] discussed the motion of dusty gas occupying the semi-infinite space above a rigid plane boundary. Sarker and Rahman [112] considered dust particles on their won works. Sinha [122] studied the effect of dust particles on the acceleration covariance of ordinary turbulence. Kishore and Sinha [59] also studied the rate of change of vorticity covariance in dusty fluid turbulence.

Deissler [27,28] developed a theory for homogeneous turbulence, which was valid for times before the final period. Following Deissler's theory Loeffler and Deissler [72] studied the decay of temperature fluctuations in homogeneous turbulence before the final period. In their study, they presented the theory which is valid during the period for which the quadruple correlation terms are neglected compared to the 2^{nd} and 3^{rd}–order correlation terms. Using Deissler's same theory Kumar and Patel [64] studied the first-order reactants in homogeneous turbulence before the final period for the case of multi-point and single-time. The problem [64] extended to the case of multi-point and multi-time concentration correlation by Kumar and Patel [65] and also the numerical result of [65] carried out by Patel [67]. Following Desiler's approach Sarker and Islam [115] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Islam and Sarker [46] also studied the first–order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Sarker and Rahman [113] studied the decay of temperature fluctuations in MHD turbulence before the final period. Sarker and Islam [116] also studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time.

They considered two and three point correlations and neglecting fourth- and higher-order correlation terms compared to the second- and third-order correlation terms. In this chapter the method of [27,28] is used and we have studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in presence of dust particles. The main purpose of this part of the chapter to derive an equation for the energy decay law of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in presence of dust particles. When the frequency f is zero, i.e. for homogeneous turbulence of a clean fluid the result reduces to the one obtained earlier by Sarker and Islam [116].

2.7 Correlation and spectral equations :

The equations of energy for an incompressible fluid with constant properties at the point P and P' are given by

$$\frac{\partial T}{\partial t}+u_i\frac{\partial T}{\partial x_i}=\left(\frac{\nu}{p_r}\right)\frac{\partial^2 T}{\partial x_i.\partial x_i} \qquad \text{---------}(2.7.1)$$

and

$$\frac{\partial T'}{\partial t'}+u_i'\frac{\partial T'}{\partial x_i'}=\left(\frac{\nu}{p_r}\right)\frac{\partial^2 T'}{\partial x_i'\partial x_i'}. \qquad \text{----------}(2.7.2)$$

The subscripts can take the values 1, 2, or 3.

Here u_i turbulent velocity component P_r, Prandtl number; ν, kinematic viscosity

Multiplying equation (2.7.1) by T', equation (2.7.2) by T and taking ensemble average, result in

$$\frac{\partial\langle TT'\rangle}{\partial t}+\frac{\partial\langle TT'u_i\rangle}{\partial x_i}=\left(\frac{\nu}{p_r}\right)\frac{\partial^2\langle TT'\rangle}{\partial x_i\partial x_i} \qquad \text{---------}(2.7.3)$$

and $$\frac{\partial\langle TT'\rangle}{\partial t'}+\frac{\partial\langle TT'u_i'\rangle}{\partial x_i'}=\left(\frac{\nu}{p_r}\right)\frac{\partial^2\langle TT'\rangle}{\partial x_i'\partial x_i'} \qquad \text{---------}(2.7.4)$$

with the continuity equation

$$\frac{\partial u_i}{\partial x}=\frac{\partial u_i'}{\partial x_i'}=0. \qquad \text{---------}(2.7.5)$$

Angular bracket $\langle ---\rangle$ is used to denote an ensemble average. Using the transformations

$$\frac{\partial}{\partial x_i}=-\frac{\partial}{\partial r_i},\frac{\partial}{\partial x_i'}=\frac{\partial}{\partial r_i},\left(\frac{\partial}{\partial t}\right)_{t'}=\left(\frac{\partial}{\partial t}\right)_{\Delta t}-\frac{\partial}{\partial \Delta t},\frac{\partial}{\partial t'}=\frac{\partial}{\partial \Delta t}$$

into equation (2.7.3) and (2.7.4), one obtains

$$\frac{\partial\langle TT'\rangle}{\partial t}-\frac{\partial\langle u_i'TT'\rangle}{\partial r_i}(-r_i-\Delta t,t+\Delta t)+\frac{\partial\langle TT'u_i'\rangle}{\partial r_i}(\hat{r},\Delta t,t)=2\left(\frac{\nu}{p_r}\right)\frac{\partial^2\langle TT'\rangle}{\partial r_i\partial r_i} \qquad \text{-------}(2.7.6)$$

and $$\frac{\partial\langle TT'\rangle}{\partial \Delta t}+\frac{\partial\langle u_iTT'\rangle}{\partial r_i}(-\hat{r},-\Delta t,t+\Delta t)=\left(\frac{\nu}{p_r}\right)\frac{\partial^2\langle TT'\rangle}{\partial r_i\partial r_i}. \qquad \text{-------}(2.7.7)$$

It is convenient to write this equation in spectral form by use of the following three-dimensional Fourier transforms.

$$\langle TT'(\hat{r},\Delta t,t)\rangle = \int_{-\infty}^{\infty}\langle \tau\tau'(\hat{K},\Delta t,t)\rangle \exp[\hat{i}(\hat{K}.\hat{r})]d\hat{K}\,, \qquad \text{------ (2.7.8)}$$

$$\langle u_i TT'(\hat{r},\Delta t,t)\rangle = \int_{-\infty}^{\infty}\langle \phi_i\ \tau\tau'(\hat{K},\Delta t,t)\rangle \exp[\hat{i}(\hat{K}.\hat{r})]d\hat{K} \qquad \text{------- (2.7.9)}$$

and $\langle u_i' TT'(\hat{r},\Delta t,t)\rangle = \langle u_i TT'(-\hat{r},-\Delta t,t+\Delta t)\rangle$

$$= \int_{-\infty}^{\infty}\langle \phi_i \tau\tau'(-\hat{K},-\Delta t,t+\Delta t)\rangle \exp[\hat{i}(\hat{K}.\hat{r})]d\hat{K} \qquad \text{------ (2.7.10)}$$

(Interchange are made between the points p and p')

where $\hat{K}$ is known as a wave number vector and magnitude of $\hat{K}$ has the dimension 1/length and can be considered to be the reciprocal of an eddy size.

Substituting the equations (2.7.8)-(2.7.10) into equations (2.7.6) and (2.7.7) leads to the spectral equations.

$$\frac{\partial\langle \tau\tau\rangle}{\partial t} + 2\left(\frac{\nu}{p_r}\right)k^2\langle \tau\tau'\rangle = iK_i\left[\langle \phi_i\tau\tau'(\hat{K},\Delta t,t)\rangle - \langle \phi_i\tau\tau'(-\hat{K},-\Delta t,t+\Delta t)\rangle\ \right], \qquad \text{------- (2.7.11)}$$

$$\frac{\partial\langle \tau\tau'\rangle}{\partial \Delta t} + 2\left(\frac{\nu}{p_r}\right)k^2 = -iK_i\langle \phi_i\tau\tau'\rangle(-\hat{K},-\Delta t,t+\Delta t). \qquad \text{------- (2.7.12)}$$

In equations (2.7.11)-(2.7.12) the quantity $\tau\tau'(\hat{K})$ may be interpreted as a temperature fluctuation "energy" contribution of thermal eddies of size 1/k. The time derivative of this "energy' as a function of the convective transfer to the wave numbers and the "dissipation" due to the action of the thermal conductivity. The term on the right hand side of equation (2.7.11) is also called transfer term while the second term on the left hand side is the "dissipation" term.

2.8 Three-Point, Three-Time Correlation and Spectral Equations :

In order to obtain the three-point, three-time correlation and spectral equations, we write the Navier-Stockes equation for turbulent flow of dusty incompressible fluid at the point P, energy equations at the points p' and p'' separated by the vectors $\hat{r}$ and $\hat{r}'$

$$\frac{\partial u_j}{\partial t}+\frac{\partial}{\partial x_i}\left(u_j u_i\right)=-\frac{1}{\rho}\frac{\partial p}{\partial x_j}+\nu\frac{\partial^2 u_j}{\partial x_i \partial x_i}+f\left(u_j-v_j\right), \qquad \text{------- (2.8.1)}$$

$$\frac{\partial T'}{\partial t}+u_i'\frac{\partial T'}{\partial x_i'}=(\frac{\nu}{P_r})\frac{\partial^2 T'}{\partial x_i'\partial x_i'} \qquad \text{------ (2.8.2)}$$

and $$\frac{\partial T''}{\partial t}+u_i''\frac{\partial T''}{\partial x_i''}=(\frac{\nu}{P_r})\frac{\partial^2 T''}{\partial x_i''\partial x_i''}. \qquad \text{------ (2.8.3)}$$

Multiplying equations (2.8.1) – (2.8.3) by $T'T''$, u_jT'' and u_jT' respectively and then taking ensemble average, we obtained

$$\frac{\partial\langle u_jT'T''\rangle}{\partial t}+\frac{\partial\langle u_jT'T''u_i\rangle}{\partial x_i}=-\frac{1}{\rho}\frac{\partial\langle PT'T''\rangle}{\partial x_j}+\nu\frac{\partial^2\langle u_jT'T''\rangle}{\partial x_i\partial x_i}+f(\langle u_jT'T''\rangle-\langle v_jT'T''\rangle), \qquad \text{-- (2.8.4)}$$

$$\frac{\partial\langle T'u_jT''\rangle}{\partial t'}+\frac{\partial\langle u_i'T'u_jT''\rangle}{\partial x_i'}=\left(\frac{\nu}{P_r}\right)\frac{\partial^2\langle T'u_jT''\rangle}{\partial x_i'\partial x_i'} \qquad \text{------ (2.8.5)}$$

and $$\frac{\partial\langle T''u_jT'\rangle}{\partial t''}+\frac{\partial\langle u_i''T''u_jT'\rangle}{\partial x_i''}=\left(\frac{\nu}{P_r}\right)\frac{\partial^2\langle T''u_jT'\rangle}{\partial x_i''\partial x_i''}. \qquad \text{------ (2.8.6)}$$

Using the transformations

$$\frac{\partial}{\partial x_i}=-\left(\frac{\partial}{\partial r_i}+\frac{\partial}{\partial r_i'}\right),\frac{\partial}{\partial x_i'}=\frac{\partial}{\partial r_i},\frac{\partial}{\partial x_i''}=\frac{\partial}{\partial r_i'}$$

$$\left(\frac{\partial}{\partial t}\right)_{t',t''}=\left(\frac{\partial}{\partial t}\right)_{\Delta t,\Delta t'}-\frac{\partial}{\partial \Delta t}-\frac{\partial}{\partial \Delta t'},\frac{\partial}{\partial \Delta t'}=\frac{\partial}{\partial \Delta t},\frac{\partial}{\partial \Delta t''}=\frac{\partial}{\partial \Delta t'}$$

into equations (2.8.4) – (2.8.6), we have

$$\frac{\partial\langle u_j T'T''\rangle}{\partial t}-\left(\frac{\partial}{\partial r_i}+\frac{\partial}{\partial r_i'}\right)\langle u_j T'T''u_i\rangle+\frac{\partial\langle u_i'T'u_jT''\rangle}{\partial r_i}+\frac{\partial\langle u_i''T''u_jT'\rangle}{\partial r_i'}$$

$$=\frac{1}{\rho}\left(\frac{\partial}{\partial r_j}+\frac{\partial}{\partial r_j'}\right)\langle PT'T''\rangle+\nu\left(\frac{\partial}{\partial r_i}+\frac{\partial}{\partial r_i'}\right)^2\langle u_jT'T''\rangle+\left(\frac{\nu}{P_r}\right)\left(\frac{\partial^2}{\partial r_i\partial r_i}+\frac{\partial^2}{\partial r_i'\partial r_i'}\right)\langle u_jT'T''\rangle$$

$$+\mathrm{f}\left(\langle \mathrm{u}_j T'T''\rangle-\langle \mathrm{v}_j T'T''\rangle\right), \qquad \text{-------- (2.8.7)}$$

$$\frac{\partial\langle u_jT'T''\rangle}{\partial\Delta t}+\frac{\partial\langle u_i'T'u_jT''\rangle}{\partial r_i}=\left(\frac{\nu}{P_r}\right)\frac{\partial^2\langle T'u_jT''\rangle}{\partial r_i\partial r_i}, \qquad \text{-------- (2.8.8)}$$

$$\frac{\partial\langle T'u_jT''\rangle}{\partial\Delta t'}+\frac{\partial\langle u_i''T''u_jT'\rangle}{\partial r_i'}=\left(\frac{\nu}{P_r}\right)\frac{\partial^2\langle u_jT'T''\rangle}{\partial r_i'\partial r_i'}. \qquad \text{-------- (2.8.9)}$$

The six-dimensional Fourier transforms for quantities in the equations (2.8.7)-(2.8.9) may be defined as

$$\langle u_jT'T''\rangle(\hat r,\hat r',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\beta_j\theta'\theta''\rangle\exp[\hat i(\hat K.\hat r+\hat K'.\hat r')]d\hat K dK', \qquad \text{------- (2.8.10)}$$

$$\langle u_iu_jT'T''\rangle(\hat r,\hat r',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\beta_i\beta_j\theta'\theta''\rangle\exp[\hat i(\hat K.\hat r+\hat K'.\hat r')]d\hat K d\hat K', \qquad \text{------- (2.8.11)}$$

$$\langle pT'T''\rangle(\hat r,\hat r',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\langle\alpha\theta'\theta''\rangle\exp[\hat i(\hat K.\hat r+\hat K'.\hat r')]d\hat K d\hat K'. \qquad \text{------ (2.8.12)}$$

Interchanging the points p' and p'' shows that

$$\langle u_ju_i''T'T''\rangle=\langle u_ju_i'T'T''\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\beta_j\beta_i'\theta'\theta''\rangle\exp[\hat i(\hat K.\hat r+\hat K'.\hat r')]d\hat K d\hat K', \qquad \text{------ (2.8.13)}$$

$$\langle v_jT'T''\rangle(\hat r,\hat r',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\gamma_j\theta'\theta''\rangle\exp[\hat i(\hat K.\hat r+\hat K'.\hat r')]d\hat K dK'. \qquad \text{------ (2.8.14)}$$

In these equations, $\hat K$ and $\hat K'$ are known as wave number vectors, and $d\hat k = dk_1dk_2dk_3$. The quantities $\beta_j\theta'\theta''$ etc, are spectral tensors in wave number space corresponding to the correlation tensors in physical space.

By use of these facts and equations (2.8.10)-(2.8.12), the equations (2.8.7)-(2.8.9) may be transformed as

$$\frac{\partial\langle\beta_j\theta'\theta''\rangle}{\partial t}(\hat{K},\hat{K}',\Delta t,\Delta t',t)+\frac{\nu}{p_r}[(1+p_r)k^2+2p_rk_ik_i'+(1+p_r)k'^2-\frac{p_rf}{\nu}]\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)$$

$$=\frac{1}{\rho}i\left(k_j+k_j'\right)\langle\alpha\theta'\theta''\rangle-f\langle\gamma_j\theta'\theta''\rangle+i(k_i+k_i')(\langle\beta_i\beta_j\theta'\theta''\rangle-\langle\beta_i'\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\,,$$

-------- (2.8.15)

$$\frac{\partial\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)}{\partial\Delta t}+\left(\frac{\nu}{p_r}\right)k^2\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=-ik_i\langle\beta_i'\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\,,$$

-------- (2.8.16)

$$\frac{\partial\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)}{\partial\Delta t'}+\left(\frac{\nu}{p_r}\right)k'^2\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=-ik_i'\langle\beta_i'\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\,.$$

-------- (2.8.17)

If the derivative with respect to x_j is taken of the momentum equation (2.8.1) for the point p, the equation multiplied by $T'T''$ and taken the ensemble average, the resulting equation is

$$\frac{\partial^2\langle u_iu_jT'T''\rangle}{\partial x_j\partial x_i}=-\frac{1}{\rho}\frac{\partial^2\langle pT'T''\rangle}{\partial x_j\partial x_j}\,.$$ -------- (2.8.18)

Writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$[\frac{\partial^2}{\partial r_i\partial r_i}+\frac{\partial^2}{\partial r_i\partial r_i'}+\frac{\partial^2}{\partial r_i'\partial r_j}+\frac{\partial^2}{\partial r_i'\partial r_j'}]\langle u_iu_jT'T''\rangle=-\frac{1}{\rho}[\frac{\partial^2}{\partial r_j\partial r_j}+2\frac{\partial^2}{\partial r_j\partial r_j'}+\frac{\partial^2}{\partial r_j'}]\langle pT'T''\rangle\,.$$ - (2.8.19)

Taking the Fourier transforms of equation (2.8.19)

$$\langle\alpha\theta'\theta''\rangle=\frac{-\rho(k_ik_j+k_ik_j'+k_i'k_j+k_i'k_j')\langle\beta_i\beta_j\theta'\theta''\rangle}{k_jk_j+2k_jk_j'+k_j'k_j'}\,.$$ -------- (2.8.20)

Equation (2.8.20) can be used to eliminate $\langle\alpha\theta'\theta''\rangle$ from equation (2.8.15).

2.9 Solution for times before the final period :

To obtain the equation for times before the final period of decay, the third-order fluctuation terms are neglected compared to the second-order terms. Analogously, it would be anticipated that for times before but sufficiently near to the final period the fourth-order terms should be negligible in comparison with the third-order terms. If this assumption is made the equation (2.8.20) shows that the term $\langle \alpha\theta'\theta'' \rangle$ associated with the pressure fluctuations should also be neglected. Thus neglecting all the terms on the right hand side of equations (2.8.15)-(2.8.17), we can write

$$\frac{\partial \langle \beta_j \theta'\theta'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)}{\partial t} + \frac{\nu}{p_r}\left[(1+p_r)k^2 + 2p_r k_i k_i' - \frac{p_r}{\nu} fs\right] \langle \beta_j \theta'\theta'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0, \quad \text{-------- (2.9.1)}$$

$$\frac{\partial \langle \beta_j \theta'\theta'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)}{\partial \Delta t} + \left(\frac{\nu}{p_r}\right) k^2 \langle \beta_j \theta'\theta'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0, \quad \text{-------- (2.9.2)}$$

$$\frac{\partial \langle \beta_j \theta'\theta'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)}{\partial \Delta t'} + \left(\frac{\nu}{p_r}\right) k'^2 \langle \beta_j \theta'\theta'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0, \quad \text{-------- (2.9.3)}$$

where $\langle \gamma_j \theta'\theta'' \rangle = R \langle \beta_j \theta'\theta'' \rangle$ and 1-R=S here R and S arbitrary constant.

Inner multiplication of equations (2.9.1), (2.9.2) and (2.9.3) by k_j and integrating between t_o and t we obtain

$$k_j \langle \beta_j \theta'\theta'' \rangle = f_j \exp\left\{ -\frac{\nu}{p_r}\left[(1+p_r)(k^2 + k'^2) + 2p_r kk' \cos\theta - \frac{p_r}{\nu} fs\right](t-t_0) \right\}, \quad \text{------- (2.9.4)}$$

$$k_j \langle \beta_j \theta'\theta'' \rangle = g_j \exp\left(-\frac{\nu}{p_r} k^2 \Delta t\right) \quad \text{-------- (2.9.5)}$$

and $$k_j \langle \beta_j \theta'\theta'' \rangle = q_j \exp\left(-\frac{\nu}{p_r} k'^2 \Delta t'\right). \quad \text{-------- (2.9.6)}$$

For these relations to be consistent, we have

$$k_j \langle \beta_j \theta'\theta'' \rangle = k_j \langle \beta_j \theta'\theta'' \rangle_0 \exp\left[-\frac{\nu}{p_r}\{(1+p_r)(k^2 + k'^2)(t-t_0) + k^2 \Delta t + k'^2 \Delta t' \right.$$

$$\left. + 2p_r kk' \cos\theta (t-t_0) - \frac{p_r}{\nu} fs (t-t_0) \right], \quad \text{-------- (2.9.7)}$$

where θ is the angle between k and k' and $\langle \beta_j \theta' \theta'' \rangle_0$ is the value of $\langle \beta_j \theta' \theta'' \rangle$ at t=t₀ $, \Delta t = \Delta t' = 0$. Letting $\hat{r}' = 0, \Delta t' = 0$ in the equation (2.8.10) and comparing the result with the equation (2.7.11) shows that

$$\langle k_j \phi_j \tau\tau'(\hat{K}, \Delta t, t) \rangle = \int_{-\infty}^{\infty} \langle k_j \beta_j \theta' \theta''(\hat{K}, \hat{K}', \Delta t, 0, t) \rangle d\hat{K}. \quad \text{------------- (2.9.8)}$$

Substituting the equation (2.9.7) and (2.9.8) into the equation (2.7.13) we obtain

$$\frac{\partial \langle \tau\tau' \rangle (\hat{K}, \Delta t, t)}{\partial t} + 2\frac{\nu}{p_r} k^2 \langle \tau\tau' \rangle (\hat{K}, \Delta t, t) = \int_{-\infty}^{\infty} ik_j \left[\langle \beta_j \theta' \theta'' \rangle (\hat{K}, K', \Delta t, 0, t) - \langle \beta_j \theta' \theta'' \rangle \right.$$

$$\left. (-\hat{K}, -\hat{K}', \Delta t, 0, t) \right]_0 \exp\left[-\frac{\nu}{p_r} \{ (1+p_r)(k^2 + K'^2)(t - t_0) + k^2 \Delta t + k'^2 \Delta t' + 2p_r kk'(t - t_0)\cos\theta \right.$$

$$\left. - \frac{p_r}{\nu} fs(t - t_0) \} \right] d\hat{K}'. \quad \text{----------- (2.9.9)}$$

Now, $d\hat{K}' (\equiv dK_1' dK_2' dK_3')$ can be expressed in terms of k' and θ as $-2\pi k'^2 d(\cos\theta) d\hat{k}'$ (cf. Deissler [28])

$$d\hat{K}' = -2\pi k'^2 d(\cos\theta) dk'. \quad \text{--------- (2.9.9a)}$$

Substitution of equation (2.9.9a) in equation (2.9.9) yields

$$\frac{\partial \langle \tau\tau' \rangle (\hat{K}, \Delta t, t)}{\partial t} + 2\frac{\nu}{p_r} k^2 \langle \tau\tau' \rangle (\hat{K}, \Delta t, t)$$

$$= 2\int_{\infty}^{\infty} 2\pi i k_j \left[\langle \beta_j \theta' \theta'' \rangle (k, k') - \langle \beta_j \theta' \theta'' \rangle (-\hat{K}, \hat{K}') \right]_0 k'^2 \left[\int_{-1}^{1} \exp\left\{ -\frac{\nu}{p_r} [(1 + p_r)(k^2 + k'^2)(t - t_0) \right. \right.$$

$$\left. \left. + k^2 \Delta t + k'^2 \Delta t' + 2p_r kk'(t - t_0)\cos\theta - \frac{p_r}{\nu} fs(t - t_0) \right] \right\} d(\cos\theta) \Bigg] d\hat{k}'. \quad \text{-------- (2.9.10)}$$

In order to find the solution completely and following Loeffler and Dessiler [72], we assume that

$$ik_j \left[\langle \beta_j \theta' \theta'' \rangle (\hat{K}, \hat{K}') - \langle \beta_j \theta' \theta'' \rangle (-\hat{K}, -\hat{K}') \right]_0 = -\frac{\delta_0}{(2\pi)^2} (k^2 k'^4 - k^4 k'^2), \quad \text{----- (2.9.11)}$$

where δ_0 is a constant depending on the initial condition. The negative sign is placed in front of δ_0 in order to make the transfer of energy from small to large wave numbers for positive value of δ_0.

Substituting equation (2.9.11) into equation (2.9.10), we get

$$\frac{\partial}{\partial t}\langle\tau\tau'\rangle(\hat{K},\Delta t,t)2\pi + 2\frac{\nu}{p_r}2\pi k^2\langle\tau\tau'\rangle(\hat{K},\Delta t,t) = -2\delta_o\int_0^\infty(k^2k'^4 - k^4k'^2)k'^2$$

$$\times\left[\int_{-1}^{1}\exp\left\{-\frac{\nu}{p_r}[(1+p_r)(k^2+k'^2)(t-t_o)+k^2\Delta t + k'^2\Delta t' + 2p_r kk'(t-t_o)\cos\theta\right.\right.$$

$$\left.\left.-\frac{p_r}{\nu}fs(t-t_o)\ ]\right\}d(\cos\theta)\ \right]d\hat{k}'. \qquad \text{--------- (2.9.12)}$$

Multiplying both sides of equation (2.9.12) by k^2, we get

$$\frac{\partial E}{\partial t} + 2\frac{\nu}{p_r}k^2E = w, \qquad \text{--------- (2.9.13)}$$

where $E = 2\pi k^2\langle\tau\tau'\rangle$ is the energy spectrum function and w is the energy transfer term given by

$$w = -2\delta_o\int_0^\infty(k^2k'^4 - k^4k'^2)k^2k'^2\left[\int_{-1}^{1}\exp\left\{-\frac{\nu}{p_r}[(1+p_r)(k^2+k'^2)(t-t_o)+k^2\Delta t\right.\right.$$

$$\left.\left.+2p_r kk'(t-t_o)\cos\theta\ -\frac{p_r}{\nu}fs(t-t_o)\ ]\ \right\}d(\cos\theta)\right]d\hat{k}'. \qquad \text{--------- (2.9.14)}$$

Integrating equation (2.9.12) with respect to θ, we have

$$w = -\frac{\delta_o}{\nu(t-t_o)}\int_0^\infty(k^3k'^5 - k^5k'^3)\left[\exp\{-\frac{\nu}{p_r}[(1+p_r)(k^2+k'^2)(t-t_o)+k^2\Delta t\right.$$

$$\left.-2p_r kk'(t-t_o) - \frac{p_r}{\nu}fs(t-t_o)]]\}\right]dk' + \frac{\delta_o}{\nu(t-t_o)}\int_0^\infty(k^3k'^5 - k^5k'^3)\times$$

$$\left[\exp\left\{-\frac{\nu}{p_r}[(1+p_r)(k^2+k'^2)(t-t_o)+k^2\Delta t + 2p_r kk'(t-t_o) - \frac{p_r}{\nu}fs(t-t_o)]\right\}\right]d\hat{k}' \quad . \text{--- (2.9.15)}$$

Again integrating equation (2.9.15) with respect to k', we have

$$w = -\frac{\delta_o\sqrt{\pi}\,p_r^{5/2}}{4\nu^{3/2}(t-t_o)^{3/2}(1+p_r)^{5/2}}\exp\{\frac{p_r}{\nu}fs(t-t_o)\}\times\exp[\frac{-k^2\nu(1+2p_r)}{p_r(1+p_r)}(t-t_o+\frac{1+p_r}{1+2p_r}\Delta t]$$

$$\times[\frac{15p_rk^4}{4\nu^2(t-t_o)^2(1+p_r)}+\{\frac{5p_r^2}{(1+p_r)^2}-\frac{3}{2}\}\frac{k^6}{\nu(t-t_o)}+\{\frac{p_r^3}{(1+p_r)^3}-\frac{p_r}{1+p_r}\}k^8]$$

$$-\frac{\delta_o\sqrt{\pi}\,p_r^{5/2}}{4\nu^{3/2}(t-t_o+\Delta t)^{3/2}(1+p_r)^{5/2}}\exp\{\frac{p_r}{\nu}fs(t-t_o)\}\times\exp[\frac{-k^2\nu(1+2p_r)}{p_r(1+p_r)}(t-t_o+\frac{p_r}{1+2p_r}\Delta t)]$$

$$\times[\frac{15p_rk^4}{4\nu^2(t-t_o+\Delta t)^2(1+p_r)}+\{\frac{5p_r^2}{(1+p_r)^2}-\frac{3}{2}\}\frac{k^6}{\nu(t-t_o+\Delta t)}+\{\frac{p_r^3}{(1+p_r)^3}-\frac{p_r}{1+p_r}\}k^8]. \quad \text{----- (2.9.16)}$$

The series of equation (2.9.16) contains only even power of k and start with k^4. The quantity w is the contribution to the energy transfer arising from consideration of the three-point correlation equation.

If we integrate equation (2.9.16) for $\Delta t=0$ over all wave numbers, it can be easily shown that

$$\int_0^\infty w\,dk = 0. \quad \text{-------- (2.9.17)}$$

which indicates that the expression for w satisfies the condition of continuity and homogeneity. Physically it was to be expected, since w is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero.

The linear equation (2.9.13) can be solved to give

$$E = \exp[-\frac{2\nu}{p_r}k^2(t-t_o+\frac{\Delta t}{2})]\int w\exp[2\frac{\nu}{p_r}k^2(t-t_o+\frac{\Delta t}{2})]dt + J(k)\exp[-\frac{2\nu}{p_r}k^2(t-t_o+\frac{\Delta t}{2})], \quad (2.9.18)$$

where $J(k)=\frac{N_ok^2}{\pi}$ is a constant of integration and can be obtained as by Corrsin [24].

Substituting the values of w from (2.9.16) and J(k) into the equation (2.9.18) and integrating with respect to t_0 we get

$$E=\frac{N_o k^2}{\pi}\exp[-2\frac{v}{p_r}k^2(t-t_o+\frac{\Delta t}{2})]+\frac{\delta_o\sqrt{\pi}\,p_r^{5/2}}{4v^{3/2}(1+p_r)^{7/2}}\exp[fs(t-t_o)]$$

$$\times\exp[\frac{-k^2v(1+2p_r)}{p_r(1+p_r)}(t-t_o+\frac{1+p_r}{1+2p_r}\Delta t)]\times[\frac{3p_r k^4}{2v^2(t-t_o)^{5/2}}+\frac{p_r(7p_r-6)k^6}{3v(1+p_r)(t-t_o)^{3/2}}$$

$$-\frac{4(3p_r^2-2p_r+3)k^8}{3(1+p_r)^2(t-t_o)^{1/2}}+\frac{8\sqrt{v}(3p_r^2-2p_r+3)k^9}{3(1+p_r)^{5/2}p_r^{1/2}}F(\eta)+\frac{\delta_o\sqrt{\pi}\,p_r^{5/2}}{4v^{3/2}(1+p_r)^{7/2}}$$

$$\times\exp[fs(t-t_o)]\times\exp[\frac{-vk^2(1+2p_r)}{p_r(1+p_r)}(t-t_o+\frac{p_r}{1+2p_r}\Delta t)\times[\frac{3p_r k^4}{2v^2(t-t_o+\Delta t)^{5/2}}]$$

$$+\frac{p_r(7p_r-6)k^6}{3v(1+p_r)(t-t_o+\Delta t)^{3/2}}-\frac{4}{3}\frac{(3p_r^2-2p_r+3)k^8}{(1+p_r)^2(t-t_o+\Delta t)^{1/2}}+\frac{8\sqrt{v}(3p_r^2-2p_r+3)k^9}{3(1+p_r)^{5/2}p_r^{1/2}}F(\eta), \quad (2.9.19)$$

where $F(\eta)=\bar{e}^{\eta^2}\int_0^{\eta}e^{x^2}dx$,

$$\eta=k\sqrt{\frac{v(t-t_o)}{p_r(1+p_r)}} \quad \text{or} \quad \eta=k\sqrt{\frac{v(t-t_o+\Delta t)}{p_r(1+p_r)}}.$$

By setting $\hat{r}=0$ in equation (2.7.10) and use is made of the definition of E, the result is

$$\frac{\langle TT'\rangle}{2}=\frac{\langle T^2\rangle}{2}=\int_0^{\infty}E\,dk\,\cdot \quad \text{---------} \ (2.9.20)$$

Substituting equation (2.9.19) into equation (2.9.20) and integrating with respect to k, gives

$$\frac{\langle T^2\rangle}{2}=\frac{N_o p_r^{3/2}(T_0+\frac{\Delta T_0}{2})^{-3/2}}{8\sqrt{2\pi}v^{3/2}}+\frac{\pi\delta_o p_r^6}{4v^6(1+p_r)(1+2p_r)^{5/2}}\exp[fs]\left[\frac{9}{16T_0^{5/2}(T_0+\frac{1+p_r}{1+2p_r}\Delta T_0)^{5/2}}\right.$$

$$+\frac{9}{16(T_0+\Delta T_0)^{5/2}(T_0+\frac{p_r}{1+2p_r}\Delta T_0)^{5/2}}+\frac{5p_r(7p_r-6)}{16(1+2p_r)T_0^{3/2}(T_0+\frac{1+p_r}{1+2p_r}\Delta T_0)^{7/2}}$$

$$+\frac{5p_r(7p_r-6)}{16(1+2p_r)(T_0+\Delta T_0)^{3/2}(T_0+\frac{p_r}{1+2p_r}\Delta T_0)^{7/2}}+\frac{35p_r(3p_r^2-2p_r+3)}{8(1+2p_r)T_0^{1/2}(T_0+\frac{1+p_r}{1+2p_r}\Delta T_0)^{9/2}}$$

$$+\frac{35p_r(3p_r^2-2p_r+3)}{8(1+2p_r)(T_0+\Delta T_0)^{1/2}(T_0+\frac{p_r}{1+2p_r}\Delta T_0)^{9/2}}+\frac{8p_r(3p_r^2-2p_r+3)(1+2p_r)^{5/2}}{3.2^{23/2}(1+p_r)^{11/2}}$$

$$\sum_{n=0}^{\infty}\frac{1.3.5..........(2n+9)}{n!(2n+1)2^{2n}(1+p_r)^n}\times\{\frac{T_0^{(2n+1)/2}}{(T_0+\Delta T_0/2)^{(2n+1)/2}}+\frac{(T_0+\Delta T_0)^{(2n+1)/2}}{(T_0+\Delta T_0/2)^{(2n+1)/2}}\}\Bigg], \quad \text{---------} \quad (2.9.21)$$

where $T_0=t-t_0$.

Equation (2.9.21) is the decay law of temperature energy fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in presence of dust particles.

2.10 Concluding Remarks :

In equation (2.9.21) we obtained the decay law of temperature fluctuations in homogeneous turbulence before the final period in presence of dust particles by neglecting the quadruple correlation terms in comparison with the third–order terms for the case of multi-point and multi-time. If the fluid is clean then f=0, the equation (2.9.21) becomes

$$\frac{\langle T^2\rangle}{2}=\frac{N_o p_r^{3/2}(T_0+\frac{\Delta T_0}{2})^{-3/2}}{8\nu^{3/2}\sqrt{2\pi}}+\frac{\pi\delta_o p_r^6}{4\nu^6(1+p_r)(1+2p_r)^{5/2}}\times[\frac{9}{16T_0^{5/2}(T_0+\frac{1+p_r}{1+2p_r}\Delta T_0)^{5/2}}$$

$$+\frac{9}{16(T_0+\Delta T_0)^{5/2}(T_0+\frac{p_r}{1+2p_r}\Delta T_0)^{5/2}}+\frac{5p_r(7p_r-6)}{16(1+2p_r)T_0^{3/2}(T_0+\frac{1+p_r}{1+2p_r}\Delta T_0)^{7/2}}$$

$$+\frac{5p_r(7p_r-6)}{16(1+2p_r)(T_0+\Delta T_0)^{3/2}(T_0+\frac{p_r}{1+2p_r}\Delta T_0)^{7/2}}+\frac{35p_r(3p_r^2-2p_r+3)}{8(1+2p_r)T_0^{1/2}(T_0+\frac{1+p_r}{1+2p_r}\Delta T_0)^{9/2}}$$

$$+\frac{35p_r(3p_r^2-2p_r+3)}{8(1+2p_r)(T_0+\Delta T_0)^{1/2}(T_0+\frac{p_r}{1+2p_r}\Delta T_0)^{9/2}}+\frac{8p_r(3p_r^2-2p_r+3)(1+2p_r)^{5/2}}{3.2^{23/2}(1+p_r)^{11/2}}$$

$$\sum_{n=0}^{\infty}\frac{1.3.5..........(2n+9)}{n!(2n+1)2^{2n}(1+p_r)^n}\times\{\frac{T_0^{(2n+1)/2}}{(T_0+\Delta T_0/2)^{(2n+1)/2}}+\frac{(T_0+\Delta T_0)^{(2n+1)/2}}{(T_0+\Delta T_0/2)^{(2n+1)/2}}\}], \quad \text{--------} (2.10.1)$$

which was obtained earlier by Sarker and Islam [116].

If we put $\Delta T_0 = 0$ in equation (2.10.1), we can easily find out

$$\frac{\langle T^2\rangle}{2}=\frac{N_o p_r^{3/2} T_0^{-3/2}}{8\sqrt{2\pi}v^{3/2}}+\frac{\pi\delta_o p_r^6 T_0^{-5}}{4v^6(1+p_r)(1+2p_r)^{5/2}}\times[\frac{9}{16}+\frac{5}{16}.\frac{p_r(7p_r-6)}{1+2p_r}+\frac{35}{8}\frac{p_r(3p_r^2-2p_r+3)}{(1+2p_r)^2}+--]$$

$$=AT_0^{-3/2}+BT_0^{-5}=A(t-t_0)^{-3/2}+B(t-t_0)^{-5}, \quad \text{--------} (2.10.2)$$

where $A=\frac{N_o p_r^{3/2}}{8\sqrt{2\pi}v^{3/2}}$ and

$$B=\frac{\pi\delta_o p_r^6}{2v^6(1+p_r)(1+2p_r)^{5/2}}\times[\frac{9}{16}+\frac{5}{16}.\frac{p_r(7p_r-6)}{1+2p_r}+\frac{35}{8}.\frac{p_r(3p_r^2-2p_r+3)}{(1+2p_r)^2}+..........],$$

which was obtained earlier by Loeffler and Deissler [72].

This study shows that the effect of dust particles in homogeneous turbulence, the temperature energy fluctuations decays more rapidly than the energy for clean fluid for times before the final period.

If higher order correlation equations were considered in the analysis it appears that more terms of higher power of time would be added to the equation (2.9.21). For large times, the second term in the equation becomes negligible leaving the -3/2 power decay law for the final period.

CHAPTER-II

PART-C

DECAY OF TEMPERATURE FLUCTUATIONS IN HOMOGENEOUS TURBULENCE BEFORE THE FINAL PERIOD FOR THE CASE OF MULTI-POINT AND MULTI-TIME IN A ROTATING SYSTEM IN PRESENCE OF DUST PARTICLES

2.11 Introduction :

In recent years, the motion of dusty viscous fluids has developed rapidly. Such a situation occurs in the moment of dust-laden air, in problems of fluidization, in the use of dust in a gas cooling system and in the Sedimentation problem of tidal rivers. Saffman [106] derived an equation that described the motion of a fluid containing small dust particles, which is applicable to laminar flows as well as turbulent flow. Michael and Miller [83] discussed the motion of dusty gas occupying the semi-infinite space above a rigid plane boundary. Kishore and Sinha [59] studied the rate of change of vorticity covariance in dusty fluid turbulence. Sinha [122] also studied the effect of dust particles on the acceleration covariance of ordinary turbulence. Sarker [110], Sarker and Rahman [112] considered dust particles on their won works. Deissler [27,28] developed a theory for homogeneous turbulence, which was valid for times before the final period. Following Deissler's theory Loeffler and Deissler [72] studied the decay of temperature fluctuations in homogeneous turbulence before the final period. In their study, they presented the theory, which is valid during the period for which the quadruple correlation terms are neglected compared to the second- and third–order correlation terms. Using Deissler's same theory Kumar and Patel [64] studied the first-order reactants in homogeneous turbulence before the final period for the case of multi-point and single-time. The problem [64] extended to the case of multi-point and multi-time concentration correlation by Kumar and Patel [65] and also the numerical result of [65] carried out by Patel [97]. Following Deissler's approach Sarker and Islam [115] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Islam and Sarker [46] also studied the first –order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Sarker and Rahman [113] studied the decay of temperature fluctuations in MHD turbulence before the final period. Sarker and Islam [116] also studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time. In their approach, they used two and three point correlations and neglecting fourth- and higher-order correlation terms compared to the second- and third-order correlation terms.

In this chapter the method of [27,28] is used and we have studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in a rotating system in presence of dust particles. It is the extension work of Part-A and Part-B of this chapter and we have derived an equation for decaying energy of temperature fluctuations in homogeneous turbulence for times before the final period. Here we have considered correlations between fluctuating quantities at two- and three-point and the set of equation is made to determinate by neglecting the higher correlations in comparison to the second- and third-order correlations.

2.12 Correlation and spectral equations :

From the equation (2.2.3) and (2.2.4), we can write the equations of energy for an incompressible fluid with constant properties at the point P and P' as

$$\frac{\partial T}{\partial t}+u_i\frac{\partial T}{\partial x_i}=\left(\frac{\nu}{p_r}\right)\frac{\partial^2 T}{\partial x_i \partial x_i} \qquad \text{----------} (2.12.1)$$

and

$$\frac{\partial T'}{\partial t'}+u_i'\frac{\partial T'}{\partial x_i'}=\left(\frac{\nu}{p_r}\right)\frac{\partial^2 T'}{\partial x_i' \partial x_i'}. \qquad \text{----------} (2.12.2)$$

Here u_i turbulent velocity component $P_r=\frac{\nu}{\gamma}$, Prandtl number; ν, kinematic viscosity

The subscripts can take on the values 1, 2 or 3.

Multiplying equation (2.12.1) by T', equation (2.12.2) by T and taking ensemble average, result in

$$\frac{\partial\langle TT'\rangle}{\partial t}+\frac{\partial\langle TT'u_i\rangle}{\partial x_i}=\left(\frac{\nu}{p_r}\right)\frac{\partial^2\langle TT'\rangle}{\partial x_i\partial x_i}, \qquad \text{----------} (2.12.3)$$

$$\frac{\partial\langle TT'\rangle}{\partial t'}+\frac{\partial\langle TT'u'_i\rangle}{\partial x'_i}=\left(\frac{\nu}{p_r}\right)\frac{\partial^2\langle TT'\rangle}{\partial x'_i\partial x'_i} \qquad \text{----------} (2.12.4)$$

with the continuity equation

$$\frac{\partial u_i}{\partial x}=\frac{\partial u'_i}{\partial x'_i}=0\,. \qquad \text{----------} (2.12.5)$$

Angular bracket $\langle---\rangle$ is used to denote an ensemble average. Using the transformations

$$\frac{\partial}{\partial x_i}=-\frac{\partial}{\partial r_i},\frac{\partial}{\partial x'_i}=\frac{\partial}{\partial r_i},\left(\frac{\partial}{\partial t}\right)_{t'}=\left(\frac{\partial}{\partial t}\right)_{\Delta t}-\frac{\partial}{\partial \Delta t},\frac{\partial}{\partial t'}=\frac{\partial}{\partial \Delta t}$$

into equations (2.12.3) and (2.12.4), one obtains

$$\frac{\partial\langle TT'\rangle}{\partial t}-\frac{\partial\langle u'_iTT'\rangle}{\partial r_i}(-r_{.}-\Delta t,t+\Delta t)+\frac{\partial\langle TT'u'_i\rangle}{\partial r_i}(\hat{r},\Delta t,t)=2\left(\frac{\nu}{p_r}\right)\frac{\partial^2\langle TT'\rangle}{\partial r_i\partial r_i}, \qquad \text{---------} (2.12.6)$$

$$\frac{\partial\langle TT'\rangle}{\partial \Delta t}+\frac{\partial\langle u_iTT'\rangle}{\partial r_i}(-\hat{r},-\Delta t,t+\Delta t)=\left(\frac{\nu}{p_r}\right)\frac{\partial^2\langle TT'\rangle}{\partial r_i\partial r_i}. \qquad \text{---------} (2.12.7)$$

It is convenient to write this equation in spectral form by use of the following three-dimensional Fourier transforms.

$$\langle TT'(\hat{r},\Delta t,t)\rangle=\int_{-\infty}^{\infty}\langle\tau\tau'(\hat{K},\Delta t,t)\rangle\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K}\,, \qquad \text{---------} (2.12.8)$$

$$\langle u_iTT'(\hat{r},\Delta t,t)\rangle=\int_{-\infty}^{\infty}\langle\phi_i\ \tau\tau'(\hat{K},\Delta t,t)\rangle\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K} \qquad \text{----------} (2.12.9)$$

and $\langle u'_iTT'(\hat{r},\Delta t,t)\rangle=\langle u_iTT'(-\hat{r},-\Delta t,t+\Delta t)\rangle$

$$=\int_{-\infty}^{\infty}\langle\phi_i\tau\tau'(-\hat{K},-\Delta t,t+\Delta t)\rangle\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K} \qquad \text{---------} (2.12.10)$$

(Interchange are made between the points p and p'), where $\hat{K}$ is known as a wave number vector and magnitude of $\hat{K}$ has the dimension 1/length and can be considered to be the reciprocal of an eddy size.

Substituting the equations (2.12.8)-(2.12.10) into equations (2.12.6) and (2.12.7) leads to the spectral equations.

$$\frac{\partial\langle\tau\tau'\rangle}{\partial t}+2\left(\frac{\nu}{P_r}\right)k^2\langle\tau\tau'\rangle=iK_i\left[\langle\phi_i\tau\tau'(\hat{K},\Delta t,t)\rangle-\langle\phi_i\tau\tau'(-\hat{K},-\Delta t,t+\Delta t)\rangle\right], \qquad \text{---------} (2.12.11)$$

$$\frac{\partial\langle\tau\tau'\rangle}{\partial\Delta t}+2\left(\frac{\nu}{P_r}\right)k^2=-iK_i\langle\phi_i\tau\tau'\rangle\left(-\hat{K},-\Delta t,t+\Delta t\right). \qquad \text{---------} (2.12.12)$$

In equations (2.12.11)-(2.12.12) the quantity $\tau\tau'(\hat{K})$ may be interpreted as a temperature fluctuation "energy" contribution of thermal eddies of size 1/k. The time derivative of this "energy' as a function of the convective transfer to the wave numbers and the "dissipation" due to the action of the thermal conductivity. The term on the right hand side of equation (2.12.11) is also called transfer term while the second term on the left hand side is the "dissipation" term.

2.13 Three-Point, Three-Time Correlation and Spectral Equations :

In order to obtain the three-point, three-time correlation and spectral equations, we write the Navier-Stockes equation for turbulent flow of dusty incompressible fluid in a rotating system at the point P, energy equations at the points p' and p'' separated by the vectors $\hat{r}$ and $\hat{r}'$

$$\frac{\partial u_j}{\partial t}+\frac{\partial}{\partial x_i}\left(u_ju_i\right)=-\frac{1}{\rho}\frac{\partial p}{\partial x_j}+\nu\frac{\partial^2u_j}{\partial x_i\partial x_i}-2\in_{mij}\Omega_mu_j+f\left(u_j-v_j\right), \qquad \text{----------} (2.13.1)$$

$$\frac{\partial T'}{\partial t}+u_i'\frac{\partial T'}{\partial x_i'}=(\frac{\nu}{P_r})\frac{\partial^2T'}{\partial x_i'\partial x_i'} \qquad \text{----------} (2.13.2)$$

and $$\frac{\partial T''}{\partial t}+u_i''\frac{\partial T''}{\partial x_i''}=(\frac{\nu}{P_r})\frac{\partial^2T''}{\partial x_i''\partial x_i''}. \qquad \text{----------} (2.13.3)$$

Multiplying equations (2.13.1)–(2.13.3) by $T'T''$, u_jT'' and u_jT' respectively and then taking ensemble average, we obtained

$$\frac{\partial\langle u_j T'T''\rangle}{\partial t}+\frac{\partial\langle u_j T'T''u_i\rangle}{\partial x_i}=-\frac{1}{\rho}\frac{\partial\langle PT'T''\rangle}{\partial x_j}+\nu\frac{\partial^2\langle u_j T'T''\rangle}{\partial x_i\partial x_i}-2\in_{mij}\Omega_m\langle u_j T'T''\rangle+f(\langle u_j T'T''\rangle-\langle v_j T'T''\rangle),$$

--------- (2.13.4)

$$\frac{\partial\langle T'u_j T''\rangle}{\partial t'}+\frac{\partial\langle u_i' T'u_j T''\rangle}{\partial x_i'}=\left(\frac{\nu}{P_r}\right)\frac{\partial^2\langle T'u_j T''\rangle}{\partial x_i'\partial x_i'} \qquad \text{--------- (2.13.5)}$$

and $$\frac{\partial\langle T''u_j T'\rangle}{\partial t''}+\frac{\partial\langle u_i'' T''u_j T'\rangle}{\partial x_i''}=\left(\frac{\nu}{P_r}\right)\frac{\partial^2\langle T''u_j T'\rangle}{\partial x_i''\partial x_i''}. \qquad \text{---------- (2.13.6)}$$

Using the transformations

$$\frac{\partial}{\partial x_i}=-\left(\frac{\partial}{\partial r_i}+\frac{\partial}{\partial r_i'}\right),\frac{\partial}{\partial x_i'}=\frac{\partial}{\partial r_i},\frac{\partial}{\partial x_i''}=\frac{\partial}{\partial r_i'}$$

$$\left(\frac{\partial}{\partial t}\right)_{t',t''}=\left(\frac{\partial}{\partial t}\right)_{\Delta t,\Delta t'}-\frac{\partial}{\partial \Delta t}-\frac{\partial}{\partial \Delta t'},\frac{\partial}{\partial \Delta t'}=\frac{\partial}{\partial \Delta t},\frac{\partial}{\partial \Delta t''}=\frac{\partial}{\partial \Delta t'}$$

into equations (2.13.4) – (2.13.6), we have

$$\frac{\partial\langle u_j T'T''\rangle}{\partial t}-\left(\frac{\partial}{\partial r_i}+\frac{\partial}{\partial r_i'}\right)\langle u_j T'T''u_i\rangle+\frac{\partial\langle u_i' T'u_j T''\rangle}{\partial r_i}+\frac{\partial\langle u_i'' T''u_j T'\rangle}{\partial r_i'}$$

$$=\frac{1}{\rho}\left(\frac{\partial}{\partial r_j}+\frac{\partial}{\partial r_j'}\right)\langle PT'T''\rangle+\nu\left(\frac{\partial}{\partial r_i}+\frac{\partial}{\partial r_i'}\right)^2\langle u_j T'T''\rangle+\left(\frac{\nu}{P_r}\right)\left(\frac{\partial^2}{\partial r_i\partial r_i}+\frac{\partial^2}{\partial r_i'\partial r_i'}\right)\langle u_j T'T''\rangle$$

$$-2\in_{mij}\Omega_m\langle \mathrm{u}_j T'T''\rangle+\mathrm{f}(\langle \mathrm{u}_j T'T''\rangle-\langle \mathrm{v}_j T'T''\rangle) \quad , \qquad \text{---------- (2.13.7)}$$

$$\frac{\partial\langle u_j T'T''\rangle}{\partial \Delta t}+\frac{\partial\langle u_i' T'u_j T''\rangle}{\partial r_i}=\left(\frac{\nu}{P_r}\right)\frac{\partial^2\langle T'u_j T''\rangle}{\partial r_i\partial r_i}, \qquad \text{---------- (2.13.8)}$$

$$\frac{\partial\langle T'u_j T''\rangle}{\partial \Delta t'}+\frac{\partial\langle u_i' T''u_j T'\rangle}{\partial r_i'}=\left(\frac{\nu}{P_r}\right)\frac{\partial^2\langle u_j T'T''\rangle}{\partial r_i'\partial r_i'}. \qquad \text{---------- (2.13.9)}$$

The six-dimensional Fourier transforms for quantities in the equations (2.13.7)-(2.13.9) may be defined as

$$\langle u_j T'T''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\beta_j\theta'\theta''\rangle\exp[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}dK', \qquad \text{---------} (2.13.10)$$

$$\langle u_i u_j T'T''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\beta_i\beta_j\theta'\theta''\rangle\exp[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}d\hat{K}', \qquad \text{----------} (2.13.11)$$

$$\langle pT'T''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\langle\alpha\theta'\theta''\rangle\exp[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}d\hat{K}'. \qquad \text{----------} (2.13.12)$$

Interchanging the points p' and p'' shows that

$$\langle u_j u_i'' T'T''\rangle=\langle u_j u_i' T'T''\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\beta_j\beta_i'\theta'\theta''\rangle\exp[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}d\hat{K}' \qquad \text{---------} (2.13.13)$$

$$\langle v_j T'T''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\gamma_j\theta'\theta''\rangle\exp[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}dK' \qquad \text{---------} (2.13.14)$$

Substituting the preceding relations (2.13.10)-(2.13.14) into equations (2.13.7)-(2.13.9) give in the forms

$$\frac{\partial\langle\beta_j\theta'\theta''\rangle}{\partial t}(\hat{K},\hat{K}',\Delta t,\Delta t',t)+\frac{v}{p_r}[(1+p_r)k^2+2p_r k_i k_i'+(1+p_r)k'^2+\frac{p_r}{v}(2\in_{mij}\Omega_m-f)]$$

$$\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=\frac{1}{\rho}i(k_j+k_j')\langle\alpha\theta'\theta''\rangle-f\langle\gamma_j\theta'\theta''\rangle+i(k_i+k_i')[\langle\beta_i\beta_j\theta'\theta''\rangle\ ,$$

$$-\langle\beta_i'\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)] \qquad \text{---------} (2.13.15)$$

$$\frac{\partial\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)}{\partial\Delta t}+\left(\frac{v}{p_r}\right)k^2\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=-ik_i\langle\beta_i'\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t),$$

$$\text{---------} (2.13.16)$$

$$\frac{\partial\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)}{\partial\Delta t'}+\left(\frac{v}{p_r}\right)k'^2\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=-ik_i'\langle\beta_i'\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t).$$

$$\text{--------} (2.13.17)$$

If the derivative with respect to x_j is taken of the momentum equation (2.13.1) for the point p, the equation multiplied by $T'T''$ and taken the ensemble average, the resulting equation is

$$\frac{\partial^2\langle u_i u_j T'T''\rangle}{\partial x_j \partial x_i} = -\frac{1}{\rho}\frac{\partial^2\langle pT'T''\rangle}{\partial x_j \partial x_j}. \qquad \text{-------- (2.13.18)}$$

Writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$\left[\frac{\partial^2}{\partial r_i \partial r_i}+\frac{\partial^2}{\partial r_i \partial r_i'}+\frac{\partial^2}{\partial r_i' \partial r_j}+\frac{\partial^2}{\partial r_i' \partial r_j'}\right]\langle u_i u_j T'T''\rangle = -\frac{1}{\rho}\left[\frac{\partial^2}{\partial r_j \partial r_j}+2\frac{\partial^2}{\partial r_j \partial r_j'}+\frac{\partial^2}{\partial r_j'}\right]\langle pT'T''\rangle. \quad \text{-- (2.13.19)}$$

Taking the Fourier transforms of equation (2.13.19).

$$\langle\alpha\theta'\theta''\rangle = \frac{-\rho\left(k_i k_j + k_i k_j' + k_i' k_j + k_i' k_j'\right)\langle\beta_i\beta_j\theta'\theta''\rangle}{k_j k_j + 2k_j k_j' + k_j' k_j'}. \qquad \text{--------- (2.13.20)}$$

Equation (2.13.20) can be used to eliminate $\langle\alpha\theta'\theta''\rangle$ from equation (2.13.15).

2.14 Solution for times before the final period :

To obtain the equation for times before the final period of decay, the three point correlations are considered and the quadruple correlation terms are neglected in comparison with the third-order correlation terms. Because, the quadruple correlation terms decay faster than the third-order correlation terms. If this assumption is made the equation (2.13.20) shows that the term $\langle\alpha\theta'\theta''\rangle$ associated with the pressure fluctuations should also be neglected. Thus neglecting all the terms on the right hand side of equations (2.13.15)-(2.13.17), we can write

$$\frac{\partial\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)}{\partial t}+\frac{\nu}{P_r}\left[(1+p_r)k^2+2p_r k_i k_i'+P_r/\nu\left(2\in_{mij}\Omega_m-fs\right)\right]\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=0,$$

--------- (2.14.1)

$$\frac{\partial\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)}{\partial\Delta t}+\left(\frac{\nu}{P_r}\right)k^2\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=0, \qquad \text{--------- (2.14.2)}$$

$$\frac{\partial\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)}{\partial\Delta t'}+\left(\frac{\nu}{P_r}\right)k'^2\langle\beta_j\theta'\theta''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=0, \qquad \text{--------- (2.13.3)}$$

where $\langle \gamma_j \theta'\theta'' \rangle = R\langle \beta_j \theta'\theta'' \rangle$ and 1-R=S here R and S arbitrary constant.

Inner multiplication of equations (2.14.1), (2.14.2) and (2.14.3) by k_j and integrating between t_o and t we obtain

$$k_j \langle \beta_j \theta'\theta'' \rangle = f_j \exp\{ -\frac{\nu}{p_r}\left[(1+p_r)(k^2+k'^2)+2p_r kk'\cos\theta + p_r/\nu(2\in_{mij}\Omega_m - fs)\right](t-t_0) \},$$

--------- (2.14.4)

$$k_j \langle \beta_j \theta'\theta'' \rangle = g_j \exp\left(-\frac{\nu}{p_r}k^2\Delta t\right) \quad \text{--------- (2.14.5)}$$

and $k_j \langle \beta_j \theta'\theta'' \rangle = q_j \exp\left(-\frac{\nu}{p_r}k'^2\Delta t'\right)$ --------- (2.14.6)

For these relations to be consistent, we have

$$k_j \langle \beta_j \theta'\theta'' \rangle = k_j \langle \beta_j \theta'\theta'' \rangle_0 \exp\left[-\frac{\nu}{p_r}\{(1+p_r)(k^2+k'^2)(t-t_0)+k^2\Delta t + k'^2\Delta t' + 2p_r kk'\cos\theta(t-t_0)\right.$$

$$\left. + \frac{p_r}{\nu}\left(2\in_{mij}\Omega_m - fs\right)(t-t_0)\ \}\right] \quad \text{--------- (2.14.7)}$$

where θ is the angle between k and k' and $\langle \beta_j \theta'\theta'' \rangle_0$ is the value of $\langle \beta_j \theta'\theta'' \rangle$ at $t=t_o$, $\Delta t = \Delta t' = 0$. Letting $\hat{r}' = 0, \Delta t' = 0$ in the equation (2.13.10) and comparing the result with the equation (2.12.9) shows that

$$\langle k_j \phi_j \tau\tau'(\hat{K}, \Delta t, t) \rangle = \int_{-\infty}^{\infty} \langle k_j \beta_j \theta'\theta''(\hat{K}, \hat{K}', \Delta t, 0, t) \rangle d\hat{K}. \quad \text{--------- (2.14.8)}$$

Substituting the equation (2.14.7) and (2.14.8) into the equation (2.12.11) we obtain

$$\frac{\partial\langle \tau\tau' \rangle(\hat{K}, \Delta t, t)}{\partial t} + 2\frac{\nu}{p_r}k^2\langle \tau\tau' \rangle(\hat{K}, \Delta t, t) = \int_{-\infty}^{\infty} ik_j\left[\langle \beta_j \theta'\theta'' \rangle(\hat{K}, K', \Delta t, 0, t) - \langle \beta_j \theta'\theta'' \rangle\right.$$

$$\left(-\hat{K},-\hat{K}',\Delta t,0,t\right)\Big]_0 \exp\left[-\frac{\nu}{p_r}\left\{(1+p_r)\left(k^2+K'^2\right)(t-t_0)+k^2\Delta t+k'^2\Delta t'+2p_r kk'(t-t_0)\cos\theta\right.\right.$$

$$\left.\left.+\frac{p_r}{\nu}\left(2\in_{mij}\Omega_m-fs\right)(t-t_0)\right\}\right]d\hat{K}'. \qquad \text{--------- (2.14.9)}$$

Now, $d\hat{K}'$ can be expressed in terms of k' and θ as $-2\pi k'^2 d(\cos\theta)d\hat{k}'$ (cf. Deissler [28]).

Hence, $d\hat{K}'=-2\pi k'^2 d(\cos\theta)d\hat{k}'$ --------- (2.14.9a)

$$\frac{\partial\langle\tau\tau'\rangle\left(\hat{K},\Delta t,t\right)}{\partial t}+2\frac{\nu}{p_r}k^2\langle\tau\tau'\rangle\left(\hat{K},\Delta t,t\right)$$

$$=2\int_{\infty}^{\infty}2\pi ik_j\left[\langle\beta_j\theta'\theta''\rangle(k,k')-\langle\beta_j\theta'\theta''\rangle\left(-\hat{K},\hat{K}'\right)\right]_0 k'^2\left[\int_{-1}^{1}\exp\left\{-\frac{\nu}{p_r}\left[(1+p_r)\left(k^2+k'^2\right)(t-t_0)\right.\right.\right.$$

$$\left.\left.\left.+k^2\Delta t+k'^2\Delta t'+2p_r kk'(t-t_0)\cos\theta+\frac{p_r}{\nu}\left(2\in_{mij}\Omega_m-fs\right)(t-t_0)\right]\right\}d(\cos\theta)\right]d\hat{k}'. \qquad \text{-- (2.14.10)}$$

In order to find the solution completely and following Loeffler and Deissler [72] we assume that

$$ik_j\left[\langle\beta_j\theta'\theta''\rangle\left(\hat{K},\hat{K}'\right)-\langle\beta_j\theta'\theta''\rangle\left(-\hat{K},-\hat{K}'\right)\right]_0=-\frac{\delta_0}{(2\pi)^2}\left(k^2k'^4-k^4k'^2\right), \qquad \text{------- (2.14.11)}$$

where δ_0 is a constant depending on the initial condition. The negative sign is placed in front of δ_0 in order to make the transfer of energy from small to large wave numbers for positive value of δ_0. The quantity $\left[\langle\beta_j\theta'\theta''\rangle\left(\hat{K},k'\right)-\langle\beta_i'\theta'\theta''\rangle\left(-\hat{K},-\hat{K}'\right)\right]_0$ depends on the initial conditions of the turbulence.

Substituting equation (2.14.11) into equation (2.14.10), we get

$$\frac{\partial}{\partial t}\langle\tau\tau'\rangle(\hat{K},\Delta t,t)2\pi+2\frac{\nu}{p_r}2\pi k^2\langle\tau\tau'\rangle(\hat{K},\Delta t,t)=-2\delta_o\int_0^\infty(k^2k'^4-k^4k'^2)k'^2$$

$$\times\left[\int_{-1}^{1}\exp\left\{-\frac{\nu}{p_r}[(1+p_r)(k^2+k'^2)(t-t_o)+k^2\Delta t+k'^2\Delta t'+2p_r kk'(t-t_o)\cos\theta\right.\right.$$

$$\left.\left.+\frac{p_r}{\nu}(2\in_{mij}\Omega_m-fs)(t-t_o)\ ]\right\}d(\cos\theta)\right]d\hat{k}'. \qquad \text{----------- (2.14.12)}$$

Multiplying both sides of equation (2.14.12) by k^2, we get.

$$\frac{\partial E}{\partial t}+2\frac{\nu}{p_r}k^2E=w, \qquad \text{----------- (2.14.13)}$$

where $E=2\pi k^2\langle\tau\tau'\rangle$, the energy spectrum function and w is the energy transfer term given by

$$w=-2\delta_o\int_0^\infty(k^2k'^4-k^4k'^2)k^2k'^2\left[\int_{-1}^{1}\exp\left\{-\frac{\nu}{p_r}[(1+p_r)(k^2+k'^2)(t-t_o)+k^2\Delta t\right.\right.$$

$$\left.\left.+2p_r kk'(t-t_o)\cos\theta+\frac{p_r}{\nu}(2\in_{mij}\Omega_m-fs)(t-t_o)\ ]\right\}d(\cos\theta)\right]d\hat{k}'. \qquad \text{---------- (2.14.14)}$$

Integrating equation (2.14.12) with respect to θ, we have

$$w=-\frac{\delta_o}{\nu(t-t_o)}\int_0^\infty(k^3k'^5-k^5k'^3)\left[\exp\{-\frac{\nu}{p_r}[(1+p_r)(k^2+k'^2)(t-t_o)+k^2\Delta t\right.$$

$$\left.-2p_r kk'(t-t_o)+\frac{p_r}{\nu}(2\in_{mij}\Omega_m-fs)(t-t_o)]\}\right]dk'+\frac{\delta_o}{\nu(t-t_o)}\int_0^\infty(k^3k'^5-k^5k'^3)$$

$$\left[\exp\left\{-\frac{\nu}{p_r}[(1+p_r)(k^2+k'^2)(t-t_o)+k^2\Delta t+2p_r kk'(t-t_o)+\frac{p_r}{\nu}(2\in_{mij}\Omega_m-fs)(t-t_o)]\right\}\right]d\hat{k}'.$$

$$\text{------------- (2.14.15)}$$

Again integrating equation (2.14.15) with respect to k', we have

$$w = -\frac{\delta_o\sqrt{\pi}\,p_r^{5/2}}{4\nu^{3/2}(t-t_o)^{3/2}(1+p_r)^{5/2}}\exp\{-\frac{p_r}{\nu}\left(2\in_{mij}\Omega_m - fs\right)(t-t_o)\}\times\exp[\frac{-k^2\nu(1+2p_r)}{p_r(1+p_r)}$$

$$\times(t-t_o+\frac{1+p_r}{1+2p_r}\Delta t)]\times[\frac{15p_rk^4}{4\nu^2(t-t_o)^2(1+p_r)}+\{\frac{5p_r^2}{(1+p_r)^2}-\frac{3}{2}\}\frac{k^6}{\nu(t-t_o)}$$

$$+\{\frac{p_r^3}{(1+p_r)^3}-\frac{p_r}{1+p_r}\}k^8-\frac{\delta_o\sqrt{\pi}p_r^{5/2}}{4\nu^{3/2}(t-t_o+\Delta t)^{3/2}(1+p_r)^{5/2}}\exp\{\frac{p_r}{\nu}\left(2\in_{mij}\Omega_m - fs\right)$$

$$\times(t-t_o)\}\times\exp[\frac{-k^2\nu(1+2p_r)}{p_r(1+p_r)}\times(t-t_o+\frac{p_r}{1+2p_r}\Delta t)]\times[\frac{15p_rk^4}{4\nu^2(t-t_o+\Delta t)^2(1+p_r)}$$

$$+\{\frac{5p_r^2}{(1+p_r)^2}-\frac{3}{2}\}\frac{k^6}{\nu(t-t_o+\Delta t)}+\{\frac{p_r^3}{(1+p_r)^3}-\frac{p_r}{1+p_r}\}k^8]. \qquad \text{---------- (2.14.16)}$$

The series of equation (2.14.16) contains only even power of k and start with k^4. The quantity w is the contribution to the energy transfer arising from consideration of the three-point correlation equation.

If we integrate equation (2.14.16) for $\Delta t=0$ over all wave numbers, we find that

$$\int_0^\infty w\,dk = 0 \qquad \text{----------- (2.14.17)}$$

which indicates that the expression for w satisfies the condition of continuity and homogeneity. Physically it was to be expected, since w is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero.

The linear equation (2.14.13) can be solved to give

$$E=\exp[-\frac{2\nu}{p_r}k^2(t-t_o+\frac{\Delta t}{2})]\int w\exp[2\frac{\nu}{p_r}k^2(t-t_o+\frac{\Delta t}{2})]dt+J(k)\exp[-\frac{2\nu}{p_r}k^2(t-t_o+\frac{\Delta t}{2})] \quad \text{-(2.14.18)}$$

where $J(k)=\dfrac{N_o k^2}{\pi}$ is a constant of integration and can be obtained as by Corrsin [24].

Substituting the values of w from (2.14.16) and J(k) into the equation (2.14.18) and integrating with respect to t_0 we get

$$E=\frac{N_o k^2}{\pi}\exp[-2\frac{\nu}{p_r}k^2(t-t_o+\frac{\Delta t}{2})]+\frac{\delta_o\sqrt{\pi}p_r^{5/2}}{4\nu^{3/2}(1+p_r)^{7/2}}\exp[-(2\in_{mij}\Omega_m-fs)(t-t_o)]$$

$$\times\exp[\frac{-k^2\nu(1+2p_r)}{p_r(1+p_r)}(t-t_o+\frac{1+p_r}{1+2p_r}\Delta t)]\times[\frac{3p_r k^4}{2\nu^2(t-t_o)^{5/2}}+\frac{p_r(7p_r-6)k^6}{3\nu(1+p_r)(t-t_o)^{3/2}}$$

$$-\frac{4(3p^2{}_r-2p_r+3)k^8}{3(1+p_r)^2(t-t_o)^{1/2}}+\frac{8\sqrt{\nu}(3p_r^2-2p_r+3)k^9}{3(1+p_r)^{5/2}p_r^{1/2}}F(\eta)+\frac{\delta_o\sqrt{\pi}p_r^{5/2}}{4\nu^{3/2}(1+p_r)^{7/2}}$$

$$\times\exp[-(2\in_{mij}\Omega_m-fs)(t-t_o)]\times\exp[\frac{-\nu k^2(1+2p_r)}{p_r(1+p_r)}(t-t_o+\frac{p_r}{1+2p_r}\Delta t)\times[\frac{3p_r k^4}{2\nu^2(t-t_o+\Delta t)^{5/2}}]$$

$$+\frac{p_r(7p_r-6)k^6}{3\nu(1+p_r)(t-t_o+\Delta t)^{3/2}}-\frac{4}{3}\frac{(3p^2{}_r-2p_r+3)k^8}{(1+p_r)^2(t-t_o+\Delta t)^{1/2}}+\frac{8\sqrt{\nu}(3p_r^2-2p_r+3)k^9}{3(1+p_r)^{5/2}p_r^{1/2}}F(\eta),\quad \text{-(2.14.19)}$$

where $F(\eta)=\bar{e}^{\eta^2}\int_0^{\eta}e^{x^2}dx$,

$$\eta=k\sqrt{\frac{\nu(t-t_o)}{p_r(1+p_r)}}\qquad\text{or}\qquad\eta=k\sqrt{\frac{\nu(t-t_o+\Delta t)}{p_r(1+p_r)}}.$$

By letting $\hat{r}=0$ in equation (2.12.8) and use is made of the definition of E, the result is

$$\frac{\langle TT'\rangle}{2}=\frac{\langle T^2\rangle}{2}=\int_0^{\infty}Edk. \qquad \text{--------- (2.14.20)}$$

Substituting equation (2.14.19) into equation (2.14.20) and integrating with respect to k, gives

$$\frac{\langle T^2\rangle}{2}=\frac{N_o p_r^{3/2}(T_0+\frac{\Delta T_0}{2})^{-3/2}}{8\sqrt{2\pi}\nu^{3/2}}+\frac{\pi\delta_o p_r^6}{4\nu^6(1+p_r)(1+2p_r)^{5/2}}\exp[-(2\in_{mij}\Omega_m-fs)]$$

$$[\frac{9}{16T^{5/2}(T_0+\frac{1+p_r}{1+2p_r}\Delta T_0)^{5/2}}+\frac{9}{16(T_0+\Delta T_0)^{5/2}(T_0+\frac{p_r}{1+2p_r}\Delta T_0)^{5/2}}$$

$$+\frac{5p_r(7p_r-6)}{16(1+2p_r)(T_0+\Delta T_0)^{3/2}(T_0+\frac{p_r}{1+2p_r}\Delta T_0)^{7/2}}+\frac{5p_r(7p_r-6)}{16(1+2p_r)T_0^{3/2}(T_0+\frac{1+p_r}{1+2p_r}\Delta T_0)^{7/2}}$$

$$+\frac{35p_r(3p_r^2-2p_r+3)}{8(1+2p_r)(T_0+\Delta T_0)^{1/2}(T_0+\frac{p_r}{1+2p_r}\Delta T_0)^{9/2}}+\frac{35p_r(3p_r^2-2p_r+3)}{8(1+2p_r)T_0^{1/2}(T_0+\frac{1+p_r}{1+2p_r}\Delta T_0)^{9/2}}$$

$$+\frac{8p_r(3p_r^2-2p_r+3)(1+2p_r)^{5/2}}{3.2^{23/2}(1+p_r)^{11/2}}\sum_{n=0}^{\infty}\frac{1.3.5..........(2n+9)}{n!(2n+1)2^{2n}(1+p_r)^n}$$

$$\times\{\frac{T_0^{(2n+1)/2}}{(T_0+\Delta T_0/2)^{(2n+1)/2}}+\frac{(T_0+\Delta T_0)^{(2n+1)/2}}{(T_0+\Delta T_0/2)^{(2n+1)/2}}\}], \qquad \text{-------------} (2.14.21)$$

where $T_0=t-t_0$.

Equation (2.14.21) is the decay law of temperature energy fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in a rotating system in presence of dust particles.

2.15 Concluding Remarks :

In equation (2.14.21) we obtained the decay law of temperature fluctuation in homogeneous turbulence before the final period in a rotating system in presence of dust particles by neglecting the quadruple correlation terms in comparison with the third–order terms for the case of multi-point and multi-time. If the fluid is clean and the system is non rotating then f=0, and $\Omega_m=0$ the equation (2.14.21) becomes

$$\frac{\langle T^2\rangle}{2}=\frac{N_o p_r^{3/2}(T_0+\frac{\Delta T_0}{2})^{-3/2}}{8\nu^{3/2}\sqrt{2\pi}}+\frac{\pi\delta_o p_r^6}{4\nu^6(1+p_r)(1+2p_r)^{5/2}}\times[\frac{9}{16T^{5/2}(T_0+\frac{1+p_r}{1+2p_r}\Delta T_0)^{5/2}}$$

$$+\frac{9}{16(T_0+\Delta T_0)^{5/2}(T_0+\frac{p_r}{1+2p_r}\Delta T_0)^{5/2}}+\frac{5p_r(7p_r-6)}{16(1+2p_r)T_0^{3/2}(T_0+\frac{1+p_r}{1+2p_r}\Delta T_0)^{7/2}}$$

$$+\frac{5p_r(7p_r-6)}{16(1+2p_r)(T_0+\Delta T_0)^{3/2}(T_0+\frac{p_r}{1+2p_r}\Delta T_0)^{7/2}}+\frac{35p_r(3p_r^2-2p_r+3)}{8(1+2p_r)T_0^{1/2}(T_0+\frac{1+p_r}{1+2p_r}\Delta T_0)^{9/2}}$$

$$+\frac{35p_r(3p_r^2-2p_r+3)}{8(1+2p_r)(T_0+\Delta T_0)^{1/2}(T_0+\frac{p_r}{1+2p_r}\Delta T_0)^{9/2}}+\frac{8p_r(3p_r^2-2p_r+3)(1+2p_r)^{5/2}}{3.2^{23/2}(1+p_r)^{11/2}}$$

$$\times\sum_{n=0}^{\infty}\frac{1.3.5..........(2n+9)}{n!(2n+1)2^{2n}(1+p_r)^n}\times\{\frac{T_0^{(2n+1)/2}}{(T_0+\Delta T_0/2)^{(2n+1)/2}}+\frac{(T_0+\Delta T_0)^{(2n+1)/2}}{(T_0+\Delta T_0/2)^{(2n+1)/2}}\}], \quad \text{---------} (2.15.1)$$

which was obtained earlier by Sarker and Islam [116].

If we put $\Delta T_0 = 0$ in equation (2.15.1), we can easily find out

$$\frac{\langle T^2\rangle}{2}=\frac{N_o p_r^{3/2}T_0^{-3/2}}{8\sqrt{2\pi}v^{3/2}}+\frac{\pi\delta_o p_r^6 T_0^{-5}}{4v^6(1+p_r)(1+2p_r)^{5/2}}\times[\frac{9}{16}+\frac{5}{16}.\frac{p_r(7p_r-6)}{1+2p_r}+\frac{35}{8}\frac{p_r(3p_r^2-2p_r+3)}{(1+2p_r)^2}+--]$$

$$=AT_0^{-3/2}+BT_0^{-5}=A(t-t_0)^{-3/2}+B(t-t_0)^{-5}, \quad \text{------------} (2.15.2)$$

where $A=\frac{N_o p_r^{3/2}}{8\sqrt{2\pi}v^{3/2}}$ and

$$B=\frac{\pi\delta_o p_r^6}{2v^6(1+p_r)(1+2p_r)^{5/2}}\times[\frac{9}{16}+\frac{5}{16}.\frac{p_r(7p_r-6)}{1+2p_r}+\frac{35}{8}.\frac{p_r(3p_r^2-2p_r+3)}{(1+2p_r)^2}+..........]$$

which was obtained earlier by Loeffler and Deissler [72].

In this problem, due to rotation (of the fluid) in presence of dust particles, the temperature energy fluctuation decays more rapidly than the energy for non-rotating clean fluid for times before the final period.

If higher order correlation equations were considered in the analysis it appears that more terms of higher power of time would be added to the equation (2.14.21). For large times, the second term in the equation becomes negligible leaving the -3/2 power decay law for the final period.

CHAPTER-III

STATISTICAL THEORY OF CERTAIN DISTRIBUTION FUNCTIONS IN MHD TURBULENT FLOW IN A ROTATING SYSTEM IN PRESENCE OF DUST PARTICLES

3.1 Introduction:

Several authors discuss the distribution functions in the statistical theory of turbulence in the past. Lundgren [74] derived a hierarchy of coupled equations for multi-point turbulence velocity distribution functions, which resemble with BBGKY hierarchy of equations of Ta-Yu-Wu [131] in the kinetic theory of gasses; Kishore [51] studied the distributions functions in the statistical theory of MHD turbulence of an incompressible fluid. Pope [100] derived the transport equation for the joint probability density function of velocity and scalars in turbulent flow. Kishore and Singh [53] derived the transport equation for the bivariate joint distribution function of velocity and temperature in turbulent flow. Also Kishore and Singh [55] have been derived the transport equation for the joint distribution function of velocity, temperature and concentration in convective turbulent flow. Dixit and Upadhyay [31] considered the distribution functions in the statistical theory of MHD turbulence of an incompressible fluid in the presence of the coriolis force. Kollman and Janicka [66] derived the transport equation for the probability density function of a scalar in turbulent shear flow and considered a closure model based on gradient –flux model.

But at this stage, one is met with the difficulty that the N-point distribution function depends upon the N+1-point distribution function and thus result is an unclosed system. This so-called "closer problem" is encountered in turbulence, kinetic theory and other non-linear system. Sarker and Kishore [107] discussed the distribution functions in the statistical theory of convective MHD turbulence of an incompressible fluid. Also Sarker and Kishore [114] studied the distribution functions in the statistical theory of convective MHD turbulence of mixture of a miscible incompressible fluid.

In this paper, an attempt is made to study the distribution function for simultaneous velocity, magnetic, temperature and concentration fields in MHD turbulence in a rotating system in presence of dust particles. Finally, the transport equations for evolution of distribution functions have been derived and various properties of the distribution function have been discussed. The resulting one-point equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

3.2 Basic Equations:

The equations of motion and continuity for viscous incompressible dusty fluid MHD turbulent flow, the diffusion equations for the temperature and concentration in a rotating system are given by

$$\frac{\partial u_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(u_\alpha u_\beta-h_\alpha h_\beta\right)=-\frac{\partial w}{\partial x_\alpha}+\nu\nabla^2 u_\alpha-2\in_{m\alpha\beta}\Omega_m u_\alpha+f\left(u_\alpha-v_\alpha\right), \quad \text{---------} (3.2.1)$$

$$\frac{\partial h_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(h_\alpha u_\beta-u_\alpha h_\beta\right)=\lambda\nabla^2 h_\alpha\ , \quad \text{---------} (3.2.2)$$

$$\frac{\partial \theta}{\partial t}+u_\beta\frac{\partial \theta}{\partial x_\beta}=\gamma\nabla^2\theta\ , \quad \text{---------} (3.2.3)$$

$$\frac{\partial c}{\partial t}+u_\beta\frac{\partial c}{\partial x_\beta}=D\nabla^2 c \quad \text{---------} (3.2.4)$$

with $\frac{\partial u_\alpha}{\partial x_\alpha}=\frac{\partial v_\alpha}{\partial x_\alpha}=\frac{\partial h_\alpha}{\partial x_\alpha}=0\ ,$ ---------- (3.2.5)

where

$u_\alpha(x,t)$, α – component of turbulent velocity

$h_\alpha(x,t)$, α – component of magnetic field

$\theta(x,t)$, temperature fluctuation

c, concentration of contaminants

v_α, dust particle velocity

$\in_{m\alpha\beta}$, alternating tensor

$f = \frac{KN}{\rho}$, dimension of frequency

N, constant number of density of the dust particle

$w(\hat{x},t) = P/_{\rho} + \frac{1}{2}|\hat{h}|^2 + \frac{1}{2}|\hat{\Omega} \times \hat{x}|^2$, total pressure

$P(\hat{x},t)$, hydrodynamic pressure

ρ, fluid density

Ω, angular velocity of a uniform rotation

ν, Kinetic viscosity

$\lambda = (4\pi\mu\sigma)^{-1}$, magnetic diffusivity

$\gamma = \frac{k_T}{\rho c_p}$, thermal diffusivity,

c_p, specific heat at constant pressure,

k_T, thermal conductivity

σ, electrical conductivity

μ, magnetic permeability

D, diffusive co-efficient for contaminants.

The repeated suffices are assumed over the values 1, 2 and 3 and unrepeated suffices may take any of these values. Here u, h and x are vector quantities in the whole process.

The total pressure w which, occurs in equation (3.2.1) may be eliminated with the help of the equation obtained by taking the divergence of equation (3.2.1)

$$\nabla^2 w = -\frac{\partial^2}{\partial x_\alpha \partial x_\beta}(u_\alpha u_\beta - h_\alpha h_\beta) = -[\frac{\partial u_\alpha}{\partial x_\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \frac{\partial h_\alpha}{\partial x_\beta}\frac{\partial h_\beta}{\partial x_\alpha}]. \quad \text{---------} (3.2.6)$$

In a conducting infinite fluid only the particular solution of the Equation (3.2.6) is related, so that

$$w = \frac{1}{4\pi}\int [\frac{\partial u'_\alpha}{\partial x'_\beta}\frac{\partial u'_\beta}{\partial x'_\alpha} - \frac{\partial h'_\alpha}{\partial x'_\beta}\frac{\partial h'_\beta}{\partial x'_\alpha}]\frac{\partial \bar{x}'}{|\bar{x}' - \bar{x}|}. \quad \text{----------} (3.2.7)$$

Hence equation (3.2.1) – (3.2.4) becomes

$$\frac{\partial u_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(u_\alpha u_\beta - h_\alpha h_\beta\right)=-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha}\int\left[\frac{\partial u'_\alpha}{\partial x'_\beta}\frac{\partial u'_\beta}{\partial x'_\alpha}-\frac{\partial h'_\alpha}{\partial x'_\beta}\frac{\partial h'_\beta}{\partial x'_\alpha}\right]\frac{d\bar{x}'}{|\bar{x}'-\bar{x}|}+\nu\nabla^2 u_\alpha$$

$$-2\in_{m\alpha\beta}\Omega_m u_\alpha + f\left(u_\alpha - v_\alpha\right), \qquad \text{------------- (3.2.8)}$$

$$\frac{\partial h_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(h_\alpha u_\beta - u_\alpha h_\beta\right)=\lambda\nabla^2 h_\alpha, \qquad \text{------------- (3.2.9)}$$

$$\frac{\partial \theta}{\partial t}+u_\beta\frac{\partial \theta}{\partial x_\beta}=\gamma\nabla^2\theta, \qquad \text{----------- (3.2.10)}$$

$$\frac{\partial c}{\partial t}+u_\beta\frac{\partial c}{\partial x_\beta}=D\nabla^2 c. \qquad \text{----------- (3.2.11)}$$

3.3 Formulation of the Problem:

We consider a large ensemble of identical fluids in which each member is an infinite incompressible heat conducting fluid in turbulent state. The fluid velocity u, Alfven velocity h, temperature θ and concentration c, are randomly distributed functions of position and time and satisfy their field. Different members of ensemble are subjected to different initial conditions and our aim is to find out a way by which we can determine the ensemble averages at the initial time.

Certain microscopic properties of conducting fluids, such as total energy, total pressure, stress tensor which are nothing but ensemble averages at a particular time, can be determined with the help of the bivariate distribution functions (defined as the averaged distribution functions with the help of Dirac delta-functions). Our present aim is to construct the bivariate distribution functions, study its properties and derive an equation for its evolution of this distribution function.

3.4 Distribution Function in MHD Turbulence and Their Properties:

Lundgren [74] has studied the flow field on the basis of one variable character only (namely the fluid u), but we can study it for two or more variable characters as well. In MHD turbulence, we may consider the fluid velocity u, Alfven velocity h, temperature θ and concentration c at each point of the flow field. Then corresponding to each point of the flow field, we have four measurable characteristics. We represent the four variables by v, g, ϕ and ψ and denote the pairs of these variables at the points $\bar{x}^{(1)}, \bar{x}^{(2)}, ------, \bar{x}^{(n)}$ as $\left(\bar{v}^{(1)}, \bar{g}^{(1)}, \phi^{(1)}, \psi^{(1)}\right), \left(\bar{v}^{(2)}, \bar{g}^{(2)}, \phi^{(2)}, \psi^{(2)}\right), ------ \left(\bar{v}^{(n)}, \bar{g}^{(n)}, \phi^{(n)}, \psi^{(n)}\right)$ at a fixed instant of time.

It is possible that the same pair may be occur more than once; therefore, we simplify the problem by an assumption that the distribution is discrete (in the sense that no pairs occur more than once). Symbolically we can express the bivariate distribution as

$$\left\{ \left(\bar{v}^{(1)}, \bar{g}^{(1)}, \phi^{(1)}, \psi^{(1)}\right), \left(\bar{v}^{(2)}, \bar{g}^{(2)}, \phi^{(2)}, \psi^{(2)}\right), ------ \left(\bar{v}^{(n)}, \bar{g}^{(n)}, \phi^{(n)}, \psi^{(n)}\right) \right\}.$$

Instead of considering discrete points in the flow field, if we consider the continuous distribution of the variables $\bar{v}, \bar{g}, \phi$ and ψ over the entire flow field, statistically behaviour of the fluid may be described by the distribution function $F(\bar{v}, \bar{g}, \phi, \psi)$ which is normalized so that

$$\int F(\bar{v}, \bar{g}, \phi, \psi) d\bar{v}, d\bar{g} d\phi d\psi = 1$$

where the integration ranges over all the possible values of v, g, ϕ and ψ. We shall make use of the same normalization condition for the discrete distributions also.

The distribution functions of the above quantities can be defined in terms of Dirac delta function.

The one-point distribution function $F_1^{(1)}\left(v^{(1)}, g^{(1)}, \phi^{(1)}, \psi^{(1)}\right)$, defined so that $F_1^{(1)}\left(v^{(1)}, g^{(1)}, \phi^{(1)}, \psi^{(1)}\right) dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)}$ is the probability that the fluid velocity, Alfven velocity, temperature and concentration at a time t are in the element $dv^{(1)}$ about $v^{(1)}$, $dg^{(1)}$ about $g^{(1)}$, $d\phi^{(1)}$ about $\phi^{(1)}$ and $d\psi^{(1)}$ about $\psi^{(1)}$ respectively and is given by

$$F_1^{(1)}\left(v^{(1)}, g^{(1)}, \phi^{(1)}, \psi^{(1)}\right) = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \right\rangle \quad \text{--------} (3.4.1)$$

where δ is the Dirac delta-function defined as

$$\int \delta(\bar{u} - \bar{v}) d\bar{v} = \begin{cases} 1 & \text{at the point } \bar{u} = \bar{v} \\ 0 & \text{elsewhere} \end{cases}.$$

Two-point distribution function is given by

$$F_2^{(1,2)} = \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})$$

$$\delta(\theta^{(2)} - \phi^{(2)})\delta(c^{(2)} - \psi^{(2)}) \rangle \qquad \text{------------ (3.4.2)}$$

and three point distribution function is given by

$$F_3^{(1,2,3)} = \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})$$

$$\times \delta(\theta^{(2)} - \phi^{(2)})\delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \phi^{(3)})\delta(c^{(3)} - \psi^{(3)}) \rangle . \qquad \text{-- (3.4.3)}$$

Similarly, we can define an infinite numbers of multi-point distribution functions $F_4^{(1,2,3,4)}$, $F_5^{(1,2,3,4,5)}$ and so on.

The following properties of the constructed distribution functions can be deduced from the above definitions:

(A) Reduction Properties:

Integration with respect to pair of variables at one-point, lowers the order of distribution function by one. For example,

$$\int F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} = 1 ,$$

$$\int F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} = F_1^{(1)} ,$$

$$\int F_3^{(1,2,3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} = F_2^{(1,2)}$$

and so on. Also the integration with respect to any one of the variables, reduces the number of Delta-functions from the distribution function by one as

$$\int F_1^{(1)} dv^{(1)} = \langle \delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)}) \rangle ,$$

$$\int F_1^{(1)} dg^{(1)} = \langle \delta(u^{(1)} - v^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)}) \rangle ,$$

$$\int F_1^{(1)} d\phi^{(1)} = \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(c^{(1)} - \psi^{(1)}) \rangle$$

and

$$\int F_2^{(1,2)} dv^{(2)} = \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)})$$

$$\delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \rangle .$$

(B) Separation Properties:

If two points are far apart from each other in the flow field, the pairs of variables at these points are statistically independent of each other i.e.,

$$\lim_{|\bar{x}^{(2)} \to \bar{x}^{(1)}| \to \infty} F_2^{(1,2)} = F_1^{(1)} F_1^{(2)}$$

and similarly,

$$\lim_{|\bar{x}^{(3)} \to \bar{x}^{(2)}| \to \infty} F_3^{(1,2,3)} = F_2^{(1,2)} F_1^{(3)} \quad \text{etc.}$$

(C) Co-incidence Property:

When two points coincide in the flow field, the components at these points should be obviously the same that is $F_2^{(1,2)}$ must be zero. Thus $\bar{v}^{(2)} = \bar{v}^{(1)}$, $g^{(2)} = g^{(1)}$, $\phi^{(2)} = \phi^{(1)}$ and $\psi^{(2)} = \psi^{(1)}$, but $F_1^{(1,2)}$ must also have the property.

$$\int F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} = F_1^{(1)}$$

and hence it follows that

$$\lim_{|\bar{x}^{(2)} \to \bar{x}^{(1)}| \to \infty} \int F_2^{(1,2)} = F_1^{(1)} \delta(v^{(2)} - v^{(1)}) \delta(g^{(2)} - g^{(1)}) \delta(\phi^{(2)} - \phi^{(1)}) \delta(\psi^{(2)} - \psi^{(1)}).$$

Similarly,

$$\lim_{|\bar{x}^{(3)} \to \bar{x}^{(2)}| \to \infty} \int F_3^{(1,2,3)} = F_2^{(1,2)} \delta(v^{(3)} - v^{(1)}) \delta(g^{(3)} - g^{(1)}) \delta(\phi^{(3)} - \phi^{(1)}) \delta(\psi^{(3)} - \psi^{(1)}) \quad \text{etc.}$$

(D) Symmetric Conditions:

$$F_n^{(1,2,r,------s,-----n)} = F_n^{(1,2,-----s,---r,---n)}.$$

(E) Incompressibility Conditions:

(i) $$\int \frac{\partial F_n^{(1,2,---n)}}{\partial x_\alpha^{(r)}} v_\alpha^{(r)} d\overline{v}^{(r)} d\overline{h}^{(r)} = 0,$$

(ii) $$\int \frac{\partial F_n^{(1,2,---n)}}{\partial x_\alpha^{(r)}} h_\alpha^{(r)} d\overline{v}^{(r)} d\overline{h}^{(r)} = 0.$$

3.5 Continuity Equation in Terms of Distribution Functions:

The continuity equations can be easily expressed in terms of distribution functions. An infinite number of continuity equations can be derived for the convective MHD turbulent flow and are obtained directly by div $u = 0$

Taking ensemble average of equation (3.2.5)

$$0 = \left\langle \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \right\rangle = \left\langle \frac{\partial}{\partial x_\alpha^{(1)}} u_\alpha^{(1)} \int F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \right\rangle$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \left\langle u_\alpha^{(1)} \int F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \right\rangle$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \int \langle u_\alpha^{(1)} \rangle \langle F_1^{(1)} \rangle dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)}$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \int v_\alpha^{(1)} F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)}$$

$$= \int \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} v_\alpha^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \qquad \text{------------ (3.5.1)}$$

and similarly,

$$0 = \int \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} g_\alpha^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \qquad \text{------------ (3.5.2)}$$

which are the first order continuity equations in which only one point distribution function is involved.

For second-order continuity equations, if we multiply the continuity equation by

$$\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

and if we take the ensemble average, we obtain

$$o=\left\langle\ \delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}}\ \right\rangle$$

$$=\frac{\partial}{\partial x_\alpha^{(1)}}\left\langle\ \delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)u_\alpha^{(1)}\ \right\rangle$$

$$=\frac{\partial}{\partial x_\alpha^{(1)}}\int\left\langle\ u_\alpha^{(1)}\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\right.$$

$$\left.\times\,\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\right\rangle \qquad \text{----------}\ (3.5.3)$$

and similarly,

$$o=\frac{\partial}{\partial x_\alpha^{(1)}}\int g_\alpha^{(1)}F_2^{(1,2)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)}. \qquad \text{-----------}\ (3.5.4)$$

The Nth – order continuity equations are

$$o=\frac{\partial}{\partial x_\alpha^{(1)}}\int v_\alpha^{(1)}F_N^{(1,2,---,N)}dv^{(1)}d\phi^{(1)}d\psi^{(1)} \qquad \text{----------}\ (3.5.5)$$

and

$$o=\frac{\partial}{\partial x_\alpha^{(1)}}\int g_\alpha^{(1)}F_N^{(1,2,.........,N)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)}. \qquad \text{-----------}\ (3.5.6)$$

The continuity equations are symmetric in their arguments i.e.;

$$\frac{\partial}{\partial x_\alpha^{(r)}}\left(v_\alpha^{(r)}F_N^{(1,2,..........;r.N)}dv^{(r)}dg^{(r)}d\psi^{(r)}\right)=\frac{\partial}{\partial x_\alpha^{(s)}}\int v_\alpha^{(s)}F_N^{(1,2,.....r,s,.....N)}dv^{(s)}dg^{(s)}d\phi^{(s)}d\psi^{(s)}. \text{-}(3.5.7)$$

Since the divergence property is an important property and it is easily verified by the use of the property of distribution function as

$$\frac{\partial}{\partial x_\alpha}\int v_\alpha^{(1)} F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \frac{\partial}{\partial x_\alpha^{(1)}}\langle\ u_\alpha^{(1)}\ \rangle = \langle\ \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha(1)}\ \rangle = o$$

and all the properties of the distribution function obtained in section (4) can also be verified.

3.6 Equations for Evolution of Distribution Functions:

We shall make use of equation (3.2.8)-(3.2.11) to convert these into a set of equations for the variation of the distribution function with time. This, in fact, is done by making use of the definitions of the constructed distribution functions, differentiating them partially with respect to time, making some suitable operations on the right-hand side of the equation so obtained and lastly replacing the time derivative of v, h, θ and c from the equations (3.2.8)-(3.2.11).

Differentiating equation (3.4.1), and then using equations (3.2.8)-(3.2.11) we get,

$$\frac{\partial F_1^{(1)}}{\partial t} = \frac{\partial}{\partial t}\langle\ \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\ \rangle$$

$$= \langle\ \delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(u^{(1)} - v^{(1)}\right)\ \rangle + \langle\ \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)$$

$$\times\delta\left(c^{(1)} - \psi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(h^{(1)} - g^{(1)}\right)\ \rangle + \langle\ \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(c^{(1)} - \psi^{(1)}\right)\ \rangle$$

$$= \langle -\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\frac{\partial u^{(1)}}{\partial t}\frac{\partial}{\partial v^{(1)}}\delta\left(u^{(1)} - v^{(1)}\right)\ \rangle$$

$$+ \langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\frac{\partial h^{(1)}}{\partial t}\frac{\partial}{\partial g^{(1)}}\delta\left(h^{(1)} - g^{(1)}\right)\ \rangle$$

$$+ \langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\frac{\partial \theta^{(1)}}{\partial t}\frac{\partial}{\partial \phi^{(1)}}\delta\left(\theta^{(1)} - \phi^{(1)}\right)\ \rangle$$

$$+ \langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\frac{\partial c^{(1)}}{\partial t}\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)} - \psi^{(1)}\right)\ \rangle. \qquad \text{------ -- (3.6.1)}$$

Using equations (3.2.8) – (3.2.11) in the equation (3.6.1), we get

$$\frac{\partial F_1^{(1)}}{\partial t} = \langle -\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\{ \; -\frac{\partial}{\partial x_\beta}\left(u_\alpha^{(1)} u_\beta^{(1)} - h_\alpha^{(1)} h_\beta^{(1)}\right)$$

$$-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha}\int [\; \frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}} - \frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}} \;]\frac{d\bar{x}'}{|\bar{x}' - \bar{x}|} + \nu\nabla^2 u_\alpha^{(1)} - 2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)} + f\left(u_\alpha^{(1)} - v_\alpha^{(1)}\right) \}$$

$$\times \frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)} - v^{(1)}\right) \rangle + \langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\{ \; -\frac{\partial}{\partial x_\beta^{(1)}}\left(h_\alpha^{(1)} u_\beta^{(1)} - u_\alpha^{(1)} h_\beta^{(1)}\right)$$

$$+ \lambda\nabla^2 h_\alpha^{(1)} \; \}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)} - g^{(1)}\right) \rangle + \langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)$$

$$\times \{ -u_\beta^{(1)}\frac{\partial \theta^{(1)}}{\partial x_\beta} + \gamma\nabla^2\theta^{(1)} \; \} \times \frac{\partial}{\partial \phi^{(1)}}\delta\left(\theta^{(1)} - \phi^{(1)}\right) \rangle + \langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)$$

$$\times \delta\left(\theta^{(1)} - \phi^{(1)}\right)\{ -u_\beta^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}} + D\nabla^2 c \; \}\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)} - \psi^{(1)}\right) \rangle$$

$$= \langle \; \delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\frac{\partial u_\alpha^{(1)} u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)} - v^{(1)}\right) \rangle$$

$$+ \langle -\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\frac{\partial h_\alpha^{(1)} h_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha}\delta\left(u^{(1)} - v^{(1)}\right) \rangle$$

$$+ \langle \; \delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}\int [\; \frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}} - \frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}} \;]$$

$$\times \frac{d\bar{x}'}{|\bar{x}' - \bar{x}|}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)} - v^{(1)}\right) \rangle + \langle -\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\nu\nabla^2 u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)} - v^{(1)}\right) \rangle$$

$$+ \langle \; \delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)} - v^{(1)}\right) \rangle$$

$$+ \langle -\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)f\left(u_\alpha^{(1)} - v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)} - v^{(1)}\right) \rangle$$

$$+ \langle \; \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\frac{\partial h_\alpha^{(1)} u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)} - g^{(1)}\right) \rangle$$

$$+\langle -\delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\frac{\partial u_\alpha^{(1)}h_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha}\delta(h^{(1)}-g^{(1)})\rangle$$

$$+\langle -\delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\lambda\nabla^2 h_\alpha^{(1)}\frac{\partial}{\partial g_\alpha^{(1)}}\delta(h^{(1)}-g^{(1)})\rangle$$

$$+\langle \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(c^{(1)}-\psi^{(1)})u_\beta^{(1)}\frac{\partial \theta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial \phi^{(1)}}\delta(\theta^{(1)}-\phi^{(1)})\rangle$$

$$+\langle -\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(c^{(1)}-\psi^{(1)})\gamma\nabla^2\theta^{(1)}\frac{\partial}{\partial \phi^{(1)}}\delta(\theta^{(1)}-\phi^{(1)})\rangle$$

$$+\langle \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})u_\beta^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial \psi^{(1)}}\delta(c^{(1)}-\psi^{(1)})\rangle$$

$$+\langle -\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})D\nabla^2 c^{(1)}\frac{\partial}{\partial \psi^{(1)}}\delta(c^{(1)}-\psi^{(1)})\rangle. \qquad \text{----------- (3.6.2)}$$

Various terms in the above equation can be simplified as that they may be expressed in terms of one point and two point distribution functions.

The 1[st] term in the above equation is simplified as follows:

$$\langle \delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\frac{\partial u_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta(u^{(1)}-v^{(1)})\rangle$$

$$=\langle u_\beta^{(1)}\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta(u^{(1)}-v^{(1)})\rangle$$

$$=\langle -u_\beta^{(1)}\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}\delta(u^{(1)}-v^{(1)})\rangle$$

$$=\langle -u_\beta^{(1)}\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\frac{\partial}{\partial x_\beta^{(1)}}\delta(u^{(1)}-v^{(1)})\rangle, \quad \left(\text{since } \frac{\partial u_\alpha^{(1)}}{\partial v_\alpha^{(1)}}=1\right)$$

$$=\langle -\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})u_\beta^{(1)}\frac{\partial}{\partial x_\alpha^{(1)}}\delta(u^{(1)}-v^{(1)})\rangle. \qquad \text{----------- (3.6.3)}$$

Similarly, seventh, tenth and twelfth terms of right hand-side of equation (3.6.2) can be simplified as follows;

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial h_{\alpha}^{(1)}u_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial g_{\alpha}^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

$$=\left\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\right\rangle . \quad \text{----------- (3.6.4)}$$

Tenth term,

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial\theta^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle$$

$$=\left\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right\rangle \quad \text{----------- (3.6.5)}$$

and twelfth term

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial c^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$=\left\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle . \quad \text{----------- (3.6.6)}$$

Adding (3.6.3) – (3.6.6), we get

$$\left\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$

$$+\left\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\right\rangle$$

$$+\left\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right\rangle$$

$$+\left\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle$$

$$=-\frac{\partial}{\partial x_{\beta}^{(1)}}\left\langle u_{\beta}^{(1)}\left\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle\right.$$

$$=-\frac{\partial}{\partial x_{\beta}^{(1)}}v_{\beta}^{(1)}F_{1}^{(1)}$$ [Applying the properties of distribution functions]

$$= -v_\beta^{(1)} \frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}} \quad . \qquad \text{-----------} (3.6.7)$$

Similarly second and eighth terms on the right hand-side of the equation (3.6.2) can be simplified as

$$\left\langle -\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\frac{\partial h_\alpha^{(1)} h_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial v_\alpha^{(1)}} \delta\left(u^{(1)} - v^{(1)}\right) \right\rangle$$

$$= -g_\beta^{(1)} \frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}} \frac{\partial}{\partial x_\beta^{(1)}} F_1^{(1)} \qquad \text{-----------} (3.6.8)$$

and

$$\left\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\frac{\partial u_\alpha^{(1)} h_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial g_\alpha} \delta\left(h^{(1)} - g^{(1)}\right) \right\rangle \qquad \text{-----------} (3.6.9)$$

$$= -g_\beta^{(1)} \frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}} \frac{\partial}{\partial x_\beta^{(1)}} F_1^{(1)} \quad .$$

Fourth term can be reduced as

$$\left\langle -\nu\nabla^2 u_\alpha^{(1)} \delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(1)}} \delta\left(u^{(1)} - v^{(1)}\right) \right\rangle$$

$$= -\nu \frac{\partial}{\partial v_\alpha^{(1)}} \left\langle \nabla^2 u_\alpha^{(1)} \left[\ \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\ \right] \right\rangle$$

$$= -\nu \frac{\partial}{\partial v_\alpha^{(1)}} \frac{\partial^2}{\partial x_\beta^{(1)} \partial x_\beta^{(1)}} \left\langle u_\alpha^{(1)} \left[\ \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\ \right] \right\rangle$$

$$= -\nu \frac{\partial}{\partial v_\alpha^{(1)}} \lim_{\bar{x}^{(2)} \to \bar{x}^{(1)}} \frac{\partial^2}{\partial x_\beta^{(2)} \partial x_\beta^{(2)}} \left\langle u_\alpha^{(2)} \left[\ \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\ \right] \right\rangle$$

$$= -\nu \frac{\partial}{\partial v_\alpha^{(1)}} \lim_{\bar{x}^{(2)} \to \bar{x}^{(1)}} \frac{\partial^2}{\partial x_\beta^{(2)} \partial x_\beta^{(2)}} \left\langle \int u_\alpha^{(2)} \delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right) \right.$$

$$\times \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right) dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)}$$

lim

$$= -\nu \frac{\partial}{\partial v_\alpha^{(1)}} \lim_{\bar{x}^{(2)} \to \bar{x}^{(1)}} \frac{\partial^2}{\partial x_\beta^{(2)} \partial x_\beta^{(2)}} \int v_\alpha^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} . \qquad \text{---------- (3.6.10)}$$

Ninth, eleventh and thirteen terms of the right hand side of equation (3.6.2)

$$\langle -\delta(u^{(1)} - v^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)})\lambda\nabla^2 h_\alpha^{(1)} \frac{\partial}{\partial g_\alpha^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle$$

$$= \langle -\lambda\nabla^2 h_\alpha^{(1)} \frac{\partial}{\partial g_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)})\delta(\theta^{(1)} - \phi^{(1)})\delta(c^{(1)} - \psi^{(1)}) \frac{\partial}{\partial g_\alpha^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle$$

$$= -\lambda \frac{\partial}{\partial g_\alpha^{(1)}} \lim_{\bar{x}^{(2)} \to \bar{x}^{(1)}} \frac{\partial^2}{\partial x_\beta^{(2)} \partial x_\beta^{(2)}} \int g_\alpha^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} , \qquad \text{----------- (3.6.11)}$$

$$\langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(c^{(1)} - \psi^{(1)})\gamma\nabla^2 \theta^{(1)} \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle$$

$$= \langle -\gamma\nabla^2 \theta^{(1)} \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(c^{(1)} - \psi^{(1)}) \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle$$

$$= -\gamma \frac{\partial}{\partial \phi^{(1)}} \lim_{\bar{x}^{(2)} \to \bar{x}^{(1)}} \frac{\partial^2}{\partial x_\beta^{(2)} \partial x_\beta^{(2)}} \int \phi^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} , \qquad \text{------- (3.6.12)}$$

$$\langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \phi^{(1)}) D\nabla^2 c^{(1)} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle$$

$$= \langle -D\nabla^2 c^{(1)} \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(c^{(1)} - \psi^{(1)}) \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle$$

$$= -D \frac{\partial}{\partial \psi^{(1)}} \lim_{\bar{x}^{(2)} \to \bar{x}^{(1)}} \frac{\partial^2}{\partial x_\beta^{(2)} \partial x_\beta^{(2)}} \int \psi^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} \qquad \text{--------- (3.6.13)}$$

respectively.

We reduce the third term of right hand side of equation (3.6.2)

$$\left\langle \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}\int\left[\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}}-\frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\right]\frac{d\bar{x}'}{\left|\bar{x}'-\bar{x}\right|}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$

$$=\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\int\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|\bar{x}^{(2)}-\bar{x}^{(1)}\right|}\right)\left(\frac{\partial v_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial v_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial g_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial g_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right)F_2^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}\right] \quad .(3.6.14)$$

Fifth and sixth terms of right hand side of equation (3.6.2)

$$\left\langle \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$

$$=\left\langle 2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\left[\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\right]\right\rangle$$

$$=2\in_{m\alpha\beta}\Omega_m\frac{\partial}{\partial v_\alpha^{(1)}}\left\langle u_\alpha^{(1)}\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle$$

$$=2\in_{m\alpha\beta}\Omega_m\frac{\partial u_\alpha^{(1)}}{\partial v_\alpha^{(1)}}\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle$$

$$=2\in_{m\alpha\beta}\Omega_m F_1^{(1)} \qquad \text{--------------} (3.6.15)$$

and

$$\left\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$

$$=-\left\langle f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(1)}}\left[\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\right]\right\rangle$$

$$=-f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(1)}}\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle$$

$$=-f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(1)}}F_1^{(1)} . \qquad \text{--------------} (3.6.16)$$

Substituting the results (3.6.3) – (3.6.16) in equation (3.6.2) we get the transport equation for one point distribution function $F_1^{(1)}(v,g,\phi,\psi)$ in MHD turbulent flow in a rotating system in presence of dust particles as

$$\frac{\partial F_1^{(1)}}{\partial t}+v_\beta^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\int\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|\overline{x}^{(2)}-\overline{x}^{(1)}\right|}\right)\right.$$

$$\times\left(\frac{\partial v_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial v_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial g_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial g_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right)F_2^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+v\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{\overline{x}^{(2)}\to\overline{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int v_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\lambda\frac{\partial}{\partial g_\alpha^{(1)}}\lim_{\overline{x}^{(2)}\to\overline{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int g_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\gamma\frac{\partial}{\partial \phi^{(1)}}\lim_{\overline{x}^{(2)}\to\overline{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int \phi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+D\frac{\partial}{\partial \psi^{(1)}}\lim_{\overline{x}^{(2)}\to\overline{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int \psi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+2\in_{m\alpha\beta}\Omega_m F_1^{(1)}+f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(1)}}F_1^{(1)}=0\ . \qquad \text{-------------- (3.6.17)}$$

Similarly, an equation for two-point distribution function $F_2^{(1,2)}$ in MHD dusty fluid turbulent flow in a rotating system can be derived by differentiating equation (3.4.2) and simplifying in the same manner, which is

$$\frac{\partial F_2^{(1,2)}}{\partial t}+\left(v_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}+v_\beta^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\right)F_2^{(1,2)}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial}{\partial x_\beta^{(1)}}F_2^{(1,2)}$$

$$+g_\beta^{(2)}\left(\frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}}+\frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}}\right)\frac{\partial}{\partial x_\beta^{(2)}}F_2^{(1,3)}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\int\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|\overline{x}^{(3)}-\overline{x}^{(1)}\right|}\right)\right.$$

$$\left.\times\left(\frac{\partial v_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial v_\beta^{(3)}}{\partial x_\alpha^{(3)}}-\frac{\partial g_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial g_\beta^{(3)}}{\partial x_\alpha^{(3)}}\right)F_3^{(1,2,3)}dx^{(3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{3)}\right]$$

$$-\frac{\partial}{\partial v_{\alpha}^{(2)}}\left[\ \frac{1}{4\pi}\int \frac{\partial}{\partial x_{\alpha}^{(2)}}\left(\ \frac{1}{\left|\overline{x}^{(3)}-\overline{x}^{(2)}\right|}\ \right)\left(\ \frac{\partial v_{\alpha}^{(3)}}{\partial x_{\beta}^{(3)}}\frac{\partial v_{\beta}^{(3)}}{\partial x_{\alpha}^{(3)}}-\frac{\partial g_{\alpha}^{(3)}}{\partial x_{\beta}^{(3)}}\frac{\partial g_{\beta}^{(3)}}{\partial x_{\alpha}^{(3)}}\ \right)\right.$$

$$\left.\times F_{3}^{(1,2,3)}dx^{(3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{3)}\ \right]$$

$$+\nu\left(\ \frac{\partial}{\partial v_{\alpha}^{(1)}}\lim_{\overline{x}^{(3)}\to\overline{x}^{(1)}}+\frac{\partial}{\partial v_{\alpha}^{(2)}}\lim_{\overline{x}^{(3)}\to\overline{x}^{(2)}}\ \right)\frac{\partial^{2}}{\partial x_{\beta}^{(3)}\partial x_{\beta}^{(3)}}\int v_{\alpha}^{(3)}F_{3}^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$

$$+\lambda\left(\ \frac{\partial}{\partial g_{\alpha}^{(1)}}\lim_{\overline{x}^{(3)}\to\overline{x}^{(1)}}+\frac{\partial}{\partial g_{\alpha}^{(2)}}\lim_{\overline{x}^{(3)}\to\overline{x}^{(2)}}\ \right)\frac{\partial^{2}}{\partial x_{\beta}^{(3)}\partial x_{\beta}^{(3)}}\int g_{\alpha}^{(3)}F_{3}^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{3)}d\psi^{(3)}$$

$$+\gamma\left(\ \frac{\partial}{\partial \phi^{(1)}}\lim_{\overline{x}^{(3)}\to\overline{x}^{(1)}}+\frac{\partial}{\partial \phi^{(2)}}\lim_{\overline{x}^{(3)}\to\overline{x}^{(2)}}\ \right)\frac{\partial^{2}}{\partial x_{\beta}^{(3)}\partial x_{\beta}^{(3)}}\int \phi^{(3)}F_{3}^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$

$$+D\left(\ \frac{\partial}{\partial \psi^{(1)}}\lim_{\overline{x}^{(3)}\to\overline{x}^{(1)}}+\frac{\partial}{\partial \psi^{(2)}}\lim_{\overline{x}^{(3)}\to\overline{x}^{(2)}}\ \right)\frac{\partial^{2}}{\partial x_{\beta}^{(3)}\partial x_{\beta}^{(3)}}\int \psi^{(3)}F_{3}^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$

$$+2\in_{m\alpha\beta}\Omega_{m}F_{2}^{(1,2)}\ +f\left(\ u_{\alpha}^{(1)}-v_{\alpha}^{(1)}\ \right)\frac{\partial}{\partial v_{\alpha}^{(2)}}F_{2}^{(1,2)}=0\ . \qquad \text{---------}\ (3.6.18)$$

Continuing this way, we can derive the equations for evolution of $F_{3}^{(1,2,3,)}$, $F_{4}^{(1,2,3,4)}$ and so on. Logically it is possible to have an equation for every F_n (n is an integer) but the system of equations so obtained is not closed. Certain approximations will be required thus obtained.

3.7 Concluding Remarks:

If the fluid is clean and the system is non rotating then f=0 and Ωm=0, the transport equation for one point distribution function in MHD turbulent flow (3.6.17) becomes

$$\frac{\partial F_{1}^{(1)}}{\partial t}+v_{\beta}^{(1)}\frac{\partial F_{1}^{(1)}}{\partial x_{\beta}^{(1)}}+g_{\beta}^{(1)}\left(\ \frac{\partial g_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}}+\frac{\partial v_{\alpha}^{(1)}}{\partial g_{\alpha}^{(1)}}\ \right)\frac{\partial F_{1}^{(1)}}{\partial x_{\beta}^{(1)}}-\frac{\partial}{\partial v_{\alpha}^{(1)}}\left[\ \frac{1}{4\pi}\int\left(\ \frac{\partial}{\partial x_{\alpha}^{(1)}}\frac{1}{\left|\overline{x}^{(2)}-\overline{x}^{(1)}\right|}\ \right)\right.$$

$$\times\left(\ \frac{\partial v_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}}\frac{\partial v_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}}-\frac{\partial g_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}}\frac{\partial g_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}}\ \right)F_{2}^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\nu\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int v_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\lambda\frac{\partial}{\partial g_\alpha^{(1)}}\lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int g_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\gamma\frac{\partial}{\partial \phi^{(1)}}\lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int \phi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+D\frac{\partial}{\partial \psi^{(1)}}\lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int \psi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}=0 \quad \text{-------(3.7.1)}$$

which was obtained earlier by Sarker and Kishore [114].

If we drop the viscous, magnetic and thermal diffusive and concentration terms from the one point evolution equation (3.7.1), we have

$$\frac{\partial F}{\partial t}+v_\beta^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\int\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|\bar{x}^{(2)}-\bar{x}^{(1)}\right|}\right)\right.$$

$$\left.\times\left(\frac{\partial v_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial v_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial g_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial g_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right)F_2^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}\right]=0\,. \quad \text{---------- (3.7.2)}$$

The existence of the term

$$\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}$$

can be explained on the basis that two characteristics of the flow field are related to each other and describe the interaction between the two modes (velocity and magnetic) at a single point $x^{(1)}$.

We can exhibit an analogy of this equation with the 1st equation in BBGKY hierarchy in the kinetic theory of gases. The first equation of BBGKY hierarchy is given [75] as

$$\frac{\partial F_1^{(1)}}{\partial t}+\frac{1}{m}v_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}F_1^{(1)}=n\iint\frac{\partial\psi_{1,2}}{\partial x_\alpha^{(1)}}\frac{\partial F_2^{(1,2)}}{\partial v_\alpha^{(1)}}d\overline{x}^{(2)}d\overline{v}^{(2)} \quad \text{------------ (3.7.3)}$$

where $\psi_{1,2}=\psi\left|v_\alpha^{(2)}-v_\alpha^{(1)}\right|$ is the inter molecular potential.

In order to close the system of equations for the distribution functions, some approximations are required. If we consider the collection of ionized particles, i.e. in plasma turbulence case, it can be provided closure form easily by decomposing $F_2^{(1,2)}$ as $F_1^{(1)}\,F_1^{(2)}$. But such type of approximations can be possible if there is no interaction or correlation between two particles. If we decompose $F_2^{(1,2)}$ as

$$F_2^{(1,2)}=(1+\in)\,F_1^{(1)}\,F_1^{(2)}$$

and

$$F_3^{(1,2,3)}=(1+\in)^2\,F_1^{(1)}\,F_1^{(2)}\,F_1^{(3)}$$

where $\in$ is the correlation coefficient between the particles. If there is no correlation between the particles, $\in$ will be zero and distribution function can be decomposed in usual way. Here we are considering such type of approximation only to provide closed from of the equation i.e., to approximate two-point equation as one point equation.

The transport equation for distribution function of velocity, magnetic, temperature and concentration have been shown here to provide an advantageous basis for modeling the turbulent flows in a rotating system in presence of dust particles. Here we have made an attempt for the modeling of various terms such as fluctuating pressure, viscosity and diffusivity in order to close the equation for distribution function of velocity, magnetic, temperature and concentration. It is also possible to construct such type of distribution functions in variable density follows. The advantage of constructing such type hierarchy is to provide simultaneous information about velocity, magnetic temperature and concentration without knowledge of scale of turbulence.

CHAPTER-IV

PART-A

DECAY OF TEMPERATURE FLUCTUATIONS IN MAGNETO-HYDRODYNAMIC TURBULENCE BEFORE THE FINAL PERIOD IN A ROTATING SYSTEM

4.1 Introduction:

Corrsin [24,26] made an analytical discussion on the problem of turbulent temperature fluctuations using the approaches employed in the statistical theory of turbulence. His result pertains to the final period of decay and for the case of appreciable convective effects to the energy spectral form in specific wave number ranges. Oruga [90] had been done further work along this same line. Deissler [27,28] developed a theory for homogeneous turbulence, which was valid for times before the final period. Using Deissler's theory Loeffler and Deissler [72] studied decay of the temperature fluctuations in homogeneous turbulence before the final period. Following Deissler's approach Sarker and Islam [116] also studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time. Sarker and Rahman [113] studied the decay of temperature fluctuations in MHD turbulence before the final period. Islam and Sarker [46] studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Kumar and Patel [65] also studied on fist-order reactant in homogeneous turbulence before the final period of decay for the case of multipoint and multi-time. Sarker and Islam [115] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Sarker and Kishore [108] had been done further work along this same line for the case of multi-point and single time. They considered two and three-point correlations after neglecting higher order correlation terms compared to the second-and third-order correlation terms. Also Kishore and Dixit [52], Kishore and Singh [54] discussed the effect of coriolis force on acceleration covariance in ordinary and MHD turbulence. Shimomura and Yoshizawa [119], Shimomura [120] and [121] also discussed the statistical analysis of turbulent viscosity, turbulent scalar flux and turbulent shear flows respectively in a rotating system by two-scale direct interaction approach. Sarker and Islam [117] studied the decay of dusty fluid turbulence before the final period in rotating system.

Using the above theories we have studied the decay of temperature fluctuations in MHD turbulence before the final period in a rotating system. Here two-and three-point correlation equations have been considered and fourth order correlation terms are neglected in comparison to the second-and third-order correlation terms. Finally, the energy decay law of temperature fluctuations in MHD turbulence before the final period in a rotating system is obtained.

4.2 Basic Equations:

The equation of motion and continuity for viscous, incompressible MHD turbulent flow in a rotating system are given by

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}\left(u_i u_k - h_i h_k\right)=-\frac{\partial w}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k \partial x_k}-2\in_{mki}\Omega_m u_i), \qquad \text{---------} (4.2.1)$$

$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}(h_i u_k - u_i h_k)=\frac{\nu}{P_M}\frac{\partial^2 h_i}{\partial x_k \partial x_k}, \qquad \text{---------} (4.2.2)$$

$$\frac{\partial v_i}{\partial t}+v_k\frac{\partial v_i}{\partial x_k}=-\frac{k}{m_s}(v_i-u_i) \qquad \text{---------} (4.2.3)$$

with

$$\frac{\partial u_i}{\partial x_i}=\frac{\partial v_i}{\partial x_i}=\frac{\partial h_i}{\partial x_i}=0 \qquad \text{---------} (4.2.4)$$

and the equation of energy for an incompressible fluid with constant properties and for negligible frictional heating.

$$\frac{\partial T}{\partial t}+u_i\frac{\partial T}{\partial x_i}=\left(\frac{v}{p_r}\right)\frac{\partial^2 T}{\partial x_i \partial x_i}. \qquad \text{-----------(4.2.5)}$$

The subscripts can take on the values 1, 2 or 3.

Here, u_i, turbulent velocity component; h_i, magnetic field fluctuation component

$W(\hat{x},t)=\frac{p}{\rho}+\frac{1}{2}\langle h^2\rangle+\frac{1}{2}\left|\hat{\Omega}\times\hat{x}\right|^2$, total MHD pressure inclusive of potential and centrifugal force,

$p(\hat{x},t)=$ hydro-dynamic pressure,

$\rho=$ fluid density,

$p_M=\frac{v}{\lambda}$, magnetic Prandtl number,

$p_r=\frac{v}{\gamma}$, Prandtl number,

$v=$ kinematic viscosity,

$\gamma=\frac{K}{\rho c_p}$, thermal diffusivity,

$\lambda=(4\pi\mu\sigma)^{-1}$, magnetic diffusivity,

$c_p=$ heat capacity at constant pressure ,

$\Omega_m=$ constant angular velocity components,

$\in_{mki}=$ alternating tensor,

$m_s=\frac{4}{3}\pi R_s^3\rho_s$, mass of single spherical dust particle of radius R_s ,

$\rho_s=$ constant density of the material in dust particle,

$x_k=$ Space co-ordinate, the subscripts can take on the values 1, 2 or 3.

4.3 Two-point Correlation and Spectral Equations:

The induction equation of a magnetic field at the point p is

$$\frac{\partial h_i}{\partial t}+u_k\frac{\partial h_i}{\partial x_k}-h_k\frac{\partial u_i}{\partial x_k}=\frac{\nu}{p_M}\frac{\partial^2 h_i}{\partial x_k\partial x_k} \qquad \text{---------- (4.3.1)}$$

and the energy equation at the point p' is

$$\frac{\partial T_j'}{\partial t}+u_k'\frac{\partial T_j'}{\partial x_k'}=\frac{\nu}{p_r}\frac{\partial^2 T_j'}{\partial x_k'\partial x_k'}. \qquad \text{---------- (4.3.2)}$$

Multiplying equation (4.3.1) by T_j' and (4.3.2) by h_i, adding and taking ensemble average, we get

$$\frac{\partial\langle h_iT_j'\rangle}{\partial t}+u_k\frac{\partial\langle h_iT_j'\rangle}{\partial x_k}+u_k'\frac{\partial\langle h_iT_j'\rangle}{\partial x_k'}-h_k\frac{\partial\langle u_iT_j'\rangle}{\partial x_k'}=\nu\left[\frac{1}{P_M}\frac{\partial^2\langle h_iT_j'\rangle}{\partial x_k\partial x_k}+\frac{1}{P_r}\frac{\partial^2\langle h_iT_j'\rangle}{\partial x_k'\partial x_k'}\right]. \qquad \text{---- (4.3.3)}$$

Angular bracket $\langle\ldots\ldots\ldots\ldots\rangle$ is used to denote an ensemble average and the continuity equation is

$$\frac{\partial u_k}{\partial x_k}=\frac{\partial u_k'}{\partial x_k'}=0. \qquad \text{--------- (4.3.4)}$$

Substituting equation (4.3.4) in to equation (4.3.3) yields

$$\frac{\partial\langle h_iT_j'\rangle}{\partial t}+\frac{\partial\langle u_kh_iT_j'\rangle}{\partial x_k}+\frac{\partial\langle u_k'h_iT_j'\rangle}{\partial x_k'}-\frac{\partial\langle u_ih_kT_j'\rangle}{\partial x_k}=\nu\left[\frac{1}{p_M}\frac{\partial^2\langle h_iT_j'\rangle}{\partial x_k dx_k}+\frac{1}{p_r}\frac{\partial^2\langle h_iT_j'\rangle}{\partial x_k'\partial x_k'}\right]. \qquad \text{-------- (4.3.5)}$$

Using the transformations

$$\frac{\partial}{\partial r_k}=-\frac{\partial}{\partial x_k}=\frac{\partial}{\partial x_k'}$$

and the Chandrasekhar relations [19].

$$\langle u_kh_iT_j'\rangle=-\langle u_k'h_iT_j'\rangle,$$

Equation (4.3.5) becomes

$$\frac{\partial}{\partial t}\langle h_i T_j'\rangle + 2\frac{\partial}{\partial r_k}\langle u_k' h_i T_j'\rangle + \frac{\partial\langle u_i h_k T_j'\rangle}{\partial r_k} = \nu\left[\frac{\partial^2\langle h_i T_j'\rangle}{\partial r_k \partial r_k}\left(\frac{1}{P_M}+\frac{1}{p_r}\right)\right]. \quad \text{------------ (4.3.6)}$$

Now we write equation in spectral form in order to reduce it to an ordinary differential equation by use of the following three-dimensional Fourier transforms.

$$\langle h_i T_j'(\hat{r})\rangle = \int_{-\infty}^{\infty}\langle \psi_i \tau_j'(\hat{K})\rangle \exp\left[\hat{i}(\hat{K},\hat{r})\right] d\hat{K}, \quad \text{----------- (4.3.7)}$$

$$\langle u_i h_k T_j'(r)\rangle = \int_{-\infty}^{\infty}\langle \phi_i \psi_k \tau_j'(\hat{K})\rangle \exp\left[\hat{i}(\hat{K},\hat{r},)\right] d\hat{K}, \quad \text{----------- (4.3.8)}$$

$$\langle u_k' h_i T_j'(r)\rangle = \langle u_k h_i T_j'(-\hat{r})\rangle = \int_{-\infty}^{\infty}\langle \phi_k \psi_i \tau_j'(-\hat{k})\rangle \exp\left[\hat{i}(\hat{k},\hat{r})\right] d\hat{K}. \quad \text{----------- (4.3.9)}$$

Equation (4.3.9) is obtained by interchanging the subscripts i and j and then the points p and p'.

Substituting of equation (4.3.7) to (4.3.9) in to equation (4.3.6) leads to the spectral equation

$$\frac{\partial\langle \psi_i \tau_j\rangle}{\partial t} + iK_k\left[2\langle \phi_k \psi_i \tau_j'(-\hat{K}) + \langle \phi_i \psi_k \tau_j'(\hat{K})\rangle\right] = -\nu\left[\left(\frac{1}{p_M}+\frac{1}{p_r}\right)k^2\langle \psi_i \tau_j'(\hat{K})\rangle\right]. \quad \text{---- (4.3.10)}$$

The tensor equation (4.3.10) becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial\langle \psi_i \tau_i'(\hat{K})\rangle}{\partial t} + iK_k\left[2\langle \phi_k \psi_i \tau'(-\hat{K})\rangle + \langle \phi_i \psi_k \tau_i'(\hat{K})\rangle\right] = -\nu\left[\left(\frac{1}{P_M}+\frac{1}{P_r}\right)k^2\langle \psi_i \tau_i'(\hat{k})\rangle\right]. \quad \text{-- (4.3.11)}$$

4.4 Three-point Correlation and Spectral Equations:

Similar Procedure can be used to find the three-point correlation equation. For this purpose we take the momentum equation of MHD turbulence at the point P, the induction equation at the point P' and the energy equation at P'' as

$$\frac{\partial u_i}{\partial t}+u_k\frac{\partial u_i}{\partial k_k}-h_k\frac{\partial h_i}{\partial x_k}=-\frac{\partial w}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k}-2\in_{mki}\Omega_m u_i\ , \qquad \text{---------} (4.4.1)$$

$$\frac{\partial h_i'}{\partial t}+u_k'\frac{\partial h_i'}{\partial x_k}-h_k'\frac{\partial u_i'}{\partial x_k'}=\frac{\nu}{P_M}\frac{\partial^2 h_i'}{\partial x_k'\partial x_k'} \qquad \text{---------} (4.4.2)$$

and

$$\frac{\partial T_j''}{\partial t}+u_k''\frac{\partial T_j''}{\partial x_k''}=\left(\frac{\nu}{P_r}\right)\frac{\partial^2 T_j''}{\partial x_k''\partial x_k''}, \qquad \text{----------} (4.4.3)$$

where

$W(\hat{x},t)=\dfrac{P}{\rho}+\dfrac{1}{2}\langle h^2\rangle+\dfrac{1}{2}\left|\hat{\Omega}\times\hat{x}\right|^2$, total MHD pressure inclusive of potential and centrifugal force $P(\hat{x},t)$, hydrodynamic pressure; Ωm, constant angular velocity components; $\in_{mki}$, alternating tensor.

Multiplying equation (4.4.1) by $h_i'T_j''$, (4.4.2) by u_iT_j'' and (4.4.3) by u_ih_i', adding and taking ensemble average, one obtains

$$\frac{\partial\langle u_ih_i'T_j''\rangle}{\partial t}+\frac{\partial\langle u_iu_kh_i'T_j\rangle}{\partial x_k}-\frac{\partial\langle h_ih_kh_i'T_j''\rangle}{\partial x_k''}+\frac{\partial\langle u_iu_k'h_i'T_j''\rangle}{\partial x_k'}-\frac{\partial\langle u_iu_i'h_k'T_j''\rangle}{\partial x_k'}+\frac{\partial\langle u_ih_i'u_k''T_j''\rangle}{\partial x_k''}$$

$$=-\frac{\partial\langle wh_i'T_j''\rangle}{\partial x_i}+\nu\frac{\partial^2\langle u_ih_i'T_j''\rangle}{\partial x_k\partial x_k}+\nu\left[\frac{1}{P_M}\frac{\partial^2\langle u_ih_i'T_j''\rangle}{\partial x_k'\partial x_k'}+\frac{1}{P_r}\frac{\partial^2\langle u_ih_i'T_j''\rangle}{\partial x_k''\partial x_k''}\right]-2\in_{mki}\Omega_m\langle u_ih_i'T_j''\rangle. \ (4.4.4)$$

Using the transformations

$\dfrac{\partial}{\partial x_k}=-\left(\dfrac{\partial}{\partial r_k}+\dfrac{\partial}{\partial r_k'}\right),\dfrac{\partial}{\partial x_k'}=\dfrac{\partial}{\partial r_k},\dfrac{\partial}{\partial x_k''}=\dfrac{\partial}{\partial r_k'}$ into equation (4.4.4)

$$\frac{\partial\langle u_i h_i' T_j''\rangle}{\partial t} - \nu\left[(1+\frac{1}{p_M})\frac{\partial^2\langle u_i h_i' T_j''\rangle}{\partial r_k \partial r_k} + (1+\frac{1}{p_r})\frac{\partial^2\langle u_i h_i' T_j''\rangle}{\partial r_k' \partial r_k'} + 2\frac{\partial^2\langle u_i h_i' T_j''\rangle}{\partial r_k \partial r_k'}\right]$$

$$= \frac{\partial\langle u_i u_k h_i' T_j''\rangle}{\partial r_k} + \frac{\partial\langle u_i u_k h_i' T_j''\rangle}{\partial r_k'} - \frac{\partial\langle h_i h_k h_i' T_j''\rangle}{\partial r_k} - \frac{\partial\langle h_i h_k h_i' T_j''\rangle}{\partial r_k'} - \frac{\partial\langle u_i u_k' h_i' T_j''\rangle}{\partial r_k}$$

$$+ \frac{\partial\langle u_i u_i' h_k' T_j''\rangle}{\partial r_k} - \frac{\partial\langle u_i u_k'' h_i' T_j''\rangle}{\partial r_k'} + \frac{\partial\langle w h_i' T_j''\rangle}{\partial r_i} + \frac{\partial\langle w h_i' T_j''\rangle}{\partial r_i'} - 2\in_{mki}\Omega_m\langle u_i h_i' T_j''\rangle . \qquad \text{-------- (4.4.5)}$$

In order to write the equation (4.4.5) to spectral form, we can define the following six dimensional Fourier transforms:

$$\langle u_i h_i'(\hat{r}) T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_i \beta_i'(\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}', \qquad \text{-------- (4.4.6)}$$

$$\langle u_i u_k h_i'(\hat{r}) T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_i \phi_k \beta_i'(\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}', \qquad \text{-------- (4.4.7)}$$

$$\langle h_i h_k h_i'(\hat{r}) T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \beta_i \beta_k \beta_i'(\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}', \qquad \text{--------- (4.4.8)}$$

$$\langle u_i u_k' h_i'(\hat{r}) T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_i \phi_k'(\hat{k}) \beta_i'(\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}', \qquad \text{-------- (4.4.9)}$$

$$\langle u_i u_i'(\hat{r}) h_k'(\hat{r}) T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_i \phi_i' \beta_k'(\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}', \qquad \text{------- (4.4.10)}$$

$$\langle w h_i'(\hat{r}) T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \gamma \beta_i'(\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}' . \qquad \text{------- (4.4.11)}$$

Interchanging the points p' and p'' along with the subscripts i and j,

$$\langle u_i u_k'' h_i' T_j''\rangle = \langle u_i u_k' h_i' T_j''\rangle .$$

By use this fact and the equations (4.4.6)-(4.4.11), we can write equation (4.4.5) in the form

$$\frac{\partial\langle\phi_i\beta_i'\theta_j''\rangle}{\partial t}+\nu\left[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2+2k_kk_k'+\frac{2\in_{mkl}\Omega_m}{\nu}\right]\langle\phi_i\beta_i'\theta_j''\rangle$$

$$=i(k_k+k_k')\langle\phi_i\phi_k\beta_i'\theta_j''\rangle-i(k_k+k_k')\langle\beta_i\beta_k\beta_i'\theta_j''\rangle-i(k_k+k_k')\langle\phi_i\phi_k'\beta_i'\theta_j''\rangle$$

$$+ik_k\langle\phi_i\phi_i'\beta_k'\theta_j''\rangle+i(k_i+k_i')\langle\gamma\beta_i'\theta_j''\rangle. \qquad \text{---------} (4.4.12)$$

The tensor equation (4.4.12) can be converted to scalar equation by contraction of the indices i and j

$$\frac{\langle\partial\langle\phi_i\beta_i'\theta_i''\rangle}{\partial t}+\nu\left[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2+2k_kk_k'+2\frac{\in_{mkl}\Omega_m}{\nu}\right]\langle\phi_i\beta_i'\theta_i''\rangle$$

$$=i(k_k+k_k')\langle\phi_i\phi_k\beta_i'\theta_i''\rangle-i(k_k+k_k')\langle\beta_i\beta_k\beta_i'\theta_i''\rangle-i(k_k+k_k')\langle\phi_i\phi_k\beta_i'\theta_i''\rangle$$

$$+ik_k\langle\phi_i\phi_i'\beta_k'\theta_i''\rangle+i(k_i+k_i')\langle\gamma\beta_i'\theta_i''\rangle. \qquad \text{---------} (4.4.13)$$

If the derivative with respect to x_i is taken of the momentum equation (4.4.1) for the point p, the equation multiplied through by $h_i T_j''$ and time average taken, the resulting equation

$$-\frac{\partial^2\langle w\,h_i'\,T_j''\rangle}{\partial x_i\partial x_i}=\frac{\partial^2}{\partial x_i\partial x_k}\left(\langle u_iu_kh_i'T_j''\rangle-\langle h_ih_kh_i'T_j''\rangle\right). \qquad \text{--------} (4.4.14)$$

Writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$-\left[\frac{\partial^2}{\partial r_i\partial r_i}+2\frac{\partial^2}{\partial r_i\partial r_i'}+\frac{\partial^2}{\partial r_i'\partial r_i'}\right]\langle wh_i'T_j''\rangle=\left[\frac{\partial^2}{\partial r_i\partial r_k}+\frac{\partial^2}{\partial r_i'\partial r_k}+\frac{\partial^2}{\partial r_i\partial r_k'}+\frac{\partial^2}{\partial r_i'\partial r_k'}\right]$$

$$\times\left(\langle u_i u_k h_i' T_j''\rangle - \langle h_i h_k h_i' T_j''\rangle\right). \qquad \text{------- (4.4.15)}$$

Now taking the Fourier transforms of equation (4.4.15), we get

$$-\langle \gamma \beta_i' \theta_j''\rangle = \frac{(k_i k_k + k_i' k_k + k_i k_k' + k_i' k_k')(\langle \phi_i \phi_k \beta_i' \theta_j''\rangle - \langle \beta_i \beta_k \beta_i' \theta_j''\rangle)}{k_i k_i + 2k_i' k_i + k_i' k_i'}. \qquad \text{-------- (4.4.16)}$$

Equation (4.4.16) can be used to eliminate $\langle \gamma \beta_i' \theta_j''\rangle$ from equation (4.4.12).

4.5 Solution for times before the final period :

It is known that equation for final period of decay is obtained by considering the two-point correlations after neglecting the 3rd order correlation terms. To study the decay for times before the final period, the three point correlations are considered and the quadruple correlation terms are neglected because the quadruple correlation terms decays faster than the lower-order correlation terms. Equation (4.4.16) shows that term $\langle \gamma \beta_i' \theta_j''\rangle$ associated with the pressure fluctuations should also be neglected. Thus neglecting all the terms on the right hand side of equation (4.4.13)

$$\frac{\partial \langle \phi_i \beta_i' \theta_i''\rangle}{\partial t} + \nu\left[(1+\frac{1}{p_M})k^2 + (1+\frac{1}{p_r})k'^2 + 2k_k k_k' + \frac{2\in_{mki}\Omega_m}{\nu}\right]\langle \phi_i \beta_i' \theta_i''\rangle = 0\,. \qquad \text{------- (4.5.1)}$$

Integrating the equation (4.5.1) between t_o and t with inner multiplication by k_k and gives

$$k_k \langle \phi_i \beta_i' \theta_i''\rangle = k_k \left[\phi_i \beta_i' \theta_i''\right]_0 \exp\left[-\nu\left\{(1+\frac{1}{p_M})k^2 + (1+\frac{1}{p_r})k'^2 + 2kk'\cos\theta + \frac{2\in_{mki}\Omega_m}{\nu}\right\}(t-t_0)\right],$$

--------(4.5.2)

where θ is the angle between k and k' and $\langle \phi_i \beta_i' \theta_i''\rangle_0$ is the value of $\langle \phi_i \beta_i' \theta_i''\rangle$ at t= t_o.

Now by letting $r' = 0$ in equation (4.4.6) and comparing with equations (4.3.8) and (4.3.9), we get

$$\langle\phi_i\psi_k\tau_i'(\hat{k})\rangle = \int_{-\infty}^{\infty}\langle\phi_i\beta_i'\theta_i''\rangle d\hat{k}', \qquad \text{-------- (4.5.3)}$$

$$\langle\phi_i\psi_i\tau_i'(-\hat{k})\rangle = \int_{-\infty}^{\infty}\phi_k\beta_i'(-\hat{k})\theta_i''(-\hat{k}')d\hat{k}'. \qquad \text{--------- (4.5.4)}$$

Substituting equation (4.5.2) to (4.5.4) in equation (4.3.11)

$$\frac{\partial\langle\psi_i\tau_i'(\hat{k})\rangle}{\partial t} + \nu(\frac{1}{p_M}+\frac{1}{p_r})k^2\langle\psi_i\tau_i'(\hat{k})\rangle = -\int_{-\infty}^{\infty} ik_k\left[\langle\phi_i\beta_i'\theta_i''\rangle + 2\langle\phi_k\beta_i'(-\hat{k})\theta_i''(-\hat{k}')\rangle\right]_0$$

$$\exp[-\nu(t-t_o)\left\{(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2+2kk'\cos\theta+2\frac{\in_{mki}\Omega_m}{\nu}\right\}]d\hat{k}'. \qquad \text{-------- (4.5.5)}$$

Now, $d\hat{k}'$ can be expressed in terms of k' and θ as $-2\pi k'^2 d(\cos\theta)dk'$ (cf. Deissler[27]).

Hence $d\hat{k}' = -2\pi k'^2 d(\cos\theta)d\hat{k}'$. --------- (4.5.6)

Putting equation (4.5.6) in equation (4.5.5) yields

$$\frac{\partial\langle\psi_i\tau_i'(\hat{k})\rangle}{\partial t} + \nu(\frac{1}{p_M}+\frac{1}{p_r})k^2\langle\psi_i\tau_i'(\hat{k})\rangle = -\int_0^{\infty} 2\pi ik_k\left[\langle\phi_i\beta_i'\theta_i''\rangle + 2\langle\phi_k\beta_i'(-\hat{k})\theta_i''(-\hat{k}')\rangle\right]_0 k'^2 \times$$

$$\left[\int_{-1}^{1}\exp\left\{-\nu(t-t_o)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2+2kk'\cos\theta+2\frac{\in_{mki}\Omega_m}{\nu}]\right\}\right]d(\cos\theta)\,d\hat{k}'. \text{-(4.5.7)}$$

In order to find the solution completely and following Loeffler and Deissler [72] we assume that

$$ik_k\left[\langle\phi_i\beta_i'\theta_i''\rangle + 2\langle\phi_k\beta_i'(-\hat{k})\theta_i''(-\hat{k}')\rangle\right]_0 = \frac{\beta_0}{(2\pi)^2}\left(k^2k'^4 - k^4k'^2\right), \qquad \text{-------- (4.5.8)}$$

where β_0 is a constant depending on the initial conditions. Substituting equation (4.5.8) into equation (4.5.7) and completing the integration with respect to $\cos\theta$, one obtains

$$\frac{\partial(2\pi\langle\psi_i\tau_i'(\hat{k})\rangle)}{\partial t}+\nu(\frac{1}{p_M}+\frac{1}{p_r})k^2(2\pi\langle\psi_i\tau_i'(\hat{k})\rangle)=-\frac{\beta_0}{2\nu(t-t_o)}\int_0^{\infty}\left(k^3k'^5-k^5k'^3\right)\times$$

$$\left[\exp\left\{-\nu(t-t_o)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2-2kk'+\frac{2\in_{mki}\Omega_m}{\nu}]\right\}\right.$$

$$\left.-\exp\left\{-\nu(t-t_o)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2+2kk'+2\frac{\in_{mki}\Omega_m}{\nu}]\right\}\right]dk'. \qquad \text{------------ (4.5.9)}$$

Multiplying both sides of equation (4.5.9) by k^2, we get

$$\frac{\partial Q}{\partial t}+\nu\left(\frac{1}{p_M}+\frac{1}{p_r}\right)k^2Q=F, \qquad \text{----------- (4.5.10)}$$

where, $Q=2\pi k^2\langle\psi_i\tau_i'(\hat{k})\rangle$, ----------- (4.5.11)

Q is the Magnetic energy Spectrum function.

and

$$F=-\frac{\beta_0}{2\nu(t-t_0)}\int_0^{\infty}(k^3k'^5-k^5k'^3)\times\left[\exp\left\{-\nu(t-t_0)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2-2kk'+\frac{2\in_{mki}\Omega_m}{\nu}]\right\}\right..$$

$$\left[-\exp\left\{-\nu(t-t_0)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2+2kk'+\frac{2\in_{mki}\Omega_m}{\nu}]\right\}\right]. \qquad \text{---------- (4.5.12)}$$

Integrating equation (4.5.12) with respect to k', we have

$$F=-\frac{\beta_0\sqrt{\pi}P_r^{5/2}}{2\nu^{3/2}(t-t_o)(1+p_r)^{5/2}}\exp\left[-2\frac{\in_{mki}\Omega_m}{\nu}(t-t_o)\right]\times\exp\left[-\nu(t-t_o)(1+\frac{1}{p_M}-\frac{p_r}{1+p_r})k^2\right]$$

$$\left[\frac{15p_r k^4}{4v^2(t-t_o)^2(1+p_r)}+\left\{\frac{5p_r^{\ 2}}{(1+p_r)^2}-\frac{3}{2}\right\}\frac{k^6}{v(t-t_o)}+\left\{\frac{p_r^{\ 3}}{(1+p_r)^3}-\frac{p_r}{(1+p_r)}\right\}k^8\right]. \quad \text{------ (4.5.13)}$$

The series of equation (4.5.13) contains only even powers of k and start with k^4 and the equation represents the transfer function arising owing to consideration of magnetic field at three points at a time.

It is interesting to note that if we integrate equation (4.5.13) over all wave numbers, we find that

$$\int_0^\infty F dk = 0 \quad \text{--------- (4.5.14)}$$

which is indicating that the expression for F satisfies the condition of continuity and homogeneity.

The linear equation (4.5.10) can be solved to give

$$Q=\exp\left[-vk^2\left(\frac{1}{p_M}+\frac{1}{p_r}\right)(t-t_o)\right]\int F\exp\left[vk^2\left(\frac{1}{p_M}+\frac{1}{p_r}\right)(t-t_o)\right]dt+J(k)\exp\left[-vk^2\left(\frac{1}{p_M}+\frac{1}{p_r}\right)(t-t_o)\right],$$

$$\text{----------- (4.5.15)}$$

where $J(K)=\frac{N_0 k^2}{\pi}$ is a constant of integration. Substituting the values of F from equation (4.5.13) in to equation (4.5.15) and integrating with respect to t, we get

$$\hat{Q}(k,t)=\frac{N_0 k^2}{\pi}\exp\left[-vk^2\left(\frac{1}{p_M}+\frac{1}{p_r}\right)(t-t_o)\right]+\frac{\beta_0\sqrt{\pi}\,p_r^{\ 3/2}}{2v^{3/2}(1+p_r)^{7/2}}\times$$

$$\exp[-\{2\in_{mki}\Omega_m(t-t_o)\}]\exp\left[-vk^2(t-t_o)\left\{\frac{1+p_r+p_M}{p_M(1+p_r)}\right\}\right]$$

$$\left[\frac{3p_r k^4}{2v^2(t-t_o)^{5/2}}+\frac{p_r(7p_r-6)k^6}{3v(1+p_r)(t-t_o)^{3/2}}-\frac{4(3p_r^{\ 2}-2p_r+3)k^8}{3(1+p_r)^2(t-t_o)^{1/2}}+\frac{8\sqrt{v}(3p_r^{\ 2}-2p_r+3)k^9}{3(1+p_r)^{5/2}\sqrt{p_r}}N(\omega)\right], \text{-(4.5.16)}$$

where $N(\omega)=e^{-\omega^2}\int_0^\omega e^{x^2}dx$

and $\omega = k\sqrt{\dfrac{\lambda(t-t_o)}{p_r(1+p_r)}}$.

The function $N_{(w)}$has been calculated numerically and tabulated in [24] .

By setting $\hat{r}=0,\ j=i,\ d\hat{K}=-2\pi k^2 d(\cos\theta)dk$ and $Q=2\pi k^2\left\langle \psi_i\psi_i'(\hat{K})\right\rangle$ in equation (4.3.7), we get the expression for temperature energy decay as

$$\frac{\langle T^2\rangle}{2}=\frac{T_iT_i'}{2}=\int_0^\infty \hat{Q}(\hat{k})d\hat{k}. \qquad \text{--------- (4.5.17)}$$

Substituting equations (4.5.16) in to (4.5.17) and after integration, we get

$$\frac{\langle T^2\rangle}{2}=\frac{N_o p_r^{3/2} p_M^{3/2}(t-t_o)^{-3/2}}{4\sqrt{\pi}\nu^{3/2}(p_r+p_M)^{3/2}}+\exp[-2\in_{mki}\Omega_m]\times\frac{\beta_0\pi p_r^{7/2} p_M^{5/2}(t-t_o)^{-5}}{2\nu^6(1+p_r)(1+p_r+p_M)^{5/2}}\times$$

$$\left\{\frac{9}{16}+\frac{5p_M(7p_r-6)}{16(1+p_r+p_M)}-\frac{35p_M^2(3p_r^2-2p_r+3)}{8p_r(1+p_r+p_M)^2}+\frac{8p_M^3(3p_r^2-2p_r+3)}{3.2^6.p_r^2(1+p_r+p_M)^3}\sum_{n=0}^{\infty}\frac{1.3.5....(2n+9)}{n!(2n+1)2^{2n}(1+p_r)^n}\right\}.$$

or

$$\frac{\langle T^2\rangle}{2}=\frac{N_0 P_r^{3/2} p_M^{3/2}(t-t_o)^{-3/2}}{4\sqrt{\pi}\nu^{3/2}(p_r+p_M)^{3/2}}+\beta_0 z\nu^{-6}(t-t_o)^{-5}\times\exp[-2\in_{mki}\Omega_m], \qquad \text{--------- (4.5.18)}$$

where

$$Z=\frac{\pi p_r^{7/2} p_M^{5/2}}{2(1+p_r)(1+p_r+p_M)^{5/2}}\times$$

$$\left[\frac{9}{16}+\frac{5P_M(7P_r-6)}{16(1+P_r+p_M)}-\frac{35p_M^2(3p_r^2-2p_r+3)}{8p_r(1+p_r+p_M)^2}+\frac{8p_M^3(3p_r^2-2p_r+3)}{3.2^6 p_r^2(1+p_r+p_M)^3}\sum_{n=0}^{\infty}\frac{1.3.5....(2n+9)}{n!(2n+1)2^{2n}(1+p_r)^n}\right].$$

Thus the energy decay law for temperature field fluctuations of MHD turbulence in a rotating system before the final period may be written as

$$\langle T^2\rangle = X(t-t_o)^{-3/2}+\exp[-\{2\in_{mki}\Omega_m\}]Y(t-t_o)^{-5}, \qquad \text{------------- (4.5.19)}$$

where

$$X = \frac{N_0 P_r^{3/2} p_M^{3/2}}{2\sqrt{\pi}\nu^{3/2}(p_r + p_M)^{3/2}} \text{ and } Y = 2\beta_0 Z\nu^{-6}.$$

$\langle T^2 \rangle$ is the total "energy" (the mean square of the temperature fluctuations) t is the time, x and t_o are constants determined by the initial conditions. The constant Y depends on both initial conditions and the fluid Prandtl number.

4.6 Concluding Remarks:

In equation (4.5.19) we obtained the decay law of temperature fluctuations in MHD turbulence before the final period in a rotating system considering three-point correlation equation after neglecting quadruple correlation terms. If the system is non-rotating, then $\Omega_m = 0$, the equation (4.5.19) becomes.

$$\langle T^2 \rangle = X(t - t_o)^{-3/2} + Y(t - t_o)^{-5} \quad \text{-------- (4.6.1)}$$

which was obtained earlier by Sarker and Rahman [113].

In the absence of a magnetic field, magnetic Prandtl number coincides with the Prandtl number (i.e. $p_r = p_M$) and the system is non rotating the equation (4.5.18) becomes

$$\frac{\langle T^2 \rangle}{2} = \frac{N_0 p_r^{3/2}}{8\sqrt{2\pi}\nu^{3/2}(t - t_o)^{3/2}} + \frac{\beta_0 Z}{\nu^6 (t - t_o)^5} \quad \text{--------- (4.6.2)}$$

which was obtained earlier by Loeffler and Deissler [72].

We conclude that due to the effect of rotation of fluid in the flow field, the turbulent energy decays more rapidly than the energy for non-rotating fluid. The 1st term of the right hand side of equation (4.5.19) corresponds to the temperature energy for two-point correlation and second term represents temperature energy for three-point correlation. For large times the last term in the equation (4.5.19) becomes negligible, leaving the -3/2power decay law for the final period. If we considering the higher order correlation terms in the analysis, it appears that more terms in higher power of time would be added to the equation (4.5.19).

CHAPTER-IV

PART-B

DECAY OF TEMPERATURE FLUCTUATIONS IN DUSTY FLUID MAGNETO-HYDRODYNAMIC TURBULENCE BEFORE THE FINAL PERIOD IN A ROTATING SYSTEM

4.7 Introduction:

Deissler [27,28] developed a theory for homogeneous turbulence, which was valid for times before the final period. Using Deissler's theory Loeffler and Deissler [72] studied the temperature fluctuations in homogeneous turbulence before the final period. Following Deissler's approach Sarker and Islam [116] also studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the ease of multi-point and multi-time. Sarker and Rahman [113] studied the decay of temperature fluctuations in MHD turbulence before the final period. Islam and Sarker [46] studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Kumar and Patel [65] also studied on fist-order reactant in homogeneous turbulence before the final period of decay for the case of multipoint and multi-time. Sarker and Islam [115] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Sarker and Kishore [108] had been done further work along this same line for the case of multi-point and single time. They considered two and three-point correlations after neglecting higher order correlation terms compared to the second-and third-order correlation terms. Also Kishore and Dixit [52], Kishore and Singh [54] discussed the effect of coriolis force on acceleration covariance in ordinary and MHD turbulence. Shimomura and Yoshizawa [119], Shimomura [120] and [121] also discussed the statistical analysis of turbulent viscosity, turbulent scalar flux and turbulent shear flows respectively in a rotating system by two-scale direct interaction approach. Sarker and Islam [117] studied the decay of dusty fluid turbulence before the final period in a rotating system.

In this chapter, we have studied the decay of temperature fluctuations in dusty fluid MHD turbulence before the final period in a rotating system. Here two-and three-point correlation equations have been considered after neglecting fourth-order correlation terms in comparison to the second-and third-order correlation terms. Finally, the energy decay law of temperature fluctuations in MHD dusty fluid turbulence before the final period in a rotating system is obtained.

4.8 Basic Equations:

The equation of motion and continuity for viscous, incompressible MHD dusty fluid turbulent flow in a rotating system are given by

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial w}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2\in_{mki}\Omega_m u_i + f(u_i - v_i)), \qquad \text{--------- (4.8.1)}$$

$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \frac{\nu}{P_M}\frac{\partial^2 h_i}{\partial x_k \partial x_k}, \qquad \text{----------(4.8.2)}$$

$$\frac{\partial v_i}{\partial t}+v_k\frac{\partial v_i}{\partial x_k} = -\frac{k}{m_s}(v_i - u_i), \qquad \text{---------- (4.8.3)}$$

with

$$\frac{\partial u_i}{\partial x_i}=\frac{\partial v_i}{\partial x_i}=\frac{\partial h_i}{\partial x_i}=0 \qquad \text{---------- (4.8.4)}$$

and the equation of energy for an incompressible fluid with constant properties and for negligible frictional heating

$$\frac{\partial T}{\partial t}+u_i\frac{\partial T}{\partial x_i}=\left(\frac{\nu}{p_r}\right)\frac{\partial^2 T}{\partial x_i \partial x_i}. \qquad \text{--------- (4.8.5)}$$

The subscripts can take on the values 1, 2 or 3.

Here, u_i, turbulent velocity component; h_i, magnetic field fluctuation component, v_i, dust velocity component

$W(\hat{x},t)=\frac{p}{\rho}+\frac{1}{2}\langle h^2\rangle+\frac{1}{2}\left|\hat{\Omega}\times\hat{x}\right|^2$, total MHD pressure inclusive of potential and centrifugal force;

$p(\hat{x},t)=$ hydrodynamic pressure,

$\rho=$ fluid density,

$p_M=\frac{\nu}{\lambda}$, magnetic prandtl number,

$p_r=\frac{\nu}{\gamma}$, prandtl number,

$\nu=$ kinematic viscosity,

$\gamma=\frac{K}{\rho c_p}$, thermal diffusivity,

$\lambda=(4\pi\mu\sigma)^{-1}$, magnetic diffusivity,

$c_p=$ heat capacity at constant pressure,

$\Omega_m=$ constant angular velocity components,

$\in_{mki}=$ alternating tensor,

$f=\frac{kN}{\rho}$, dimension of frequency; N, constant number density of dust particle,

$m_s=\frac{4}{3}\pi R^3{}_s\rho_s$, mass of single spherical dust particle of radius R_s,

$\rho_s=$ constant density of the material in dust particle,

$x_k=$ Space co-ordinate, the subscripts can take on the values 1, 2 or 3.

4.9 Two-point Correlation and Spectral Equations:

The induction equation of a magnetic field at the point p is

$$\frac{\partial h_i}{\partial t}+u_k\frac{\partial h_i}{\partial x_k}-h_k\frac{\partial u_i}{\partial x_k}=(\frac{\nu}{p_M})\frac{\partial^2 h_i}{\partial x_k\partial x_k} \qquad \text{---------- (4.9.1)}$$

and the energy equation at the point p' is

$$\frac{\partial T_j'}{\partial t}+u_k'\frac{\partial T_j'}{\partial x_k'}=(\frac{\nu}{p_r})\frac{\partial^2 T_j'}{\partial x_k'\partial x_k'} . \qquad \text{----------} (4.9.2)$$

Multiplying equation (4.9.1) by T_j' and (4.9.2) by h_i , adding and taking ensemble average, we get

$$\frac{\partial\langle h_iT_j'\rangle}{\partial t}+u_k\frac{\partial\langle h_iT_j'\rangle}{\partial x_k}+u_k'\frac{\partial\langle h_iT_j'\rangle}{\partial x^1{}_k}-h_k\frac{\partial\langle u_iT_j'\rangle}{\partial x_k'}=\nu\left[\frac{1}{P_M}\frac{\partial^2\langle h_iT_j'\rangle}{\partial x_k\partial x_k}+\frac{1}{P_r}\frac{\partial^2\langle h_iT_j'\rangle}{\partial x_k'\partial x_k'}\right]. \quad \text{--} (4.9.3)$$

Angular bracket $\langle\text{..........}\rangle$ is used to denote an ensemble average and the continuity equation is

$$\frac{\partial u_k}{\partial x_k}=\frac{\partial u_k'}{\partial x_k'}=0 . \qquad \text{------------} (4.9.4)$$

Substituting equation (4.9.4) in to equation (4.9.3) yields

$$\frac{\partial\langle h_iT_j'\rangle}{\partial t}+\frac{\partial\langle u_kh_iT_j'\rangle}{\partial x_k}+\frac{\partial\langle u_k'h_iT_j'\rangle}{\partial x_k'}-\frac{\partial\langle u_ih_kT_j'\rangle}{\partial x_k}=\nu\left[\frac{1}{p_M}\frac{\partial^2\langle h_iT_j'\rangle}{\partial x_kdx_k}+\frac{1}{p_r}\frac{\partial^2\langle h_iT_j'}{\partial x_k'\partial x_k'}\right]. \quad \text{-------} (4.9.5)$$

Using the transformations

$$\frac{\partial}{\partial r_k}=-\frac{\partial}{\partial x_k}=\frac{\partial}{\partial x_k'}$$

and the Chandrasekhar relation [19].

$$\langle u_kh_iT_j'\rangle=-\langle u_k'h_iT_j'\rangle .$$

Equation (4.9.5) become

$$\frac{\partial}{\partial t}\langle h_iT_j'\rangle+2\frac{\partial}{\partial r_k}\langle u_k'h_iT_j'\rangle+\frac{\partial\langle u_ih_kT_j'\rangle}{\partial r_k}=\nu\left[\frac{\partial^2\langle h_iT_j'\rangle}{\partial r_k\partial r_k}\left(\frac{1}{P_M}+\frac{1}{p_r}\right)\right]. \qquad \text{----------} (4.9.6)$$

Now we write equation (4.9.5) in spectral form in order to reduce it to an ordinary differential equation by use of the following three-dimensional Fourier transforms.

$$\langle h_iT_j'(\hat{r})\rangle=\int_{-\infty}^{\infty}\langle\psi_i\tau_j'(\hat{K})\rangle\exp\left[i(\hat{K},\hat{r})\right]d\hat{K} , \qquad \text{----------} (4.9.7)$$

$$\langle u_i h_k T_j'(r)\rangle = \int_{-\infty}^{\infty} \langle \phi_i \psi_k \tau_j'(\hat{K})\rangle \exp\left[\hat{i}(\hat{K},\hat{r},)\right] d\hat{K}, \quad \text{----------- (4.9.8)}$$

$$\langle u_k' h_i T_j'(r)\rangle = \langle u_k h_i T'_j(-\hat{r})\rangle = \int_{-\infty}^{\infty} \langle \phi_k \psi_i \tau_j'(-\hat{k})\rangle \exp\left[\hat{i}(\hat{k},\hat{r})\right] d\hat{K}. \quad \text{----------- (4.9.9)}$$

Equation (4.9.9) is obtained by interchanging the subscripts i and j and then the points p and p'.

Substituting of equation (4.9.7) to (4.9.9) in to equation (4.9.6) leads to the Spectral equation

$$\frac{\partial\langle \psi_i \tau_j\rangle}{\partial t} + iK_k\left[2\langle \phi_k \psi_i \tau_j'(-\hat{K}) + \langle \phi_i \psi_k \tau_j'(\hat{K})\rangle\right] = -\nu\left[(\frac{1}{P_M} + \frac{1}{P_r})k^2\langle \psi_i \tau_j'(\hat{K})\rangle\right]. \quad \text{-- (4.9.10)}$$

The tensor equation (4.9.10) be comes a scalar equation by contraction of the indices i and j

$$\frac{\partial\langle \psi_i \tau_i'(\hat{K})\rangle}{\partial t} + iK_k\left[2\langle \phi_k \psi_i \tau'(-\hat{K})\rangle + \langle \phi_i \psi_k \tau_i'(\hat{K})\rangle\right] = -\nu\left[(\frac{1}{P_M} + \frac{1}{P_r})k^2\langle \psi_i \tau_i'(\hat{k})\rangle\right]. \text{- (4.9.11)}$$

4.10 Three-point Correlation and Spectral Equations:

Similar Procedure can be used to find the three points correlation equation. For this purpose we take the momentum equation of MHD turbulence at the point P, the induction equation at the point P' and the energy equation at P'' as

$$\frac{\partial u_i}{\partial t} + u_k\frac{\partial u_i}{\partial k_k} - h_k\frac{\partial h_i}{\partial x_k} = -\frac{\partial w}{\partial x_i} + \nu\frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2\in_{mki}\Omega_m u_i + f(u_i - v_i), \quad \text{------- (4.10.1)}$$

$$\frac{\partial h_i'}{\partial t} + u_k'\frac{\partial h_i'}{\partial x_k} - h_k'\frac{\partial u_i'}{\partial x_k'} = (\frac{\nu}{P_M})\frac{\partial^2 h_i'}{\partial x_k' \partial x_k'} \quad \text{------- (4.10.2)}$$

and

$$\frac{\partial T_j''}{\partial t} + u_k''\frac{\partial T_j''}{\partial x_k''} = \left(\frac{\nu}{P_r}\right)\frac{\partial^2 T_j''}{\partial x_k'' \partial x_k''}, \quad \text{------- (4.10.3)}$$

where $W(\hat{x},t)=\frac{P}{\rho}+\frac{1}{2}\langle h^2\rangle+\frac{1}{2}\left|\hat{\Omega}\times\hat{X}\right|^2$, total MHD pressure inclusive of potential and centrifugal force $P(\hat{x},t)$, hydrodynamic pressure; Ωm, constant angular velocity components; $\in_{mki}$, alternating tensor, $f=\frac{kN}{\rho}$, dimension frequency; N, constant number density of dust particle.

Multiplying equation (4.10.1) by $h_i'T_j''$, (4.10.2) by u_iT_j'' and (4.10.3) by u_ih_i', adding and taking ensemble average, one obtains

$$\frac{\partial\langle u_ih_i'T_j''\rangle}{\partial t}+\frac{\partial\langle u_iu_kh_i'Tj\rangle}{\partial x_k}-\frac{\partial\langle h_ih_kh_i'T''j\rangle}{\partial x_k''}+\frac{\partial\langle u_iu_k'h_i'T''j\rangle}{\partial x_k'}-\frac{\partial\langle u_iu_i'h_k'T_j''\rangle}{\partial x_k'}+\frac{\partial\langle u_ih'u_k''T_i''\rangle}{\partial x_k''}$$

$$=-\frac{\partial\langle wh_i'T_i''\rangle}{\partial x_i}+\nu\frac{\partial^2\langle u_ih_i'T_j''\rangle}{\partial x_k\partial x_k}+\nu\left[\frac{1}{P_M}\frac{\partial^2\langle u_ih_i'T_j''\rangle}{\partial x_k'\partial x_k'}+\frac{1}{P_r}\frac{\partial^2\langle u_ih_i'T_j''\rangle}{\partial x_k''\partial x_k''}\right]$$

$$-2\in_{mki}\Omega_m\langle u_ih_i'T_j''\rangle+f(\langle u_ih_i''T_j''\rangle-\langle v_ih_i'T_j''\rangle)\,. \qquad \text{---------- (4.10.4)}$$

Using the transformations

$$\frac{\partial}{\partial x_k}=-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right),\ \frac{\partial}{\partial x_k'}=\frac{\partial}{\partial r_k},\ \frac{\partial}{\partial x_k''}=\frac{\partial}{\partial r_k'}$$

in to equations (4.10.4)

$$\frac{\partial\langle u_ih_i'T_j''\rangle}{\partial t}-\nu\left[(1+\frac{1}{p_M})\frac{\partial^2\langle u_ih_i'T_j''\rangle}{\partial r_k\partial r_k}+(1+\frac{1}{p_r})\frac{\partial^2\langle u_ih_i'T_j''\rangle}{\partial r_k'\partial r_k'}+2\frac{\partial^2\langle u_ih_i'T_j''\rangle}{\partial r_k\partial r_k'}\right]$$

$$=\frac{\partial\langle u_iu_kh_i'T_j''\rangle}{\partial r_k}+\frac{\partial\langle u_iu_kh_i'T_j''\rangle}{\partial r_k'}-\frac{\partial\langle h_ih_kh_i'T_j''\rangle}{\partial r_k}-\frac{\partial\langle h_ih_kh_i'T_j''\rangle}{\partial r_k'}-\frac{\partial\langle u_iu_k'h_i'T_j''\rangle}{\partial r_k}$$

$$+\frac{\partial\langle u_iu_i'h_k'T_j''\rangle}{\partial r_k}-\frac{\partial\langle u_iu_k''h_i'T_j''\rangle}{\partial r_k'}+\frac{\partial\langle wh_i'T_j''\rangle}{\partial r_i}+\frac{\partial\langle wh_i'T_j''\rangle}{\partial r_i'}$$

$$-2\in_{mki}\Omega_m\langle u_ih_i'T_j''\rangle+f(\langle u_ih_i'T_j''\rangle-\langle v_ih_i'T_j''\rangle)\,. \qquad \text{--------- (4.10.5)}$$

In order to write the equation (4.10.5) to spectral form, we can define the following six dimensional Fourier transforms:

$$\langle u_i h_i'(\hat{r})T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_i \beta_i'(\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right]d\hat{k}\,d\hat{k}', \qquad \text{--------- (4.10.6)}$$

$$\langle u_i u_k h_i'(\hat{r})T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_i \varphi_k \beta_i'(\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right]d\hat{k}\,d\hat{k}', \qquad \text{-------- (4.10.7)}$$

$$\langle h_i h_k h_i'(\hat{r})T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \beta_i \beta_k \beta_i'(\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right]d\hat{k}\,d\hat{k}', \qquad \text{-------- (4.10.8)}$$

$$\langle u_i u_k' h_i'(\hat{r})T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_i \phi_k(\hat{k})\beta_i'(\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right]d\hat{k}\,d\hat{k}', \qquad \text{--------- (4.10.9)}$$

$$\langle u_i u_i'(\hat{r})h_k'(\hat{r})T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_i \phi_i'(\hat{k})\beta_i'(\hat{k})\theta_k'\,\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right]d\hat{k}\,d\hat{k}', \text{-- (4.10.10)}$$

$$\langle w h_i'(\hat{r})T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \gamma \beta_i'(\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right]d\hat{k}\,d\hat{k}', \qquad \text{------- (4.10.11)}$$

$$\langle v_i h_i'(\hat{r})T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \mu_i \beta_i'(\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}'\right]d\hat{k}\,d\hat{k}'. \qquad \text{------- (4.10.12)}$$

Interchanging the points p' and p'' along with the indices i and j result in the relations

$$\langle u_i u_k'' h_i' T_j''\rangle = \langle u_i u_k' h_i' T_j''\rangle .$$

By use of this fact and equations (4.10.6)-(4.10.12), the equation (4.10.5) may be transformed as

$$\frac{\partial\langle \phi_i \beta_i' \theta_j''\rangle}{\partial t} + \nu\left[(1+\frac{1}{p_M})k^2 + (1+\frac{1}{p_r})k'^2 + 2k_k k_k' + \frac{2\in_{mki}\Omega_m}{\nu} - \frac{f}{\nu}\right]\langle \phi_i \beta_i' \theta_j''\rangle$$

$$= i(k_k + k_k')\langle \phi_i \phi_k \beta_i' \theta_j''\rangle - i(k_k + k_k')\langle \beta_i \beta_k \beta_i' \theta_j''\rangle - i(k_k + k_k')\langle \phi_i \phi_k' \beta_i' \theta_j''\rangle + ik_k\langle \phi_i \phi_i' \beta_k' \theta_j''\rangle$$

$$+ i(k_i + k_i')\langle \gamma \beta_i' \theta_j''\rangle - f\langle \mu_i \beta_i' \theta_j''\rangle . \qquad \text{------- (4.10.13)}$$

The tensor equation (4.10.12) can be converted to scalar equation by contraction of the indices i and j

$$\frac{\langle\partial\langle\phi_i\beta_i'\theta_i''\rangle}{\partial t}+\nu\left[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2+2k_kk_k'+2\frac{\in_{mki}\Omega_m}{\nu}-\frac{f}{\nu}\right]\langle\phi_i\beta_i'\theta_i''\rangle$$

$$= i(k_k + k_k')\langle\phi_i\phi_k\beta_i'\theta_i''\rangle - i(k_k + k_k')\langle\beta_i\beta_k\beta_i'\theta_i''\rangle - i(k_k + k_k')$$

$$\times\langle\phi_i\phi_k\beta_i'\theta_i''\rangle+ik_k\langle\phi_i\phi_i'\beta_k'\theta_i''\rangle+i(k_i+k_i')\langle\gamma_i\beta_i'\theta_i''\rangle-f\langle\mu_i\beta_i'\theta_i''\rangle. \qquad \text{-------}(4.10.14)$$

If derivative with respect to x_i is taken of the momentum equation (4.10.1) for the point p, the equation multiplied through by $h_i T_j''$ and time average taken, the resulting equation

$$-\frac{\partial^2\langle wh_i'\ T_j''\rangle}{\partial x_i\partial x_i}=\frac{\partial^2}{\partial x_i\partial x_k}\left(\langle u_iu_kh_i'T_j''\rangle-\langle h_ih_kh_i'T_j''\rangle\right). \qquad \text{-------}(4.10.15)$$

Writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$-\left[\frac{\partial^2}{\partial r_i\partial r_i}+2\frac{\partial^2}{\partial r_i\partial r_i'}+\frac{\partial^2}{\partial r_i'\partial r_i'}\right]\langle wh_i'T_j''\rangle=\left[\frac{\partial^2}{\partial r_i\partial r_k}+\frac{\partial^2}{\partial r_i'\partial r_k}+\frac{\partial^2}{\partial r_i\partial r_k'}+\frac{\partial^2}{\partial r_i'\partial r_k'}\right]$$

$$\times\left(\langle u_iu_kh_i'T_j''\rangle-\langle h_ih_kh_i'T_j''\rangle\right). \qquad \text{-------}(4.10.16)$$

Now taking the Fourier transforms of equation (4.10.16)

$$-\langle\gamma\beta_i'\theta_j''\rangle=\frac{(k_ik_k+k_i'k_k+k_ik_k'+k_i'k_k')(\langle\phi_i\phi_k\beta_i'\theta_j''\rangle-\langle\beta_i\beta_k\beta_i'\theta_j''\rangle)}{k_ik_i+2k_i'k_i+k_i'k_i'}. \qquad \text{-}(4.10.17)$$

Equation (4.10.17) can be used to eliminate $\langle\gamma\beta_i'\theta_j''\rangle$ from equation (4.10.13).

4.11 Solution for times before the final period:

It is known that equation for final period of decay is obtained by considering the two-point correlations after neglecting the 3rd order correlation terms. To study the decay for times before the final period, the three point correlations are considered and the quadruple correlation terms are neglected because the quadruple correlation terms decays faster than the lower-order correlation terms. Equation (4.10.17) shows that term $\langle\gamma\beta_i'\theta_i''\rangle$ associated with the pressure fluctuations should also be neglected. Thus neglecting all the terms on the right hand side of equation (4.10.14)

$$\frac{\partial\langle\phi_i\beta_i'\theta_i''\rangle}{\partial t}+\nu\left[(1+\frac{1}{P_M})k^2+(1+\frac{1}{P_r})k'^2+2k_k k_k'+\frac{2\in_{mki}\Omega_m}{\nu}-\frac{fS}{\nu}\right]\langle\phi_i\beta_i'\theta_i''\rangle=0\,,\quad\text{--- (4.11.1)}$$

where $\langle\mu_i\beta_i'\theta_i''\rangle=R\langle\phi_i\beta_i'\theta_i''\rangle$ and 1-R=S, here R and S are arbitrary constant.

Integrating the equation (4.11.1) between t_o and t with inner multiplication by k_k and gives

$$k_k\langle\phi_i\beta_i'\theta_i''\rangle=k_k\left[\phi_i\beta_i'\theta_i''\right]_0\exp\left[-\nu\left\{(1+\frac{1}{P_M})k^2+(1+\frac{1}{P_r})k'^2+2kk'\cos\theta+\frac{2\in_{mki}\Omega_m}{\nu}-\frac{fS}{\nu}\right\}(t-t_0)\right],$$

--------- (4.11.2)

where θ is the angle between k and k' and $\langle\phi_i\beta_i'\theta_i''\rangle_0$ is the value of $\langle\phi_i\beta_i'\theta_i''\rangle$ at t= t_o.

Now by letting $r'=0$ in equation (4.10.6) and comparing with equations (4.9.8) and (4.9.9), we get

$$\langle\phi_i\psi_k\tau_i'(\hat{k})\rangle=\int_{-\infty}^{\infty}\langle\phi_i\beta_i'\theta_i''\rangle d\hat{k}'\,,\quad\text{-------- (4.11.3)}$$

$$\langle\phi_i\psi_i\tau_i'(-\hat{k})\rangle=\int_{-\infty}^{\infty}\phi_k\beta_i'(-\hat{k})\theta_i''(-\hat{k}')d\hat{k}'\,.\quad\text{-------- (4.11.4)}$$

Substituting equation (4.11.2) - (4.11.4) in equation (4.9.11), we get

$$\frac{\partial\langle\psi_i\tau_i'(\hat{k})\rangle}{\partial t}+\nu(\frac{1}{P_M}+\frac{1}{P_r})k^2\langle\psi_i\tau_i'(\hat{k})\rangle=-\int_{-\infty}^{\infty}ik_k\left[\langle\phi_i\beta_i'\theta_i''\rangle+2\langle\phi_k\beta_i'(-\hat{k})\theta_i''(-\hat{k}')\rangle\right]_0$$

$$\times\exp\left[-\nu(t-t_o)\right]\{(1+\frac{1}{P_M})k^2+(1+\frac{1}{P_r})k'^2+2kk'\cos\theta+2\frac{\in_{mki}\Omega_m}{\nu}-\frac{fS}{\nu}\}]d\hat{k}'\,.\quad\text{----- (4.11.5)}$$

Now, $d\hat{k}'$ can be expressed in terms of k' and θ as $-2\pi k'^2 d(\cos\theta)dk'$ (cf. Deissler [27]).

Hence $d\hat{k}' = -2\pi k'^2 d(\cos\theta) d\hat{k}'$. --------- (4.11.6)

Putting equation (4.11.6) in equation (4.11.5) yields

$$\frac{\partial\langle\psi_i\tau_i'(\hat{k})\rangle}{\partial t} + \nu\left(\frac{1}{p_M}+\frac{1}{p_r}\right)k^2\langle\psi_i\tau_i'(\hat{k})\rangle = -\int_{-\infty}^{\infty} 2\pi i k_k\left[\langle\phi_i\beta_i'\theta_i''\rangle + 2\langle\phi_k\beta_i'(-\hat{k})\theta_i''(-\hat{k}')\rangle\right]_0 k'^2$$

$$\times\left[\int_{-1}^{1}\exp\left\{-\nu(t-t_0)\left[(1+\frac{1}{p_M})k^2 + (1+\frac{1}{p_r})k'^2 + 2kk'\cos\theta + 2\frac{\in_{mki}\Omega_m}{\nu} - \frac{fS}{\nu}\right]\right\}\right]d(\cos\theta)\,d\hat{k}' \text{ .-(4.11.7)}$$

In order to find the solution completely and following Loeffler and Deissler [72] we assume that

$$ik_k\left[\langle\phi_i\beta_i'(\hat{k})\theta_i''(\hat{k}')\rangle + 2\langle\phi_k\beta_i'(-\hat{k})\theta_i''(-\hat{k}')\rangle\right]_0 = \frac{\beta_0}{(2\pi)^2}\left(k^2k'^4 - k^4k'^2\right), \quad \text{----------- (4.11.8)}$$

where β_0 is a constant depending on the initial conditions. Substituting equation (4.11.8) into equation (4.11.7) and completing the integration with respect to $\cos\theta$, one obtains

$$\frac{\partial 2\pi\langle\psi_i\tau_i'(\hat{k})\rangle}{\partial t} + \nu\left(\frac{1}{p_M}+\frac{1}{p_r}\right)k^2 2\pi\langle\psi_i\tau_i'(\hat{k})\rangle = -\frac{\beta_0}{2\nu(t-t_0)}\int_0^{\infty}\left(k^3k'^5 - k^5k'^3\right)$$

$$[\exp\{-\nu(t-t_0)[(1+\frac{1}{p_M})k^2 + (1+\frac{1}{p_r})k'^2 - 2kk' + \frac{2\in_{mki}\Omega_m}{\nu} - \frac{fS}{\nu}]\}$$

$$+ -\frac{\beta_0}{2\nu(t-t_0)}\int_0^{\infty}\left(k^3k'^5 - k^5k'^3\right)\exp\{-\nu(t-t_0)[(1+\frac{1}{p_M})k^2 + (1+\frac{1}{p_r})k'^2 + 2kk' + 2\frac{\in_{mki}\Omega_m}{\nu} - \frac{fS}{\nu}]dk'.$$

---------- (4.11.9)

Multiplying both sides of equation (4.11.9) by k^2, we get

$$\frac{\partial Q}{\partial t} + \nu\left(\frac{1}{p_M}+\frac{1}{p_r}\right)k^2 Q = F, \quad \text{-------- (4.11.10)}$$

where, $Q=2\pi k^2 \langle \psi_i \tau'(\hat{k})\rangle$. -------- (4.11.11)

Q is the Magnetic energy Spectrum function.

and

$$F=-\frac{\beta_0}{2\nu(t-t_0)}\int_0^{\infty}(k^3k'^5-k^5k'^3)\exp\left\{-\nu(t-t_0)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2-2kk'+\frac{2\in_{mki}\Omega_m}{\nu}-\frac{fS}{\nu}]\right\}$$

$$+\frac{\beta_0}{2\nu(t-t_0)}\int_0^{\infty}(k^3k'^5-k^5k'^3)\exp\left\{-\nu(t-t_0)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2+2kk'+\frac{2\in_{mki}\Omega_m}{\nu}-\frac{fS}{\nu}]\right\}d\hat{k}'.$$

-------- (4.11.12)

Integrating equation (4.11.12) with respect to k', we have

$$F=-\frac{\beta_0\sqrt{\pi}\,p_r^{5/2}}{2\nu^{3/2}(t-t_o)(1+p_r)^{5/2}}\exp\left[-\left\{\frac{2\in_{mki}\Omega_m}{\nu}-\frac{fS}{\nu}\right\}(t-t_o)\right]$$

$$\times\exp\left[-\nu(t-t_o)(1+\frac{1}{p_M}-\frac{p_r}{1+p_r})k^2\right]$$

$$\left[\frac{15p_rk^4}{4\nu^2(t-t_o)^2(1+p_r)}+\left\{\frac{5p_r^2}{(1+p_r)^2}-\frac{3}{2}\right\}\frac{k^6}{\nu(t-t_o)}+\left\{\frac{p_r^3}{(1+p_r)^3}-\frac{p_r}{(1+p_r)}\right\}k^8\right]. \quad \text{--- (4.11.13)}$$

The series of equation (4.11.13) contains only even powers of k and start with k^4 and the equation represents the transfer function arising owing to consideration of magnetic field at three points at a time.

It is interesting to note that if we integrate equation (4.11.13) over all ware numbers, we find that

$$\int_0^{\infty}F\,dk=0 \qquad \text{-------- (4.11.14)}$$

which indicates that the expression for F satisfies the condition of continuity and homogeneity.

The linear equation (4.11.10) can be solved to give

$$Q = \exp\left[-\nu k^2\left(\frac{1}{p_M}+\frac{1}{p_r}\right)(t-t_o)\right]\int F\exp\left[\nu k^2\left(\frac{1}{p_M}+\frac{1}{p_r}\right)(t-t_o)\right]dt$$

$$+J(k)\exp\left[-\nu k^2\left(\frac{1}{p_M}+\frac{1}{p_r}\right)(t-t_o)\right], \qquad \text{-------- (4.11.15)}$$

where $J(K)=\frac{N_0k^2}{\pi}$ is a constant of integration. Substituting the values of F from equation (4.11.13) in to equation (4.11.15) and integrating with respect to t, we get

$$Q(\hat{k},t) = \frac{N_0k^2}{\pi}\exp\left[-\nu k^2\left(\frac{1}{p_M}+\frac{1}{p_r}\right)(t-t_o)\right] + \frac{\beta_0\sqrt{\pi}\,p_r^{3/2}}{2\nu^{3/2}(1+p_r)^{7/2}}$$

$$\times\exp[-\{2\in_{mki}\Omega_m - fS\}(t-t_o)]\exp\left[-\nu k^2(t-t_o)\left\{\frac{1+p_r+p_M}{p_M(1+p_r)}\right\}\right]$$

$$\left[\frac{3p_rk^4}{2\nu^2(t-t_o)^{5/2}} + \frac{p_r(7p_r-6)k^6}{3\nu(1+p_r)(t-t_o)^{3/2}} - \frac{4(3p_r^2-2p_r+3)k^8}{3(1+p_r)^2(t-t_o)^{1/2}}\right.$$

$$\left.+\frac{8\sqrt{\nu}(3p_r^2-2p_r+3)k^9}{3(1+p_r)^{5/2}\sqrt{p_r}}N(\omega)\right] \qquad \text{--------- (4.11.16)}$$

where $N(\omega) = e^{-\omega^2}\int_0^{\omega} e^{x^2}dx,$

$$\omega = k\sqrt{\frac{\lambda(t-t_o)}{p_r(1+p_r)}}.$$

The function has been calculated numerically and tabulated in [24].

By setting $\hat{r}=0$, $j=i$, $d\hat{K} = -2\pi k^2 d(\cos\theta)dk$ and $Q = 2\pi k^2\left\langle \psi_i\tau_i'(\hat{K})\right\rangle$ in equation (4.9.7), we get the expression for temperature energy decay as

$$\frac{\langle T^2\rangle}{2} = \frac{T_iT_i'}{2} = \int_0^{\infty} Q(\hat{k})d\hat{k}. \qquad \text{---------- (4.11.17)}$$

Substituting equations (4.11.16) in to (4.11.17) and after integration, we get

$$\frac{\langle T^2\rangle}{2}=\frac{N_o P_r^{3/2} p_M^{3/2}(t-t_o)^{1/2}}{4\sqrt{\pi}\nu^{3/2}(p_r+p_M)^{3/2}}+\exp[-\{2\in_{mki}\Omega_m-fS\}]\quad\frac{\beta_0\pi p_r^{7/2}p_M^{5/2}(t-t_o)^{-5}}{2\nu^6(1+p_r)(1+p_r+p_M)^{5/2}}\times$$

$$\left\{\frac{9}{16}+\frac{5p_M(7p_r-6)}{16(1+p_r+p_M)}-\frac{35p_M^2(3p_M^2-2p_r+3)}{8p_r(1+p_r+p_M)^2}+\frac{8p_M^3(3p_r^2-2p_r+3)}{3.2^6.p_r^2(1+p_r+p_M)^3}\sum_{n=0}^{\infty}\frac{1.3.5....(2n+9)}{n!(2n+1)2^{2n}(1+p_r)^n}\right\}$$

or

$$\frac{\langle T^2\rangle}{2}=\frac{N_0 P_r^{3/2} p_M^{3/2}(t-t_o)^{-3/2}}{4\sqrt{\pi}\nu^{3/2}(p_r+p_M)^{3/2}}+\beta_0 z\nu^{-6}(t-t_o)^{-5}\times\exp[-\{2\in_{mki}\Omega_m-fS\}],\quad \text{- (4.11.18)}$$

where

$$Z=\frac{\pi p_r^{7/2}p_M^{5/2}}{2(1+p_r)(1+p_r+p_M)^{5/2}}$$

$$\left[\frac{9}{16}+\frac{5P_M(7P_r-6)}{16(1+P_r+p_M)}-\frac{35p_M^2(3p_r^2-2p_r+3)}{8p_r(1+p_r+p_M)^2}+\frac{8p_M^3(3p_r^2-2p_r+3)}{3.2^6 p_r^2(1+p_r+p_M)^3}\sum_{n=0}^{\infty}\frac{1.3.5....(2n+9)}{n!(2n+1)2^{2n}(1+p_r)^n}\right].$$

Thus the energy decay law for temperature field fluctuation of dusty fluid MHD turbulence in a rotating system before the final period may be written as

$$\langle T^2\rangle=X(t-t_o)^{-3/2}+\exp[-\{2\in_{mki}\Omega_m-fS\}]Y(t-t_o)^{-5},\quad \text{--------- (4.11.19)}$$

where

$$X=\frac{N_0 P_r^{3/2}p_M^{3/2}}{2\sqrt{\pi}\nu^{3/2}(p_r+p_M)}\quad \text{and}\quad Y=2\beta_0 Z\nu^{-6}.$$

$\langle T^2\rangle$ is the total "energy" (the mean square of the temperature fluctuations) t is the time, x and t_o are constants determined by the initial conditions. The constant Y depends on both initial conditions and the fluid Prandtl number.

4.12 Concluding Remarks:

In equation (4.11.19) we obtained the decay law of temperature fluctuations in MHD turbulence before the final period in a rotating system in presence of dust particle considering three-point correlation equation after neglecting quadruple correlation terms. If the fluid is clean and the system is non-rotating then f=0, $\Omega=0$ the equation (4.11.19) becomes.

$$\langle T^2\rangle = X(t-t_o)^{-3/2} + Y(t-t_o)^{-5} \qquad \text{---------- (4.12.1)}$$

which was obtained earlier by Sarker and Rahman [113]

In the absence of a magnetic field, magnetic Prandtl number coincides with the Prandtl number (i.e. p_r=p_M) and the system is non-rotating with clean fluid the equation (4.11.18) becomes

$$\frac{\langle T^2\rangle}{2} = \frac{N_0 p_r^{\;3/2}}{8\sqrt{2\pi}\nu^{3/2}(t-t_o)^{3/2}} + \frac{\beta_0 Z}{\nu^6 (t-t_o)^5} \qquad \text{----------- (4.12.2)}$$

which was obtained earlier by Loeffler and Deissler [72].

Here we conclude that due to the effect of rotation in presence of dust particles in the flow field, the turbulent energy decays more rapidly than the energy for non-rotating clean fluid.

The 1st term of the right hand side of equation (4.11.19) corresponds to the temperature energy for two-point correlation and second term represents temperature energy for three-point correlation. For large times the last term in the equation (4.11.19) becomes negligible, leaving the -3/2power decay law for the final period. If higher order correlations are considered in the analysis, it appears that more terms of higher power of time would be added to the equation (4.11.19).

CHAPTER-V

PART-A

DECAY OF MAGNETO-HYDRODYNAMIC TURBULENCE BEFORE THE FINAL PERIOD FOR THE CASE OF MULTI-POINT AND MULTI-TIME IN A ROTATING SYSTEM

5.1 Introduction:

The magneto-hydrodynamic turbulence is the study of the interaction between a magnetic field and the turbulent motions of an electricity conducting fluid. The interaction between the velocity and the magnetic fields results in a transfer of energy between the kinetic and magnetic spectra and it is through that the interstellar magnetic field is maintained by a "dynamo" action from turbulence in the interstellar gas. Modern applications of magneto–hydrodynamics in the fields of propulsion, nuclear fission and electrical power generation make the problem of magneto-hydrodynamic turbulence one of considerable interest to engineers, since turbulent phenomena seen to be inherent in almost all type of flow problems. In what follows, we consider the magneto-hydrodynamic turbulent flow in a rotating system. When the bulk of the fluid, the coriolis and centrifugal force must be supposed to act on the fluid. The coriolis force due to rotation plays an important role in a rotating system of turbulent flow, while the centrifugal force with the potential is incorporated in to the pressure. Kishore and Dixit {52}, Kishore and Singh [54], Dixit and Upadhyay [31] and Kishore and Golsefied [57] discused the effect of coriolis force on acceleration covariance in ordinary and MHD turbulent flows. Kishore and Upadhyay [63] studied the decay of MHD turbulence in rotating system. Shimomura and Yoshizawa [119], Shimomura [120] discussed the statistical analysis of turbulent viscosity, turbulent sealer flux respectively in a rotating system two-scale direct interaction approach. Sarker and Islam [117] also studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time.

Dessiler [27,28] developed a theory for homogeneous turbulence, which was valid for times before the final period. Using Deissler's theory Loeffler and Deissler[72] studied the temperature fluctuations in homogeneous turbulence before the final period. Sarker and Kishore [108] studied the decay of MHD turbulence before the final period. Sarker and Islam [115] also studied the decay of dusty fluid turbulence before the final period in a rotating system. Islam and Sarker [46] studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Kumar and Patel [65] also studied on first-order reactant in homogeneous turbulence before the final period of decay for the case of multi-point and multi-time. Sarker and Islam [116] studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time. In their approach they considered two and three-point correlations and fourth-order correlation terms are neglected in comparison to the second-and third-order correlation terms.

Here, we have studied the decay of MHD turbulence before the final period in a rotating system for the case of multi-point and multi-time using two-and three-point correlation equations after neglecting fourth-order correlation terms which are compared to the second-and third-order correlation terms. Finally the decay law of magnetic energy fluctuations of MHD turbulence in a rotating system before the final period for the case of multi-point and multi-time is obtained.

5.2 Basic Equations:

The equations of motion and continuity for viscous, incompressible MHD turbulent flow in a rotating system are given by

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial w}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2\in_{mkl}\Omega_m u_l, \qquad \text{--------- (5.2.1)}$$

$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \frac{\nu}{P_M}\frac{\partial^2 h_i}{\partial x_k \partial x_k}, \qquad \text{--------- (5.2.2)}$$

$$\frac{\partial v_i}{\partial t}+v_k\frac{\partial v_i}{\partial x_K} = -\frac{K}{m_s}(v_i - u_i) \qquad \text{--------- (5.2.3)}$$

with

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial v_i}{\partial x_i} = \frac{\partial h_i}{\partial x_i} = 0. \qquad \text{---------- (5.2.4)}$$

Here, u_i, turbulence velocity component; h_i, magnetic field fluctuation component; v_i, dust particle velocity component; $w(\hat{x},t) = \frac{P}{\rho} + \frac{1}{2}\langle h^2 \rangle + \frac{1}{2}\left|\hat{\Omega}\times\hat{x}\right|^2$, total MHD pressure $p(\hat{x},t)$, hydrodynamic pressure; ρ, fluid density; ν, Kinematic viscosity; P_M, magnetic prandtl number; x_k, space co-ordinate; the subscripts can take on the values 1, 2 or 3 and the repeated subscripts in a term indicate a summation; Ω_m, constant angular velocity component; $\in_{mkl}$, alternating tensor; $m_s = \frac{4}{3}\pi R_s^3 \rho_s$, mass of single spherical dust particle of radius R_s; ρ_s, constant density of the material in dust particle.

5.3 Two-Point, Two-Time Correlation and Spectral Equations:

Induction equations of a magnetic field at the point p and p′ separated by the vector $\hat{r}$ could be written as

$$\frac{\partial h_i}{\partial t} + u_k \frac{\partial h_i}{\partial x_k} - h_k \frac{\partial u_i}{\partial x_k} = \frac{\nu}{P_M}\frac{\partial^2 h_i}{\partial x_k \partial x_k} \qquad \text{------------ (5.3.1)}$$

and
$$\frac{\partial h'_j}{\partial t'} + u'_k \frac{\partial h'_j}{\partial x'_k} - h'_k \frac{\partial u'_j}{\partial x'_k} = \frac{\nu}{P_M}\frac{\partial^2 h'_j}{\partial x'_k \partial x'_k}. \qquad \text{------------ (5.3.2)}$$

Multiplying equation (5.3.1) by h'_j and equation (5.3.2) by h_i and taking ensemble average, we get

$$\frac{\partial\langle h_i h'_j \rangle}{\partial t} + \frac{\partial}{\partial x_k}\left[\langle u_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle\right] = \frac{\nu}{P_M}\frac{\partial^2\langle h_i h'_j \rangle}{\partial x_k \partial x_k} \qquad \text{---------- (5.3.3)}$$

and
$$\frac{\partial\langle h_i h'_j \rangle}{\partial t'} + \frac{\partial}{\partial x'_k}\left[\langle u'_k h_i h'_j \rangle - \langle u'_j h_i h'_k \rangle\right] = \frac{\nu}{P_M}\frac{\partial^2\langle h_i h'_j \rangle}{\partial x'_k \partial x'_k}. \qquad \text{----------- (5.3.4)}$$

Angular bracket $\langle - - - - - \rangle$ is used to denote an ensemble average.

Using the transformations

$$\frac{\partial}{\partial x_k} = -\frac{\partial}{\partial r_k}, \frac{\partial}{\partial x'_k} = \frac{\partial}{\partial r_k}, \left(\frac{\partial}{\partial t}\right)t' = \left(\frac{\partial}{\partial t}\right)\Delta t - \frac{\partial}{\partial \Delta t}, \frac{\partial}{\partial t'} = \frac{\partial}{\partial \Delta t}$$ ----------- (5.3.5)

into equations (5.3.3) and (5.3.4), we can write

$$\frac{\partial\langle h_i h'_j\rangle}{\partial t} + \frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle - \langle u'_j h_i h'_k\rangle\right](\hat{r}, \Delta t, t) - \frac{\partial}{\partial r_k}\left[\langle u_k h_i h'_j\rangle - \langle u_i h_k h'_j\rangle\right](\hat{r}, \Delta t, t) = \frac{2\nu}{P_M}\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k \partial r_k}$$

----------- (5.3.6)

and $$\frac{\partial\langle h_i h'_j\rangle}{\partial \Delta t} + \frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle - \langle u'_j h_i h'_k\rangle\right](\hat{r}, \Delta t, t) = \frac{\nu}{P_M}\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k \partial r_k}.$$ ----------- (5.3.7)

Using the relations of Chandrasekhar [19].

$\langle u_k h_i h'_j\rangle = -\langle u'_k h_i h'_j\rangle$, and $\langle u'_j h_i h'_k\rangle = \langle u_i h_k h'_j\rangle$.

Equations (5.3.6) and (5.3.7) become

$$\frac{\partial\langle h_i h'_j\rangle}{\partial t} + 2\frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle - \langle u_i h_k h'_j\rangle\right] = \frac{2\nu}{P_M}\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k \partial r_k}$$ ----------- (5.3.8)

and $$\frac{\partial\langle h_i h'_j\rangle}{\partial t} + \frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle - \langle u_i h_k h'_j\rangle\right] = \frac{\nu}{P_M}\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k \partial r_k}.$$ ----------- (5.3.9)

Now we write equations (5.3.8) and (5.3.9) in spectral form in order to reduce it to an ordinary differential equation by use of the following three-dimensional Fourier transforms:

$$\langle h_i h'_j\rangle(\hat{r}, \Delta t, t) = \int_{-\infty}^{\infty}\langle \psi_i \psi'_j\rangle(\hat{K}, \Delta t, t)\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K},$$ --------- (5.3.10)

$$\langle u_i h_k h'_j\rangle(\hat{r}, \Delta t, t) = \int_{-\infty}^{\infty}\langle \alpha_i \psi_k \psi'_j\rangle(\hat{K}, \Delta t, t)\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K}.$$ --------- (5.3.11)

Interchanging the subscripts i and j then interchanging the points p and p′ gives

$$\langle u'_k h_i h'_j\rangle(\hat{r}, \Delta t, t) = \langle u_k h_i h'_j\rangle(-\hat{r}, -\Delta t, t + \Delta t)$$

$$= \int_{-\infty}^{\infty} \langle \alpha_i \psi_i \psi'_j \rangle (-\hat{K}, -\Delta t, t + \Delta t) \exp[\hat{i}(\hat{K}.\hat{r})] d\hat{K} , \qquad \text{---------- (5.3.12)}$$

where $\hat{K}$ is known as a wave number vector and $d\hat{K}$=$dK_1\, dK_2\, dK_3$. The magnitude of $\hat{K}$ has the dimension 1/length and can be considered to be the reciprocal of an eddy size. Substituting of equation (5.3.10) to (5.3.12) in to equations (5.3.8) and (5.3.9) leads to the spectral equations

$$\frac{\partial \langle \psi_i \psi'_j \rangle}{\partial t} + \frac{2 v k^2}{P_M} \langle \psi_i \psi'_j \rangle = 2 i k_k \left[\langle \alpha_i \psi_k \psi'_j \rangle (\hat{K}, \Delta t, t) - \langle \alpha_k \psi_i \psi'_j \rangle (-\hat{K}, -\Delta t, t + \Delta t) \right] \quad \text{-- (5.3.13)}$$

and

$$\frac{\partial \langle \psi_i \psi'_j \rangle}{\partial \Delta t} + \frac{v k^2}{P_M} \langle \psi_i \psi'_j \rangle = i k_k \left[\langle \alpha_i \psi_k \psi'_j \rangle (\hat{K}, \Delta t, t) - \langle \alpha_k \psi_i \psi'_j \rangle (-\hat{K}, -\Delta t, t + \Delta t) \right]. \quad \text{--- (5.3.14)}$$

The tensor equations (5.3.13) and (5.3.14) becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial \langle \psi_i \psi'_i \rangle}{\partial t} + \frac{2 v k^2}{P_M} \langle \psi_i \psi'_i \rangle = 2 i k_k \left[\langle \alpha_i \psi_k \psi'_i \rangle (\hat{K}, \Delta t, t) - \langle \alpha_k \psi_i \psi'_i \rangle (-\hat{K}, -\Delta t, t + \Delta t) \right] \quad \text{---- (5.3.15)}$$

and

$$\frac{\partial \langle \psi_i \psi'_i \rangle}{\partial \Delta t} + \frac{v k^2}{P_M} \langle \psi_i \psi'_i \rangle = i k_k \left[\langle \alpha_i \psi_k \psi'_i \rangle (\hat{K}, \Delta t, t) - \langle \alpha_k \psi_i \psi'_i \rangle (-\hat{K}, -\Delta t, t + \Delta t) \right]. \quad \text{------- (5.3.16)}$$

The terms on the right side of equations (5.3.15) and (5.3.16) are collectively proportional to what is known as the magnetic energy transfer terms.

5.4 Three-Point, Three-Time Correlation and Spectral Equations:

Similar procedure can be used to find the three-point correlation equations. For this purpose we take the momentum equation of MHD turbulence at the point P, the induction equation of magnetic field fluctuations at p′ and p″ separated by the vector $\hat{r}$ and $\hat{r}'$ as

$$\frac{\partial u_l}{\partial t} + u_k \frac{\partial u_l}{\partial x_k} - h_k \frac{\partial h_l}{\partial x_k} = -\frac{\partial w}{\partial x_l} + v \frac{\partial^2 u_l}{\partial x_k \partial x_k} - 2 \in_{mkl} \Omega_m u_l , \qquad \text{----------- (5.4.1)}$$

$$\frac{\partial h_i'}{\partial t'}+u_k'\frac{\partial h_i'}{\partial x_k'}-h_k'\frac{\partial u_i'}{\partial x_k'}=\frac{\nu}{P_M}\frac{\partial^2 h_i'}{\partial x_k'\partial x_k'}, \qquad \text{----------- (5.4.2)}$$

$$\frac{\partial h_j''}{\partial t''}+u_k''\frac{\partial h_j''}{\partial x_k''}-h_k''\frac{\partial u_j''}{\partial x_k''}=\frac{\nu}{P_M}\frac{\partial^2 h_j''}{\partial x_k''\partial x_k''}. \qquad \text{----------- (5.4.3)}$$

Multiplying equation (5.4.1) by $h'_i\, h_j''$, equation (5.4.2) by $u_l h_j''$ and equation (5.4.3) by $u_l h_i'$, taking ensemble average, one obtains

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t}+\frac{\partial}{\partial x_k}\left[\langle u_k u_l h_i' h_j''\rangle-\langle h_k h_l h_i' h_j''\rangle\right]=\frac{\partial\langle w h_i' h_j''\rangle}{\partial x_l}+\nu\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k\partial x_k}-2\in_{mkl}\Omega_m\langle u_l h_i' h_j''\rangle,$$

$$\text{--------- (5.4.4)}$$

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t'}+\frac{\partial}{\partial x_k'}\left[\langle u_l u_k' h_i' h_j''\rangle-\langle u_l u_i' h_k' h_j''\rangle\right]=\frac{\nu}{P_M}\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k'\partial x_k'}, \qquad \text{--------- (5.4.5)}$$

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t''}+\frac{\partial}{\partial x_k''}\left[\langle u_l u_k'' h_i' h_j''\rangle-\langle u_l u_j'' h_i' h_k''\rangle\right]=\frac{\nu}{P_M}\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k''\partial x_k''}. \qquad \text{--------- (5.4.6)}$$

Using the transformations

$$\frac{\partial}{\partial x_k}=-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right),\ \frac{\partial}{\partial x_k'}=\frac{\partial}{\partial r_k},\ \frac{\partial}{\partial x_k''}=\frac{\partial}{\partial r_k'},$$

$$\left(\frac{\partial}{\partial t}\right)t',t''=\left(\frac{\partial}{\partial t}\right)\Delta t,\Delta t'-\frac{\partial}{\partial \Delta t}-\frac{\partial}{\partial \Delta t'},$$

$$\frac{\partial}{\partial t'}=\frac{\partial}{\partial \Delta t},\ \frac{\partial}{\partial t''}=\frac{\partial}{\partial \Delta t'}$$

into equations (5.4.4) to (5.4.6), we have

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t}-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right)\left[\langle u_k u_l h_i' h_j''\rangle-\langle h_k h_l h_i' h_j''\rangle\right]+\frac{\partial}{\partial r_k}\left[\langle u_l u_k' h_i' h_j''\rangle-\langle u_l u_i' h_k' h_j''\rangle\right]$$

$$+\frac{\partial}{\partial r_k'}\left[\langle u_l u_k'' h_i' h_j''\rangle-\langle u_l u_j'' h_i' h_k''\rangle\right]=-\left(\frac{\partial}{\partial r_l}+\frac{\partial}{\partial r_l'}\right)\langle w h_i' h_j''\rangle+\nu\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right)^2\langle u_l h_i' h_j''\rangle$$

$$+\frac{\nu}{P_M}\left[\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k\partial r_k}+\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k'\partial r_k'}\right]-2\in_{mkl}\Omega_m\langle u_l h_i' h_j''\rangle, \qquad \text{-------- (5.4.7)}$$

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial \Delta t}+\frac{\partial}{\partial r_k}\left[\langle u_l u_k' h_i' h_j''\rangle-\langle u_l u_l' h_k' h_j''\rangle\right]=\frac{\nu}{P_M}\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k \partial r_k} \quad \text{-------- (5.4.8)}$$

and

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial \Delta t'}+\frac{\partial}{\partial r_k'}\left[\langle u_l u_k'' h_i' h_j''\rangle-\langle u_l u_j'' h_i' h_k''\rangle\right]=\frac{\nu}{P_M}\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k' \partial r_k'}. \quad \text{--------- (5.4.9)}$$

In order to convert equations (5.4.7)-(5.4.9) to spectral form, we can define six dimensional Fourier transforms:

$$\langle u_l h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')\right]d\hat{K}d\hat{K}', \quad \text{-- (5.4.10)}$$

$$\langle u_l u_k' h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \phi_K' \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')\right]d\hat{K}d\hat{K}', \text{-(5.4.11)}$$

$$\langle w h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \gamma \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')\right]d\hat{K}d\hat{K}', \quad \text{-- (5.4.12)}$$

$$\langle u_k u_l h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_K \phi_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')\right]d\hat{K}d\hat{K}', \text{-(5.4.13)}$$

$$\langle h_k h_l h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \beta_K \beta_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')\right]d\hat{K}d\hat{K}', \text{- (5.4.14)}$$

$$\langle u_l u_i' h_K' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \phi_i' \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')\right]d\hat{K}d\hat{K}'. \text{- (5.4.15)}$$

Interchanging the points P′ and P″ along with the indices i and j result in the relations

$$\langle u_l u_k'' h_i' h_j''\rangle=\langle u_l u_k' h_i' h_j''\rangle.$$

By use of this fact and equations (5.4.10)-(5.4.15), we can write equations (5.4.7)-(5.4.9) in the forms

$$\frac{\partial}{\partial t}\langle \phi_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)+\frac{\nu}{P_M}\left[(1+P_M)(k^2+k'^2)+2P_M kk'+\frac{P_M}{\nu}(2\in_{mkl}\Omega_m)\right]$$

$$\langle\phi_i\beta_i'\beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=\left[i(k_k+k_k')\langle\phi_k\phi_i\beta_i'\beta''\rangle-i(k_k+k_k')\langle\beta_k\beta_i\beta_i'\beta_j''\rangle\right.$$

$$\left.-i(k_k+k_k')\langle\phi_i\phi_k'\beta_i'\beta_j''\rangle+i(k_k+k_k')\langle\phi_i\phi_i'\beta_k'\beta_j''\rangle-i(k_l+k_l')\langle\gamma\beta_i'\beta_j''\rangle\right](\hat{K},\hat{K}',\Delta t,\Delta t',t),\text{--}(5.4.16)$$

$$\frac{\partial}{\partial\Delta t}\langle\phi_i\beta_i'\beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)+\frac{\nu k^2}{P_M}\langle\phi_i\beta_i'\beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)$$

$$=-ik_k'\langle\phi_l\,\phi_k'\beta_i'\beta_j''\rangle(\hat{K},\hat{K}'\Delta t,\Delta t',t)+ik_k\langle\phi_l\phi_i'\beta_k'\beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\quad\text{------ (5.4.17)}$$

and $$\frac{\partial}{\partial\Delta t'}\langle\phi_i\beta_i'\beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)+\frac{\nu k'^2}{P_M}\langle\phi_i\beta_i'\beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)$$

$$=-ik_k\langle\phi_l\,\phi_k'\beta_i'\beta_j''\rangle(\hat{K},\hat{K}'\Delta t,\Delta t',t)+ik_k'\langle\phi_l\phi_i'\beta_k'\beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t).\quad\text{-------- (5.4.18)}$$

If the derivative with respect to x_l is taken of the momentum equation (5.4.1) for the point P, the equation multiplied by $h_i'h_j''$ and time average taken, the resulting equation.

$$-\frac{\partial^2\langle wh_i'h_j''\rangle}{\partial x_l\partial x_l}=\frac{\partial^2}{\partial x_l\partial x_k}\left(\langle u_lu_kh_i'h_j''\rangle-\langle h_lh_kh_i'h_j''\rangle\right).\quad\text{--------- (5.4.19)}$$

Writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$-\left[\frac{\partial^2}{\partial r_l\partial r_l}+2\frac{\partial^2}{\partial r_l\partial r_l'}+\frac{\partial^2}{\partial r_l'\partial r_l'}\right]\langle wh_i'h_j''\rangle=\left[\frac{\partial^2}{\partial r_l\partial r_k}+\frac{\partial^2}{\partial r_l'\partial r_k}+\frac{\partial^2}{\partial r_l\partial r_k'}+\frac{\partial^2}{\partial r_l'\partial r_k'}\right]$$

$$\times\left(\langle u_lu_kh_i'h_j''\rangle-\langle h_lh_kh_i'h_j''\rangle\right).\quad\text{---------- (5.4.20)}$$

Taking the Fourier transforms of equation (5.4.20)

$$-\langle\gamma\beta_i'\beta_j''\rangle=\frac{(k_lk_k+k_l'k_k+k_lk_k'+k_l'k_k')\left(\langle\phi_l\phi_k\beta_i'\beta_j''\rangle-\langle\beta_l\beta_k\beta_i'\beta_j''\rangle\right)}{k_lk_l+2k_lk_l'+k_l'k_l'}.\quad\text{---------- (5.4.21)}$$

Equation (5.4.21) can be used to eliminate $\langle\gamma\beta_i'\beta_j''\rangle$ from equation (5.4.16).

The tensor equations (5.4.16) to (5.4.18) can be converted to scalar equation by contraction of the indices i and j

$$\frac{\partial}{\partial t}\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \frac{\nu}{P_M}\left[(1+P_M)(k^2+k'^2)+2P_M kk'\right]$$

$$+\frac{P_M}{\nu}(2 \in_{mkl} \Omega_m)\langle \phi_l \beta_i'' \beta_i'' \rangle (\hat{K}, \hat{K}, \Delta t, \Delta t', t) = i(k_k + k_k')\langle \phi_k \phi_l \beta_i' \beta_j'' \rangle$$

$$(\hat{K}, \hat{K}', \Delta t, \Delta t', t) - i(k_k + k_k')\langle \beta_K \beta_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) - i(k_k + k_k')$$

$$\langle \phi_l \phi_k' \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + i(k_k + k_k')\langle \phi_l \phi_i' \beta_k' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) - i(k_l + k_l')$$

$$\langle \gamma \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t), \qquad \text{---------- (5.4.16a)}$$

$$\frac{\partial}{\partial \Delta t}\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \frac{\nu k^2}{P_M}\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)$$

$$= -ik_k \langle \phi_l \phi_k' \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + ik_k \langle \phi_l \phi_i' \beta_k' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) \qquad \text{---------- (5.4.17a)}$$

and $$\frac{\partial}{\partial \Delta t'}\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \frac{\nu k'^2}{P_M}\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)$$

$$= -ik_k' \langle \phi_l \phi_k' \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + ik_k' \langle \phi_l \phi_i' \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t). \qquad \text{---------- (5.4.18a)}$$

5.5 Solution for Times Before the Final Period:

It is known that the equation for final period of decay is obtained by considering the two-point correlations after neglecting third-order correlation terms. To study the decay for times before the final period, the three-point correlations are considered and the quadruple correlation terms are neglected because the quadruple correlation terms decays faster than the lower-order correlation terms. Equation (5.4.21) shows that the term $\langle \gamma \beta_i' \beta_j'' \rangle$ associated with the pressure fluctuations should also be neglected. Thus neglecting all the terms on the right hand side of equations (5.4.16a) to (5.4.18a)

$$\frac{\partial}{\partial t}\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \frac{\nu}{P_M}\left[(1+P_M)(k^2+k'^2)+2P_M kk' + \right.$$

$$\left.\frac{P_M}{\nu}(2 \in_{mkl} \Omega_m)\right]\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0, \qquad \text{--------- (5.5.1)}$$

$$\frac{\partial}{\partial \Delta t}\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \frac{\nu k^2}{P_M}\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0 \quad \text{--------- (5.5.2)}$$

and $$\frac{\partial}{\partial \Delta t'}\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \frac{\nu k'^2}{P_M}\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0. \quad \text{--------- (5.5.3)}$$

Integrating equations (5.5.1) to (5.5.3) between t_o and t, we obtain

$$k_l\langle \phi_l \beta_i' \beta_i'' \rangle = f_l \exp\left\{-\frac{\nu}{P_M}\left[(1+P_M)(k^2+k'^2) + 2P_M kk'\cos\theta + \frac{P_M}{\nu}(2\in_{mkl}\Omega_m)\right](t-t_o)\right\},$$

$$k_l\langle \phi_l \beta_i' \beta_i'' \rangle = g_l \exp\left[-\frac{\nu k^2}{P_M}\Delta t\right]$$

and $k_l\langle \phi_l \beta_i' \beta_i'' \rangle = q_l \exp\left[-\frac{\nu k'^2}{P_M}\Delta t'\right]$.

For these relations to be consistent, we have

$$k_l\langle \phi_l \beta_i' \beta_i'' \rangle = k_l\langle \phi_l \beta_i' \beta_i'' \rangle_o \exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o) + k^2\Delta t + k'^2\Delta t'$$

$$+ 2P_M kk'\cos\theta(t-t_o) + \left(\frac{2\in_{mkl}\Omega_m}{\lambda}\right)(t-t_o)\} \quad \text{---------- (5.5.4)}$$

where θ is the angle between $\hat{K}$ and $\hat{K}'$ and $\langle \phi_l \beta_i' \beta_i'' \rangle_o$ is the value of $\langle \phi_l \beta_i' \beta_i'' \rangle$ at $t = t_o$, $\Delta t = \Delta t' = 0$, $\lambda = \frac{\nu}{P_M}$.

By letting $\hat{r}' = 0$, $\Delta t' = 0$ in the equation (5.4.10) and comparing with equations (5.3.11) and (5.3.12) we get

$$\langle \alpha_i \psi_k \psi_i' \rangle(\hat{K}, \Delta t, t) = \int_{-\infty}^{\infty}\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K}, \hat{K}', \Delta t, o, t)d\hat{K}' \quad \text{---------- (5.5.5)}$$

and $$\langle \alpha_i \psi_k \psi_i' \rangle(-\hat{K}, -\Delta t, t+\Delta t) = \int_{-\infty}^{\infty}\langle \phi_l \beta_i' \beta_i'' \rangle(-\hat{K}, \hat{K}', \Delta t, o, t)d\hat{K}'. \quad \text{---------- (5.5.6)}$$

Substituting equation (5.5.4) to (5.5.6) into equation (5.3.15), one obtains

$$\frac{\partial}{\partial t}\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)+2\lambda k^2\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)=\int_{-\infty}^{\infty}2ik_l\left[\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,0,t)\right.$$

$$\left.-\langle\phi_l\beta_i'\beta_i''\rangle(-\hat{K},-\hat{K}',\Delta t,0,t)\ \right]_o\exp\left[-\lambda\left\{(1+P_M)(k^2+k'^2)(t-t_o)\right.\right.$$

$$\left.\left.+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}\right)(t-t_o)\ \right\}\ \right]d\hat{K}'. \qquad \text{---------- (5.5.7)}$$

Now, $d\hat{K}'$ can be expressed in terms of k′ and θ as $-2\pi k' d(\cos\theta)dk'$ (cf. Deissler [28]).

Hence, $d\hat{K}'=-2\pi k^2 d(\cos\theta)dk'$. ----------- (5.5.7a)

Substituting of equation (5.5.7a) in equation (5.5.7) yields

$$\frac{\partial}{\partial t}\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)+2\lambda k^2\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)=2\int_0^{\infty}2\pi ik_l\left[\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}')\right.$$

$$\left.-\langle\phi_l\beta_i'\beta_i''\rangle(-\hat{K},-\hat{K}')\ \right]_o k'^2\left[\int_{-1}^{1}\ \exp\left\{-\lambda\left[(1+P_M)(k^2+k'^2)(t-t_o)\right.\right.\right.$$

$$\left.\left.\left.+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}\right)(t-t_o)\ \right]\ \right\}d(\cos\theta)\ \right]dk'. \qquad \text{----- (5.5.8)}$$

In order to find the solution completely and following Loeffler and Deissler [72] we assume that

$$ik_l\left[\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}')-\langle\phi_l\beta_i'\beta_i''\rangle(-\hat{K},-\hat{K}')\right]_o=\frac{-\delta_0}{(2\pi)^2}\left(k^2k'^4-k^4k'^2\right) \qquad \text{----------- (5.5.9)}$$

where δ_0 is a constant determined by the initial conditions. The negative sign is placed in front of δ_0 in order to make the transfer of energy from small to large wave numbers for positive value of δ_0. The quantity $[\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}')-\langle\phi_l\beta_i'\beta_i''\rangle(-\hat{K},-\hat{K}')]_o$ depends on the initial conditions of the turbulence.

Substituting of equation (5.5.9) into equation (5.5.8) we get

$$\frac{\partial}{\partial t}2\pi\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)+2\lambda k^2 2\pi\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)=-2\delta_0\int_0^{\infty}\left(k^2k'^4-k^4k'^2\right)k'^2$$

$$\left[\int_{-1}^{1} \exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta\right.$$

$$\left.+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}\right)(t-t_o)\ ]\ \}d(\cos\theta)\ \right]d\hat{k}'. \qquad \text{----------} (5.5.10)$$

Multiplying both sides of equation (5.5.10) by k^2, we get

$$\frac{\partial E}{\partial t}+2\lambda k^2 E=F \qquad \text{----------} (5.5.11)$$

where $E=2\pi k^2\langle\psi_i\psi_i'\rangle$, E is the magnetic energy spectrum function and F is the magnetic energy transfer term and is given by

$$F=-2\delta_0\int_0^\infty(k^2k'^4-k^4k'^2)k^2k'^2\times\left[\int_{-1}^{1} \exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)\right.$$

$$\left.+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}\right)(t-t_o)\ ]\ \}d(\cos\theta)\ \right]dk'. \qquad \text{------} (5.5.12)$$

Integrating equation (5.5.12) with respect to cosθ, we have

$$F=-\frac{\delta_0}{\nu(t-t_o)}\int_0^\infty(k^3k'^5-k^5k'^3)\left[\ \exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)+k^2\Delta t\right.$$

$$\left.-2P_Mkk'(t-t_o)+\frac{2\in_{mkl}\Omega_m}{\lambda}(t-t_o)\ ]\ \}\ \right]dk'$$

$$+\frac{\delta_0}{\nu(t-t_o)}\int_0^\infty(k^3k'^5-k^5k'^3)\left[\ \exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)+k^2\Delta t\right.$$

$$\left.+2P_Mkk'(t-t_o)+\frac{2\in_{mkl}\Omega_m}{\lambda}(t-t_o)\ ]\ \}\ \right]dk'. \qquad \text{------------} (5.5.13)$$

Again integrating equation (5.5.13) with respect to k′, we have

$$F=-\frac{\delta_o P_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_o)^{3/2}(1+P_M)^{5/2}}\exp\left\{-\frac{2\in_{mkl}\Omega_m}{\lambda}(t-t_o)\right\}$$

$$\times \exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{1+P_M}{1+2P_M}\Delta t\right)\right]\times\left[\frac{15P_M k^4}{4\nu^2(t-t_o)^2(1+P_M)}\right.$$

$$+\left\{\frac{5P_M^2}{(1+P_M)^2}-\frac{3}{2}\right\}\frac{k^6}{\nu(t-t_o)}+\left\{\frac{P_M^3}{(1+P_M)^3}-\frac{P_M}{1+P_M}\right\}k^8$$

$$-\frac{\delta_o P_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_o+\Delta t)^{3/2}(1+P_M)^{5/2}}\exp\left\{-\frac{2\in_{mkl}\Omega_m}{\lambda}(t-t_o)\right\}$$

$$\times \exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{P_M}{1+P_M}\Delta t\right)\right]\times\left[\frac{15P_M k^4}{4\nu^2(t-t_o+\Delta t)^2(1+P_M)}\right.$$

$$\left.+\left\{\frac{5P_M^2}{(1+P_M)^2}-\frac{3}{2}\right\}\frac{k^6}{\nu(t-t_o+\Delta t)}+\left\{\frac{P_M^3}{(1+P_M)^3}-\frac{P_M}{1+P_M}\right\}k^8\ \right]. \qquad \text{---------- (5.5.14)}$$

The series of equation (5.5.14) contains only even power of k and start with k^4 and the equation represents the transfer function arising owing to consideration of magnetic field at three-point and three-times.

If we integrate equation (5.5.14) for $\Delta t=0$ over all wave numbers, we find that

$$\int_0^\infty F dk = 0 \qquad \text{------------- (5.5.15)}$$

which indicates that the expression for F satisfies the condition of continuity and homogeneity. Physically it was to be expected, since F is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero.

The linear equation (5.5.11) can be solved to give

$$E = \exp[-2\lambda k^2(t-t_o+\Delta t/2)]\int F\exp[2\lambda k^2(t-t_o+\Delta t/2)]dt$$

$$+J(k)\exp[-2\lambda k^2(t-t_o+\Delta t/2)]\ , \qquad \text{------------- (5.5.16)}$$

$$\left[\frac{9}{16T^{5/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta t\right)^{3/2}}+\frac{9}{16(T+\Delta T)^{5/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{5/2}}\right.$$

$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)T^{3/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{7/2}}+\frac{5P_M(7P_M-6)}{16(1+2P_M)(T+\Delta T)^{3/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{7/2}}$$

$$+\frac{35P_M(3P_M^2-2P_M+3)}{8(1+2P_M)T^{1/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{9/2}}+\frac{35P_M(3P_M^2-2P_M+3)}{8(1+2P_M)(T+\Delta T)^{1/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{9/2}}$$

$$\left.+\frac{8P_M(3P_M^2-2P_M+3)(1+2P_M)^{5/2}}{3.2^{23/2}(1+P_M)^{11/2}}\sum_{n=0}^{\infty}\frac{1.3.5............(2n+9)}{n!(2n+1)2^{2n}(1+P_M)^n}\times\left\{\frac{T^{(2n+1)/2}}{\left(T+\Delta T/2\right)^{(2n+1)/2}}+\frac{(T+\Delta T)^{(2n+1)/2}}{\left(T+\Delta T/2\right)^{(2n+1)/2}}\right\}\right],$$

------------ (5.5.19)

where T=t-t₀.

For $T_m = T + \Delta T/2$, equation (5.5.19) takes the form

$$\frac{\langle h_i h_i' \rangle}{2}=\frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}T_m^{3/2}}+\frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}}\exp[-2\in_{mkl}\Omega_m]$$

$$\times\left[\frac{9}{16\left(T_m-\Delta T/2\right)^{5/2}\left(T_m+\frac{\Delta T}{1+2P_M}\right)^{5/2}}+\frac{9}{16\left(T_m+\Delta T/2\right)^{5/2}\left(T_m-\frac{\Delta T}{2(1+2P_M)}\right)^{5/2}}\right.$$

$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m-\frac{\Delta T}{2}\right)^{3/2}\left(T_m+\frac{\Delta T}{2(1+2P_M)}\right)^{7/2}}$$

$$\left.+\frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m+\frac{\Delta T}{2}\right)^{3/2}\left(T_m-\frac{\Delta T}{2(1+2P_M)}\right)^{7/2}}+---\right].$$

---------- (5.5.20)

This is the decay law of magnetic energy fluctuations of MHD turbulence before the final period for the case of multi-point and multi-time in a rotating system.

5.6 Concluding Remarks:

In equation (5.5.20) we obtained the decay law of magnetic energy fluctuations in MHD turbulence before the final period in a rotating system considering three-point correlation terms for the case of multi-point and multi-time. If the system is non-rotating then $\Omega_m = 0$, the equation (5.5.20) becomes

$$\frac{\langle h_i h_i' \rangle}{2} = \frac{N_0}{8\sqrt{2\pi}\lambda^{3/2} T_m^{\ 3/2}} + \frac{\pi\delta_0}{4\lambda^6 (1+P_M)(1+2P_M)^{5/2}}$$

$$\times \left[\frac{9}{16\left(T_m - \Delta T/2\right)^{5/2}\left(T_m + \frac{\Delta T}{1+2P_M}\right)^{5/2}} + \frac{9}{16\left(T_m + \Delta T/2\right)^{5/2}\left(T_m - \frac{\Delta T}{2(1+2P_M)}\right)^{5/2}} \right.$$

$$+ \frac{5P_M(7P_M - 6)}{16(1+2P_M)\left(T_m - \frac{\Delta T}{2}\right)^{3/2}\left(T_m + \frac{\Delta T}{2(1+2P_M)}\right)^{7/2}}$$

$$\left. + \frac{5P_M(7P_M - 6)}{16(1+2P_M)\left(T_m + \frac{\Delta T}{2}\right)^{3/2}\left(T_m - \frac{\Delta T}{2(1+2P_M)}\right)^{7/2}} + --- \right], \qquad \text{------------ (5.6.1)}$$

which was obtained earlier by Sarker and Islam [115].

If we put $\Delta T=0$ in equation (5.6.1), we can easily find out

$$\frac{\langle h^2 \rangle}{2} = \frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}} T^{-3/2} + \frac{\pi\delta_0}{2\lambda^6 (1+P_M)(1+2P_M)^{5/2}} T^{-5} \times \left\{ \frac{9}{16} + \frac{5}{16}\frac{P_M(7P_M - 6)}{1+2P_M} + --- \right\}$$

$$= AT^{-3/2} + BT^{-5}, \qquad \text{------------- (5.6.2)}$$

where

$$A = \frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}}$$

$$B = \frac{\pi\delta_0}{2\lambda^6(1+P_M)(1+2P_M)^{5/2}} \times \left\{ \frac{9}{16} + \frac{5}{16}\frac{P_M(7P_M - 6)}{1+2P_M} + --- \right\}$$

which is same as obtained earlier by Sarker and Kishore [108].

This study indicates that the turbulent energy in the magnetic field decays more rapidly due to the effect of rotation than the energy of non-rotating fluid. From the assumption we conclude that the higher-order correlation terms may be neglected in comparison with lower-order correlation terms. By neglecting the quadruple correlation terms in three-point, three-time correlation equation the result (5.5.20) applicable to the MHD turbulence in a rotating system before the final period of decay were obtained. If higher-order correlation equations are considered in the analysis, i.e. if the quadruple correlations were not neglected, it appears that more terms of higher power of $(t\text{-}t_0)$ would be added to the equation (5.5.20). For large times the last term in the equation (5.5.20) becomes negligible, leaving the -3/2power decay law for the final period.

CHAPTER-V

PART-B

DECAY OF MAGNETO-HYDRODYNAMIC TURBULENCE BEFORE THE FINAL PERIOD FOR THE CASE OF MULTI-POINT AND MULTI-TIME IN PRESENCE OF DUST PARTICLES

5.7 Introduction:

The behavior of dust particles in a turbulent flow depends on the concentration of the particles and the size of the particles with respect to the scale of turbulent fluid. Saffman [106] derived an equation that described the motion of a fluid containing small dust particles. Sinha [122] studied the effect of dust particles on the acceleration covariance of ordinary turbulence Kishore and Sinah [59] also studied the rate of change of vorticity covariance in dusty fluid turbulence. Sarker [110], Sarker and Rahman [112] considered dust particles on their won works. Batchelor and Townsend [4] studied the decay of turbulence in the final period. Dessiler [27,28] developed a theory for homogeneous turbulence, which was valid for times before the final period. Using Deissler's theory Loeffler and Deissler [72] studied the temperature fluctuations in homogeneous turbulence before the final period. Sarker and Kishore [108] studied the decay of MHD turbulence before the final period. Sarker and Islam [115] also studied the decay of dusty fluid turbulence before the final period in a rotating system. Islam and Sarker [46] studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Kumar and Patel [65] also studied on first-order reactant in homogeneous turbulence before the final period of decay for the case of multi-point and multi-time. Sarker and Islam [116] studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time. They considered two and three-point correlations and neglecting higher order correlation terms compared to the second-and third-order correlation terms. Shimomura and Yoshizawa [119], Shimomura [120] discussed the statistical analysis of turbulent viscosity, turbulent sealer flux respectively two-scale direct interaction approach. Sarker and Islam [117] also studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time.

Here, we have studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Two-and three-point correlations have been considered and the quadruple terms are neglected in comparison to the second-and third-order correlation terms. Finally the decay law of magnetic energy fluctuations of MHD turbulence before the final period for the case of multi-point and multi-time in presence of dust particles is obtained. When the fluid is clean, the result reduces to the one obtained earlier by Sarker and Islam [115].

5.8 Basic Equations:

The equation of motion and continuity for viscous, incompressible dusty fluid MHD turbulent flow system are given by

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial w}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k \partial x_k}+f(u_i - v_i), \qquad \text{---------} (5.8.1)$$

$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \frac{\nu}{P_M}\frac{\partial^2 h_i}{\partial x_k \partial x_k}, \qquad \text{---------} (5.8.2)$$

$$\frac{\partial v_i}{\partial t}+v_k\frac{\partial v_i}{\partial x_K} = -\frac{K}{m_s}(v_i - u_i), \qquad \text{---------} (5.8.3)$$

with

$$\frac{\partial u_i}{\partial x_i}=\frac{\partial v_i}{\partial x_i}=\frac{\partial h_i}{\partial x_i}=0. \qquad \text{---------} (5.8.4)$$

Here, u_i, turbulence velocity component; h_i, magnetic field fluctuation component; v_i, dust particle velocity component; $w(\hat{x},t)=\frac{P}{\rho}+\frac{1}{2}\langle h^2 \rangle+\frac{1}{2}|\hat{\Omega}\times\hat{x}|^2$, total MHD pressure $p(\hat{x},t)$, hydrodynamic pressure; ρ, fluid density; ν, Kinematic viscosity; P_M, magnetic prandtl number; x_k, space co-ordinate; the subscripts can take on the values 1, 2 or 3 and the repeated subscripts in a term indicate a summation; $f=\frac{KN}{\rho}$, dimension of frequency; N, constant number density of dust particle $m_s = \frac{4}{3}\pi R_s^3 \rho_s$, mass of single spherical dust particle of radius R_s; ρ_s, constant density of the material in dust particle.

5.9 Two-Point, Two-Time Correlation and Spectral Equations:

Induction equations of a magnetic field at the point p and p′ separated by the vector $\hat{r}$ could be written as

$$\frac{\partial h_i}{\partial t} + u_k \frac{\partial h_i}{\partial x_k} - h_k \frac{\partial u_i}{\partial x_k} = \frac{\nu}{P_M}\frac{\partial^2 h_i}{\partial x_k \partial x_k} \qquad \text{----------- (5.9.1)}$$

and $$\frac{\partial h'_j}{\partial t'} + u'_k \frac{\partial h'_j}{\partial x'_k} - h'_k \frac{\partial u'_j}{\partial x'_k} = \frac{\nu}{P_M}\frac{\partial^2 h'_j}{\partial x'_k \partial x'_k}. \qquad \text{----------- (5.9.2)}$$

Multiplying equation (5.9.1) by h'_j and equation (5.9.2) by h_i and taking ensemble average, we get

$$\frac{\partial\langle h_i h'_j \rangle}{\partial t} + \frac{\partial}{\partial x_k}\left[\langle u_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle\right] = \frac{\nu}{P_M}\frac{\partial^2\langle h_i h'_j \rangle}{\partial x_k \partial x_k} \qquad \text{------------ (5.9.3)}$$

and $$\frac{\partial\langle h_i h'_j \rangle}{\partial t'} + \frac{\partial}{\partial x'_k}\left[\langle u'_k h_i h'_j \rangle - \langle u'_j h_i h'_k \rangle\right] = \frac{\nu}{P_M}\frac{\partial^2\langle h_i h'_j \rangle}{\partial x'_k \partial x'_k}. \qquad \text{------------ (5.9.4)}$$

Angular bracket $\langle - - - - - \rangle$ is used to denote an ensemble average.

Using the transformation

$$\frac{\partial}{\partial x_k} = -\frac{\partial}{\partial r_k}, \frac{\partial}{\partial x'_k} = \frac{\partial}{\partial r_k}, \left(\frac{\partial}{\partial t}\right)t' = \left(\frac{\partial}{\partial t}\right)\Delta t - \frac{\partial}{\partial \Delta t}, \frac{\partial}{\partial t'} = \frac{\partial}{\partial \Delta t} \qquad \text{----------- (5.9.5)}$$

into equations (5.9.3) and (5.9.4), we have

$$\frac{\partial\langle h_i h'_j\rangle}{\partial t} + \frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle - \langle u'_j h_i h'_k\rangle\right](\hat{r},\Delta t, t) - \frac{\partial}{\partial r_k}\left[\langle u_k h_i h'_j\rangle - \langle u_i h_k h'_j\rangle\right](\hat{r},\Delta t, t) = \frac{2\nu}{P_M}\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k \partial r_k} \qquad \text{-(5.9.6)}$$

and $$\frac{\partial\langle h_i h'_j\rangle}{\partial \Delta t} + \frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle - \langle u'_j h_i h'_k\rangle\right](\hat{r},\Delta t, t) = \frac{\nu}{P_M}\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k \partial r_k}. \qquad \text{---------- (5.9.7)}$$

Using the relations of Chandrasekhar [19].

$$\langle u_k h_i h'_j \rangle = -\langle u'_k h_i h'_j \rangle, \quad \langle u'_j h_i h'_k \rangle = \langle u_i h_k h'_j \rangle.$$

Equation (5.9.6) and (5.9.7) become

$$\frac{\partial \langle h_i h'_j \rangle}{\partial t} + 2\frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle\right] = \frac{2\nu}{P_M}\frac{\partial^2 \langle h_i h'_j \rangle}{\partial r_k \partial r_k} \qquad \text{------------ (5.9.8)}$$

and $$\frac{\partial \langle h_i h'_j \rangle}{\partial t} + \frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle\right] = \frac{\nu}{P_M}\frac{\partial^2 \langle h_i h'_j \rangle}{\partial r_k \partial r_k}. \qquad \text{------------ (5.9.9)}$$

Now we write equations (5.9.8) and (5.9.9) in spectral form in order to reduce it to an ordinary differential equation by use of the following three-dimensional Fourier transforms:

$$\langle h_i h'_j \rangle(\hat{r}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \psi_i \psi'_j \rangle(\hat{K}, \Delta t, t) \exp\left[\hat{i}(\hat{K}.\hat{r})\right] d\hat{K}, \qquad \text{---------- (5.9.10)}$$

$$\langle u_i h_k h'_j \rangle(\hat{r}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \alpha_i \psi_k \psi'_j \rangle(\hat{K}, \Delta t, t) \exp\left[\hat{i}(\hat{K}.\hat{r})\right] d\hat{K}. \qquad \text{---------- (5.9.11)}$$

Interchanging the subscripts i and j then interchanging the points p and p′ gives

$$\langle u'_k h_i h'_j \rangle(\hat{r}, \Delta t, t) = \langle u_k h_i h'_j \rangle(-\hat{r}, -\Delta t, t + \Delta t)$$

$$= \int_{-\infty}^{\infty} \langle \alpha_i \psi_i \psi'_j \rangle(-\hat{K}, -\Delta t, t + \Delta t) \exp\left[\hat{i}(\hat{K}.\hat{r})\right] d\hat{K} \qquad \text{---------- (5.9.12)}$$

where $\hat{K}$ is known as a wave number vector and $d\hat{K} = dK_1\, dK_2\, dK_3$. The magnitude of $\hat{K}$ has the dimension 1/length and can be considered to be the reciprocal of an eddy size. Substituting of equation (5.9.10) to (5.9.12) in to equations (5.9.8) and (5.9.9) leads to the spectral equations

$$\frac{\partial \langle \psi_i \psi'_j \rangle}{\partial t} + \frac{2\nu k^2}{P_M}\langle \psi_i \psi'_j \rangle = 2ik_k\left[\langle \alpha_i \psi_k \psi'_j \rangle(\hat{K}, \Delta t, t) - \langle \alpha_k \psi_i \psi'_j \rangle(-\hat{K}, -\Delta t, t + \Delta t)\right] \qquad \text{----- (5.9.13)}$$

and $$\frac{\partial \langle \psi_i \psi'_j \rangle}{\partial \Delta t} + \frac{\nu k^2}{P_M}\langle \psi_i \psi'_j \rangle = ik_k\left[\langle \alpha_i \psi_k \psi'_j \rangle(\hat{K}, \Delta t, t) - \langle \alpha_k \psi_i \psi'_j \rangle(-\hat{K}, -\Delta t, t + \Delta t)\right]. \qquad \text{--- (5.9.14)}$$

The tensor equations (5.9.13) and (5.9.14) becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial\langle\psi_i\psi_i'\rangle}{\partial t}+\frac{2\nu k^2}{P_M}\langle\psi_i\psi_i'\rangle=2ik_k\left[\langle\alpha_i\psi_k\psi_i'\rangle(\hat{K},\Delta t,t)-\langle\alpha_k\psi_i\psi_i'\rangle(-\hat{K},-\Delta t,t+\Delta t)\right] \quad \text{---- (5.9.15)}$$

and
$$\frac{\partial\langle\psi_i\psi_i'\rangle}{\partial\Delta t}+\frac{\nu k^2}{P_M}\langle\psi_i\psi_i'\rangle=ik_k\left[\langle\alpha_i\psi_k\psi_i'\rangle(\hat{K},\Delta t,t)-\langle\alpha_k\psi_i\psi_i'\rangle(-\hat{K},-\Delta t,t+\Delta t)\right] \quad \text{---- (5.9.16)}$$

The terms on the right side of equations (5.9.15) and (5.9.16) are collectively proportional to what is known as the magnetic energy transfer terms.

5.10 Three-Point, Three-Time Correlation and Spectral Equations:

Similar procedure can be used to find the three-point correlation equations. For this purpose we take the momentum equation of dusty fluid MHD turbulence at the point P, the induction equation of magnetic field fluctuations at p′ and p″ separated by the vector $\hat{r}$ and $\hat{r}'$ as

$$\frac{\partial u_l}{\partial t}+u_k\frac{\partial u_l}{\partial x_k}-h_k\frac{\partial h_l}{\partial x_k}=-\frac{\partial w}{\partial x_l}+\nu\frac{\partial^2 u_l}{\partial x_k\partial x_k}+f(u_l-v_l), \quad \text{----------- (5.10.1)}$$

$$\frac{\partial h_i'}{\partial t'}+u_k'\frac{\partial h_i'}{\partial x_k'}-h_k'\frac{\partial u_i'}{\partial x_k'}=\frac{\nu}{P_M}\frac{\partial^2 h_i'}{\partial x_k'\partial x_k'}, \quad \text{----------- (5.10.2)}$$

$$\frac{\partial h_j''}{\partial t''}+u_k''\frac{\partial h_j''}{\partial x_k''}-h_k''\frac{\partial u_j''}{\partial x_k''}=\frac{\nu}{P_M}\frac{\partial^2 h_j''}{\partial x_k''\partial x_k''}. \quad \text{----------- (5.10.3)}$$

Multiplying equation (5.10.1) by $h'_i\, h_j''$, equation (5.10.2) by $u_l h_j''$ and equation (5.10.3) by $u_l h_i'$, taking ensemble average, one obtains

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t}+\frac{\partial}{\partial x_k}\left[\langle u_k u_l h_i' h_j''\rangle-\langle h_k h_l h_i' h_j''\rangle\right]=\frac{\partial\langle w h_i' h_j''\rangle}{\partial x_l}$$

$$+\nu\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k\partial x_k}+f(\langle u_l h_i' h_j''\rangle-\langle v_l h_i' h_j''\rangle, \quad \text{----------- (5.10.4)}$$

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t'}+\frac{\partial}{\partial x_k'}\left[\langle u_l u_k' h_i' h_j''\rangle-\langle u_l u_i' h_k' h_j''\rangle\right]=\frac{\nu}{P_M}\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k'\partial x_k'}, \qquad \text{----------- (5.10.5)}$$

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t''}+\frac{\partial}{\partial x_k''}\left[\langle u_l u_k'' h_i' h_j''\rangle-\langle u_l u_j'' h_i' h_k''\rangle\right]=\frac{\nu}{P_M}\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k''\partial x_k''}. \qquad \text{----------- (5.10.6)}$$

Using the transformations

$$\frac{\partial}{\partial x_k}=-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right),\frac{\partial}{\partial x_k'}=\frac{\partial}{\partial r_k},\frac{\partial}{\partial x_k''}=\frac{\partial}{\partial r_k'},$$

$$\left(\frac{\partial}{\partial t}\right)t',t''=\left(\frac{\partial}{\partial t}\right)\Delta t,\Delta t'-\frac{\partial}{\partial\Delta t}-\frac{\partial}{\partial\Delta t'},$$

$$\frac{\partial}{\partial t'}=\frac{\partial}{\partial\Delta t},\frac{\partial}{\partial t''}=\frac{\partial}{\partial\Delta t'}$$

into equations (5.10.4) to (5.10.6), we have

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t}-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right)\left[\langle u_k u_l h_i' h_j''\rangle-\langle h_k h_l h_i' h_j''\rangle\right]+\frac{\partial}{\partial r_k}\left[\langle u_l u_k' h_i' h_j''\rangle-\langle u_l u_i' h_k' h_j''\rangle\right]$$

$$+\frac{\partial}{\partial r_k'}\left[\langle u_l u_k'' h_i' h_j''\rangle-\langle u_l u_j'' h_i' h_k''\rangle\right]=-\left(\frac{\partial}{\partial r_l}+\frac{\partial}{\partial r_l'}\right)\langle w h_i' h_j''\rangle+\nu\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right)^2\langle u_l h_i' h_j''\rangle$$

$$+\frac{\nu}{P_M}\left[\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k\partial r_k}+\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k'\partial r_k'}\right]+f\left(\langle u_l h_i' h_j''\rangle-\langle v_l h_i' h_j''\rangle\right), \qquad \text{----------- (5.10.7)}$$

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial\Delta t}+\frac{\partial}{\partial r_k}\left[\langle u_l u_k' h_i' h_j''\rangle-\langle u_l u_i' h_k' h_j''\rangle\right]=\frac{\nu}{P_M}\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k\partial r_k} \qquad \text{----------- (5.10.8)}$$

and $$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial\Delta t'}+\frac{\partial}{\partial r_k'}\left[\langle u_l u_k'' h_i' h_j''\rangle-\langle u_l u_j'' h_i' h_k''\rangle\right]=\frac{\nu}{P_M}\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k'\partial r_k'}. \qquad \text{----------- (5.10.9)}$$

Six dimensional Fourier transforms of the quantities in the equations (5.10.7)-(5.10.9) may be defined as

$$\langle u_l h_i' h_j'' \rangle (\hat{r}, \hat{r}', \Delta t, \Delta t', t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) \exp\left[\hat{i}\left(\hat{K}.\hat{r} + \hat{K}'.\hat{r}'\right)\right] d\hat{K} d\hat{K}' \text{, -- (5.10.10)}$$

$$\langle u_l u_k' h_i' h_j'' \rangle (\hat{r}, \hat{r}', \Delta t, \Delta t', t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \phi_l \phi_K' \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) \exp\left[\hat{i}\left(\hat{K}.\hat{r} + \hat{K}'.\hat{r}'\right)\right] d\hat{K} d\hat{K}' \text{, -(5.10.11)}$$

$$\langle w h_i' h_j'' \rangle (\hat{r}, \hat{r}', \Delta t, \Delta t', t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \gamma \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) \exp\left[\hat{i}\left(\hat{K}.\hat{r} + \hat{K}'.\hat{r}'\right)\right] d\hat{K} d\hat{K}' \text{, --- (5.10.12)}$$

$$\langle u_k u_l h_i' h_j'' \rangle (\hat{r}, \hat{r}', \Delta t, \Delta t', t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \phi_K \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) \exp\left[\hat{i}\left(\hat{K}.\hat{r} + \hat{K}'.\hat{r}'\right)\right] d\hat{K} d\hat{K}' \text{, -(5.10.13)}$$

$$\langle h_k h_l h_i' h_j'' \rangle (\hat{r}, \hat{r}', \Delta t, \Delta t', t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \beta_K \beta_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) \exp\left[\hat{i}\left(\hat{K}.\hat{r} + \hat{K}'.\hat{r}'\right)\right] d\hat{K} d\hat{K}' \text{, (5.10.14)}$$

$$\langle u_l u_i' h_K' h_j'' \rangle (\hat{r}, \hat{r}', \Delta t, \Delta t', t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \phi_l \phi_i' \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) \exp\left[\hat{i}\left(\hat{K}.\hat{r} + \hat{K}'.\hat{r}'\right)\right] d\hat{K} d\hat{K}' \text{, - (5.10.15)}$$

$$\langle v_l h_i' h_j'' \rangle (\hat{r}, \hat{r}, \Delta t, \Delta t', t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \mu_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) \exp\left[\hat{i}\left(\hat{K}.\hat{r} + \hat{K}'.\hat{r}'\right)\right] d\hat{K} d\hat{K}' \text{. --- (5.10.16)}$$

Interchanging the points P′ and P″ along with the indices i and j result in the relations

$\langle u_l u_k'' h_i' h_j'' \rangle = \langle u_l u_k' h_i' h_j'' \rangle$.

By use of these facts and equations (5.10.10)-(5.10.16), the equations (5.10.7) to (5.10.9) may be transformed as

$$\frac{\partial}{\partial t}\langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \frac{\nu}{P_M}\left[(1 + P_M)(k^2 + k'^2) + 2P_M kk' - \frac{P_M}{\nu} f\right]$$

$$\langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = \left[i(k_k + k_k')\langle \phi_k \phi_l \beta_i' \beta'' \rangle - i(k_k + k_k')\langle \beta_k \beta_l \beta_i' \beta_j'' \rangle\right.$$

$$- i(k_k + k_k')\langle \phi_l \phi_k' \beta_i' \beta_j'' \rangle + i(k_k + k_k')\langle \phi_l \phi_i' \beta_k' \beta_j'' \rangle - i(k_l + k_l')\langle \gamma \beta_i' \beta_j'' \rangle$$

$$\left. - f\langle \mu_l \beta_i' \beta_j'' \rangle \right](\hat{K}, \hat{K}', \Delta t, \Delta t', t) \text{ , -------- (5.10.17)}$$

$$\frac{\partial}{\partial \Delta t}\left\langle \phi_l \beta_i' \beta_j'' \right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)+\frac{\nu k^2}{P_M}\left\langle \phi_l \beta_i' \beta_j'' \right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)$$

$$=-ik_k\left\langle \phi_l\, \phi_k' \beta_i' \beta_j'' \right\rangle\left(\hat{K},\hat{K}'\Delta t,\Delta t',t\right)+ik_k\left\langle \phi_l \phi_l' \beta_k' \beta_j'' \right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right) \qquad \text{---------- (5.10.18)}$$

and $\frac{\partial}{\partial \Delta t'}\left\langle \phi_l \beta_i' \beta_j'' \right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)+\frac{\nu k'^2}{P_M}\left\langle \phi_l \beta_i' \beta_j'' \right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)$

$$=-ik_k'\left\langle \phi_l\, \phi_k' \beta_i' \beta_j'' \right\rangle\left(\hat{K},\hat{K}'\Delta t,\Delta t',t\right)+ik_k'\left\langle \phi_l \phi_l' \beta_k' \beta_j'' \right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right). \qquad \text{---------- (5.10.19)}$$

If the derivative with respect to x_l is taken of the momentum equation (5.10.1) for the point P, the equation multiplied by $h_i'h_j''$ and time average taken, the resulting equation

$$-\frac{\partial^2\left\langle wh_i'h_j''\right\rangle}{\partial x_l \partial x_l}=\frac{\partial^2}{\partial x_l \partial x_k}\left(\left\langle u_l u_k h_i' h_j''\right\rangle-\left\langle h_l h_k h_i' h_j''\right\rangle\right) \quad . \qquad \text{----------- (5.10.20)}$$

Writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$-\left[\frac{\partial^2}{\partial r_l \partial r_l}+2\frac{\partial^2}{\partial r_l \partial r_l'}+\frac{\partial^2}{\partial r_l' \partial r_l'}\right]\left\langle wh_i'h_j''\right\rangle=\left[\frac{\partial^2}{\partial r_l \partial r_k}+\frac{\partial^2}{\partial r_l' \partial r_k}+\frac{\partial^2}{\partial r_l \partial r_k'}+\frac{\partial^2}{\partial r_l' \partial r_k'}\right]$$

$$\times\left(\left\langle u_l u_k h_i' h_j''\right\rangle-\left\langle h_l h_k h_i' h_j''\right\rangle\right) \quad . \qquad \text{----------- (5.10.21)}$$

Taking the Fourier transforms of equation (5.10.21)

$$-\left\langle \gamma \beta_i' \beta_j''\right\rangle=\frac{\left(k_l k_k+k_l' k_k+k_l k_k'+k_l' k_k'\right)\left(\left\langle \phi_l \phi_k \beta_i' \beta_j''\right\rangle-\left\langle \beta_l \beta_k \beta_i' \beta_j''\right\rangle\right)}{k_l k_l+2k_l k_l'+k_l' k_l'} \quad . \qquad \text{------------ (5.10.22)}$$

Equation (5.10.22) can be used to eliminate $\left\langle \gamma \beta_i' \beta_j''\right\rangle$ from equation (5.10.17).

The tensor equations (5.10.17) to (5.10.19) can be converted to scalar equation by contraction of the indices i and j

$$\frac{\partial}{\partial t}\left\langle \phi_l \beta_i' \beta_i''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)+\frac{\nu}{P_M}\left[\left(1+P_M\right)\left(k^2+k'^2\right)+2P_M kk'\right]-$$

$$\frac{P_M}{\nu}f\left\langle \phi_l \beta_i'' \beta_i''\right\rangle\left(\hat{K},\hat{K},\Delta t,\Delta t',t\right)=i\left(k_k+k_k'\right)\left\langle \phi_k \phi_l \beta_i' \beta_j''\right\rangle$$

$$\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)-i\left(k_k+k_k'\right)\left\langle\beta_k\beta_l\beta_i'\beta_i''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)-i\left(k_k+k_k'\right)$$

$$\left\langle\phi_l\phi_k'\beta_i'\beta_i''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)+i\left(k_k+k_k'\right)\left\langle\phi_l\phi_i'\beta_k'\beta_i''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)-i\left(k_l+k_l'\right)$$

$$\left\langle\gamma\beta_i'\beta_i''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)-f\left\langle\mu_l\beta_i'\beta_i''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)\ , \qquad \text{---------- (5.10.17a)}$$

$$\frac{\partial}{\partial\Delta t}\left\langle\phi_l\beta_i'\beta_i''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)+\frac{\nu k^2}{P_M}\left\langle\phi_l\beta_i'\beta_i''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)$$

$$=-ik_k\left\langle\phi_l\phi_k'\beta_i'\beta_i''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)+ik_k\left\langle\phi_l\phi_i'\beta_k'\beta_i''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right) \qquad \text{---------- (5.10.18a)}$$

and $\frac{\partial}{\partial\Delta t'}\left\langle\phi_l\beta_i'\beta_i''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)+\frac{\nu k'^2}{P_M}\left\langle\phi_l\beta_i'\beta_i''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)$

$$=-ik_k'\left\langle\phi_l\phi_k'\beta_i'\beta_i''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)+ik_k'\left\langle\phi_l\phi_i'\beta_i'\beta_i''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right). \qquad \text{---------- (5.10.19a)}$$

5.11 Solution for Times Before the Final Period:

It is known that the equation for final period of decay is obtained by considering the two-point correlations after neglecting third-order correlation terms. To study the decay for times before the final period, the three-point correlations are considered and the quadruple correlation terms are neglected because the quadruple correlation terms decays faster than the lower-order correlation terms. Equation (5.10.22) shows that the term $\left\langle\gamma\beta_i'\beta_j''\right\rangle$ associated with the pressure fluctuations should also be neglected. Thus neglecting all the terms on the right hand side of equations (5.10.17a) to (5.10.19a)

$$\frac{\partial}{\partial t}\left\langle\phi_l\beta_i'\beta_i''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)+\frac{\nu}{P_M}\left[\left(1+P_M\right)\left(k^2+k'^2\right)+2P_M kk'-\right.$$

$$\left.\frac{P_M}{\nu}fs\right]\left\langle\phi_l\beta_i'\beta_i''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)=0\ , \qquad \text{----------- (5.11.1)}$$

$$\frac{\partial}{\partial\Delta t}\left\langle\phi_l\beta_i'\beta_i''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)+\frac{\nu k^2}{P_M}\left\langle\phi_l\beta_i'\beta_i''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)=0 \qquad \text{----------- (5.11.2)}$$

and

$$\frac{\partial}{\partial \Delta t'}\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \frac{\nu k'^2}{P_M}\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0 \qquad \text{----------- (5.11.3)}$$

where $\langle \mu_l \beta_i' \beta_i'' \rangle = R\langle \phi_l \beta_i' \beta_i'' \rangle$ and 1-R=S, here R and S are arbitrary constant.

Integrating equations (5.11.1) to (5.11.3) between t_o and t, we obtain

$$k_l \langle \phi_l \beta_i' \beta_i'' \rangle = f_l \exp\left\{ -\frac{\nu}{P_M}\left[(1+P_M)(k^2+k'^2) + 2P_M kk'\cos\theta - \frac{P_M}{\nu} fs\right](t-t_o) \right\},$$

$$k_l \langle \phi_l \beta_i' \beta_i'' \rangle = g_l \exp\left[-\frac{\nu k^2}{P_M}\Delta t \right]$$

and $k_l \langle \phi_l \beta_i' \beta_i'' \rangle = q_l \exp\left[-\frac{\nu k'^2}{P_M}\Delta t' \right]$.

For these relations to be consistent, we have

$$k_l \langle \phi_l \beta_i' \beta_i'' \rangle = k_l \langle \phi_l \beta_i' \beta_i'' \rangle_o \exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o) + k^2\Delta t + k'^2 \Delta t'$$

$$+ 2P_M kk'\cos\theta(t-t_o)) - \frac{fs}{\lambda}(t-t_o) \} \qquad \text{------------ (5.11.4)}$$

where θ is the angle between $\hat{K}$ and $\hat{K}'$ and $\langle \phi_l \beta_i' \beta_i'' \rangle_o$ is the value of $\langle \phi_l \beta_i' \beta_i'' \rangle$ at $t = t_o$, $\Delta t = \Delta t' = 0$, $\lambda = \frac{\nu}{P_M}$.

By setting $\hat{r}' = 0$, $\Delta t' = 0$ in the equation (5.10.10) and comparing with equations (5.9.11) and (5.9.12) we get

$$\langle \alpha_l \psi_k \psi_i' \rangle(\hat{K}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K}, \hat{K}', \Delta t, o, t) d\hat{K}' \qquad \text{----------- (5.11.5)}$$

and $\langle \alpha_l \psi_k \psi_i' \rangle(-\hat{K}, -\Delta t, t+\Delta t) = \int_{-\infty}^{\infty} \langle \phi_l \beta_i' \beta_i'' \rangle(-\hat{K}, \hat{K}', \Delta t, o, t) d\hat{K}'$. ------------ (5.11.6)

Substituting equation (5.11.4) to (5.11.6) into equation (5.9.15), one obtains

$$\frac{\partial}{\partial t}\langle \psi_i \psi_i' \rangle(\hat{K}, \Delta t, t) + 2\lambda k^2 \langle \psi_i \psi_i' \rangle(\hat{K}, \Delta t, t) = \int_{-\infty}^{\infty} 2ik_l [\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K}, \hat{K}', \Delta t, 0, t)$$

$$-\langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}', \Delta t, 0, t) \,]_o \exp[-\lambda \{(1+P_M)(k^2+k'^2)(t-t_o)$$

$$+k^2 \Delta t + 2P_M (t-t_o) kk' \cos\theta - \frac{fs}{\lambda}(t-t_o) \} \,] d\hat{K} \quad . \qquad \text{----------} (5.11.7)$$

Now, $d\hat{K}'$ can be expressed in terms of k′ and θ as $2\pi k' d(\cos\theta) dk'$ (cf. Deissler [28]).

i.e. $d\hat{K}' = -2\pi k^2 d(\cos\theta) dk'$ ----------- (5.11.7a)

Substituting of equation (5.11.7a) in equation (5.11.7) yields

$$\frac{\partial}{\partial t}\langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t) + 2\lambda k^2 \langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t) = 2\int_0^\infty 2\pi i k_l \left[\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}') \right.$$

$$\left. - \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}') \, \right]_o k'^2 \left[\int_{-1}^{1} \exp\{-\lambda [(1+P_M)(k^2+k'^2)(t-t_o) \right.$$

$$\left. + k^2 \Delta t + 2P_M (t-t_o) kk' \cos\theta - \frac{fs}{\lambda}(t-t_o) \,] \, \} d(\cos\theta) \right] dk' \quad . \qquad \text{-----------} (5.11.8)$$

In order to find the solution completely and following Loeffler and Deissler [72] we assume that

$$ik_l \left[\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}') - \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}') \right]_o = \frac{-\delta_0}{(2\pi)^2}\left(k^2 k'^4 - k^4 k'^2\right) \qquad \text{-----------} (5.11.9)$$

where δ_0 is a constant determined by the initial conditions. The negative sign is placed in front of δ_0 in order to make the transfer of energy from small to large wave numbers for positive value of δ_0. The quantity $[\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}') - \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}')]_0$ depends on the initial conditions of the turbulence.

Substituting equation (5.11.9) into equation (5.11.8) we get

$$\frac{\partial}{\partial t} 2\pi \langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t) + 2\lambda k^2 2\pi \langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t) = -2\delta_0 \int_0^\infty (k^2 k'^4 - k^4 k'^2) k'^2$$

$$\left[\int_{-1}^{1} \exp\{-\lambda [(1+P_M)(k^2+k'^2)(t-t_o) + k^2 \Delta t + 2P_M (t-t_o) kk' \cos\theta \right.$$

$$-\frac{fs}{\lambda}(t-t_o)\ ]\ \}d(\cos\theta)\ \Big]d\hat{k}' \ . \qquad \text{-------- (5.11.10)}$$

Multiplying both sides of equation (5.10) by k^2, we get

$$\frac{\partial E}{\partial t}+2\lambda k^2 E = F \qquad \text{--------- (5.11.11)}$$

where, $E = 2\pi k^2\langle\psi_i\psi_i'\rangle$, E is the magnetic energy spectrum function and F is the magnetic energy transfer term and is given by

$$F = -2\delta_0\int_0^{\infty}\left(k^2k'^4-k^4k'^2\right)k^2k'^2\times\Bigg[\int_{-1}^{1}\exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)$$

$$+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta-\frac{fs}{\lambda}(t-t_o)\ ]\ \}d(\cos\theta)\ \Bigg]dk' \ . \qquad \text{------------- (5.11.12)}$$

Integrating equation (5.11.12) with respect to cos θ, we have

$$F = -\frac{\delta_0}{v(t-t_o)}\int_0^{\infty}\left(k^3k'^5-k^5k'^3\right)\Big[\ \exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)+k^2\Delta t-2P_M kk'(t-t_o)-\frac{fs}{\lambda}(t-t_o)\ ]\ \}\ \Big]dk'$$

$$+\frac{\delta_0}{v(t-t_o)}\int_0^{\infty}\left(k^3k'^5-k^5k'^3\right)\Big[\ \exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)+k^2\Delta t+2P_M kk'(t-t_o)-\frac{fs}{\lambda}(t-t_o)\ ]\ \}\ \Big]dk' \ . \qquad \text{--------------(5.11.13)}$$

Again integrating equation (5.11.13) with respect to k′, we have

$$F = -\frac{\delta_o P_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_o)^{3/2}(1+P_M)^{5/2}}\exp[\ \frac{fs}{\lambda}(t-t_o)\ ]\times\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{1+P_M}{1+2P_M}\Delta t\right)\right]$$

$$\times\left[\frac{15P_M k^4}{4v^2(t-t_o)^2(1+P_M)}\right]+\left\{\frac{5P_M^2}{(1+P_M)^2}-\frac{3}{2}\right\}\frac{k^6}{v(t-t_o)}+\left\{\frac{P_M^3}{(1+P_M)^3}-\frac{P_M}{1+P_M}\right\}k^8$$

$$-\frac{\delta_o P_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_o+\Delta t)^{3/2}(1+P_M)^{5/2}}\exp[\ \frac{fs}{\lambda}(t-t_o)\ ]\times\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{1+P_M}{1+2P_M}\Delta t\right)\right]$$

$$\times\left[\frac{15P_M k^4}{4v^2(t-t_o+\Delta t)^2(1+P_M)}\right]+\left\{\frac{5P_M^2}{(1+P_M)^2}-\frac{3}{2}\right\}\frac{k^6}{v(t-t_o+\Delta t)}+\left\{\frac{P_M^3}{(1+P_M)^3}-\frac{P_M}{1+P_M}\right\}k^8\ ]. \qquad \text{- (5.11.14)}$$

The series of equation (5.11.14) contains only even power of k and start with k^4 and the equation represents the transfer function arising owing to consideration of magnetic field at three-point and three-times.

If we integrate equation (5.11.14) for $\Delta t=0$ over all wave numbers, we find that

$$\int_0^{\infty} F dk = 0 \qquad \text{----------- (5.11.15)}$$

which indicates that the expression for F satisfies the condition of continuity and homogeneity. Physically it was to be expected as F is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero.

The linear equation (5.11.11) can be solved to give

$$E = \exp\left[-2\lambda k^2 (t - t_o + \Delta t/2)\right] \int F \exp\left[2\lambda k^2 (t - t_o + \Delta t/2)\right] dt$$

$$+ J(k) \exp\left[-2\lambda k^2 (t - t_o + \Delta t/2)\right] \qquad \text{--------- (5.11.16)}$$

where $J(k) = \frac{N_o k^2}{\pi}$ is a constant of integration. Substituting the values of F from equation (5.11.14) into equation (5.11.16) gives the equation

$$E = \frac{N_o k^2}{\pi} \exp\left[-2\lambda k^2 (t - t_o + \Delta t/2)\right] + \frac{\delta_o P_M \sqrt{\pi}}{4\lambda^{3/2} (1 + P_M)^{7/2}} \times \exp\left[fs(t - t_o)\right]$$

$$\exp\left[\frac{-k^2 \lambda (1 + 2P_M)}{1 + P_M}\left(t - t_o + \frac{1 + P_M}{1 + 2P_M}\Delta t\right)\right] \times \left[\frac{3k^4}{2P_M \lambda^2 (t - t_0)^{5/2}} + \frac{(7P_M - 6)k^6}{3\lambda (1 + P_M)(t - t_0)^{3/2}}\right.$$

$$\left. - \frac{4(3P_M^2 - 2P_M + 3)k^8}{3(1 + P_M)^2 (t - t_0)^{1/2}} + \frac{8\sqrt{\lambda}(3P_M^2 - 2P_M + 3)k^9}{3(1 + P_M)^{5/2}} F(\omega)\right] + \frac{\delta_o P_M \sqrt{\pi}}{4\lambda^{3/2} (1 + P_M)^{7/2}} \exp\left[fs(t - t_o)\right]$$

$$\times \exp\left[\frac{-k^2 \lambda (1 + 2P_M)}{1 + P_M}\left(t - t_o + \frac{P_M}{1 + P_M}\Delta t\right)\right]\left[\frac{3k^4}{2P_M \lambda^2 (t - t_0 + \Delta t)^{5/2}} + \frac{(7P_M - 6)k^6}{3\lambda (1 + P_M)(t - t_0 + \Delta t)^{3/2}}\right.$$

$$\left. - \frac{4(3P_M^2 - 2P_M + 3)k^8}{3(1 + P_M)^2 (t - t_0 + \Delta t)^{1/2}} + \frac{8\sqrt{\lambda}(3P_M^2 - 2P_M + 3)k^9 F(\omega)}{(1 + P_M)^{5/2} P_M^{1/2}}\right] \qquad \text{----------- (5.11.17)}$$

where $F(\omega) = e^{-\omega^2}\int_0^{\omega} e^{x^2}\,dx$,

$$\omega = k\sqrt{\frac{\lambda(t-t_0)}{1+P_M}} \text{ or } k\sqrt{\frac{\lambda(t-t_0+\Delta t)}{1+P_M}} \ .$$

By setting $\hat{r}=0$, j=i, $d\hat{k} = -2\pi k^2 d(\cos\theta)d\hat{k}$ and $E = 2\pi k^2\langle\psi_i\psi'_j\rangle$ in equation (5.10.10) we get the expression for magnetic energy decay law as

$$\frac{\langle h_i h'_i\rangle}{2} = \int_0^{\infty} E dk \ . \qquad \text{----------- (5.11.18)}$$

Substituting equation (5.11.17) into equation (5.11.18) and after integration, we get

$$\frac{\langle h_i h'_i\rangle}{2} = \frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}(T+\Delta T/2)^{3/2}} + \frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}}\exp[fs]$$

$$\left[\frac{9}{16T^{5/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta t\right)^{3/2}} + \frac{9}{16(T+\Delta T)^{5/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{5/2}} + \frac{5P_M(7P_M-6)}{16(1+2P_M)T^{3/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{7/2}}\right.$$

$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)(T+\Delta T)^{3/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{7/2}} + \frac{35P_M(3P_M^2-2P_M+3)}{8(1+2P_M)T^{1/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{9/2}} +$$

$$\frac{35P_M(3P_M^2-2P_M+3)}{8(1+2P_M)(T+\Delta T)^{1/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{9/2}} + \frac{8P_M(3P_M^2-2P_M+3)(1+2P_M)^{5/2}}{3.2^{23/2}(1+P_M)^{11/2}}\times$$

$$\left.\sum_{n=0}^{\infty}\frac{1.3.5\ldots\ldots\ldots\ldots(2n+9)}{n!(2n+1)2^{2n}(1+P_M)^n}\times\left\{\frac{T^{(2n+1)/2}}{\left(T+\Delta T/2\right)^{(2n+1)/2}} + \frac{(T+\Delta T)^{(2n+1)/2}}{\left(T+\Delta T/2\right)^{(2n+1)/2}}\right\}\right], \qquad \text{------- (5.11.19)}$$

where T=t-t$_0$.

For $T_m = T + \Delta T/2$, equation (5.11.19) takes the form

$$\frac{\langle h_i h_i' \rangle}{2} = \frac{N_0}{8\sqrt{2\pi}\lambda^{3/2} T_m^{3/2}} + \frac{\pi \delta_0}{4\lambda^6 (1+P_M)(1+2P_M)^{5/2}} \exp[fs)]$$

$$\times [\frac{9}{16\left(T_m - \Delta T/2\right)^{5/2}\left(T_m + \frac{\Delta T}{1+2P_M}\right)^{5/2}} + \frac{9}{16\left(T_m + \Delta T/2\right)^{5/2}\left(T_m - \frac{\Delta T}{2(1+2P_M)}\right)^{5/2}}$$

$$+ \frac{5P_M(7P_M - 6)}{16(1+2P_M)\left(T_m - \frac{\Delta T}{2}\right)^{3/2}\left(T_m + \frac{\Delta T}{2(1+2P_M)}\right)^{7/2}} + \frac{5P_M(7P_M - 6)}{16(1+2P_M)\left(T_m + \frac{\Delta T}{2}\right)^{3/2}\left(T_m - \frac{\Delta T}{2(1+2P_M)}\right)^{7/2}} + ---]. \quad -(5.11.20)$$

This is the decay law of magnetic energy fluctuations of MHD turbulence before the final period for the case of multi-point and multi-time in presence of dust particles.

5.12 Concluding Remarks:

In equation (5.11.20) we obtained the decay law of magnetic energy fluctuations in MHD turbulence before the final period in presence of dust particles considering three-point correlation terms for the case of multi-point and multi-time. If the fluid is clean then $f = 0$, the equation (5.11.20) becomes

$$\frac{\langle h_i h_i' \rangle}{2} = \frac{N_0}{8\sqrt{2\pi}\lambda^{3/2} T_m^{3/2}} + \frac{\pi \delta_0}{4\lambda^6 (1+P_M)(1+2P_M)^{5/2}}$$

$$\times \left[\frac{9}{16\left(T_m - \Delta T/2\right)^{5/2}\left(T_m + \frac{\Delta T}{1+2P_M}\right)^{5/2}} + \frac{9}{16\left(T_m + \Delta T/2\right)^{5/2}\left(T_m - \frac{\Delta T}{2(1+2P_M)}\right)^{5/2}} \right.$$

$$\left. + \frac{5P_M(7P_M - 6)}{16(1+2P_M)\left(T_m - \frac{\Delta T}{2}\right)^{3/2}\left(T_m + \frac{\Delta T}{2(1+2P_M)}\right)^{7/2}} + \frac{5P_M(7P_M - 6)}{16(1+2P_M)\left(T_m + \frac{\Delta T}{2}\right)^{3/2}\left(T_m - \frac{\Delta T}{2(1+2P_M)}\right)^{7/2}} + --- \right], \quad -(5.12.1)$$

which was obtained earlier by Sarker and Islam [115].

If we put $\Delta T = 0$, we can easily find out

$$\frac{\langle h^2\rangle}{2}=\frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}}T^{-3/2}+\frac{\pi\delta_0}{2\lambda^6(1+P_M)(1+2P_M)^{5/2}}T^{-5}\times\left\{\frac{9}{16}+\frac{5}{16}\frac{P_M(7P_M-6)}{1+2P_M}+---\right\}=AT^{-3/2}+BT^{-5} \quad (5.12.2)$$

where

$$A=\frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}},\quad B=\frac{\pi\delta_0}{2\lambda^6(1+P_M)(1+2P_M)^{5/2}}\times\left\{\frac{9}{16}+\frac{5}{16}\frac{P_M(7P_M-6)}{1+2P_M}+---\right\}$$

which is same as obtained earlier by Sarker and Kishore [108].

From the result (equation 5.11.20) of the study we conclude that due to the effect of dust particles in the magnetic field, the turbulent energy decays more rapidly than the energy for clean fluid. From the assumption we conclude that the higher-order correlation terms may be neglected in comparison with lower-order correlation terms. By neglecting the quadruple correlation terms in three-point, three-time correlation equation the result (5.11.20) applicable to the dusty fluid MHD turbulence before the final period of decay were obtained. If higher-order correlation equations are considered in the analysis, it appears that more terms of higher power of time would be added to the equation (5.11.20). For large times the last term in the equation (5.11.20) becomes negligible, leaving the -3/2power decay law for the final period.

CHAPTER-V

PART-C

DECAY OF DUSTY FLUID MAGNETO-HYDRODYNAMIC TURBULENCE BEFORE THE FINAL PERIOD IN A ROTATING SYSTEM FOR THE CASE OF MULTI-POINT AND MULTI-TIME

5.13 Introduction:

From historical point of view it seems that the first attempt to study the problem of MHD is due to Faraday. Later on in 1937 Hartmann took up Faraday's idea in understood conditions. There are two basic approaches to the problem, the macroscopic fluid continuum model known as MHD, and the microscopic statistical model known as plasma dynamics we shall be concerned here only with the MHD, that is electrically conducting fluids. Funada, Tuitiya and Ohji [37] considered the effect of coriolis force on turbulent motion in presence of strong magnetic field with the assumption that the coriolis force term is balanced by the geostropic wind approximation Sarker and Islam [117] studied the decay of dusty fluid turbulence before the final period in a rotating system. Kishore and Sinha [59] studied the rate of change of vorticity covariance in dusty fluid turbulence. Sinha [122] also studied the effect of dust particles on the acceleration covariance of ordinary turbulence. Deissler [27,28] developed a theory for homogeneous turbulence, which was valid for times before the final period. Using Deissler's theory Loeffler and Deissler [72] studied the temperature fluctuations in homogeneous turbulence before the final period. Sarker and Kishore [108] studied the decay of MHD turbulence before the final period. Sarker and Islam [117] also studied the decay of dusty fluid turbulence before the final period in a rotating system. Islam and Sarker [46] studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Kumar and Patel [65] also studied on first-order reactant in homogeneous turbulence before the final period of decay for the case of multi-point and multi-time. Sarker and Islam [116] studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time. Shimomura and Yoshizawa [119], Shimomura [120] discussed the statistical analysis of turbulent viscosity, turbulent sealer flux respectively in a rotating system two-scale direct interaction approach. Sarker and Islam [115] also studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. In their approach they considered two and three-point correlations and neglecting higher order correlation terms compared to the second-and third-order correlation terms.

In this chapter, we have studied the decay of dusty fluid MHD turbulence before the final period in a rotating system for the case of multi-point and multi-time. Here two-and three-point correlation terms have been considered and the fourth order correlation terms are neglected in comparison to the second-and third-order correlation terms. Finally the decay law of magnetic energy fluctuations of dusty fluid MHD turbulence in a rotating system before the final period for the case of multi-point and multi-time is obtained. If the fluid is clean and the system is non-rotating, the equation reduces to one obtained earlier by Sarker and Islam [115]. It is an extension work of the part-A and part-B of this chapter. In part-A, we have considered the rotating system and in part-B, have considered the dust particles. But in this part, we have considered both the rotating system and dust particles.

5.14 Basic Equations:

The equations of motion and continuity for viscous, incompressible dusty fluid MHD turbulent flow in a rotating system are given by

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial w}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2\in_{mkl}\Omega_m u_i + f(u_i - v_i), \qquad \text{-----------} (5.14.1)$$

$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \frac{\nu}{P_M}\frac{\partial^2 h_i}{\partial x_k \partial x_k}, \qquad \text{-----------} (5.14.2)$$

$$\frac{\partial v_i}{\partial t}+v_k\frac{\partial v_i}{\partial x_K} = -\frac{K}{m_s}(v_i - u_i) \qquad \text{------------} (5.14.3)$$

with

$$\frac{\partial u_i}{\partial x_i}=\frac{\partial v_i}{\partial x_i}=\frac{\partial h_i}{\partial x_i}=0\ . \qquad \text{------------ (5.14.4)}$$

Here, u_i, turbulence velocity component; h_i, magnetic field fluctuation component; v_i, dust particle velocity component; $w(\hat{x},t)=\frac{P}{\rho}+\frac{1}{2}\langle h^2\ \rangle+\frac{1}{2}\left|\hat{\Omega}\times\hat{x}\right|^2$, total MHD pressure $p(\hat{x},t)$, hydrodynamic pressure; ρ, fluid density; ν, Kinematic viscosity; P_M, magnetic prandtl number; x_k, space co-ordinate; the subscripts can take on the values 1, 2 or 3 and the repeated subscripts in a term indicate a summation; Ω_m, constant angular velocity component; $\in_{mkl}$, alternating tensor; $f=\frac{KN}{\rho}$, dimension of frequency ; N, constant number density of dust particle $m_s=\frac{4}{3}\pi R_s^3\rho_s$, mass of single spherical dust particle of radius R_s; ρ_s, constant density of the material in dust particle.

5.15 Two-Point, Two-Time Correlation and Spectral Equations:

Induction equations of a magnetic field at the point p and p′ separated by the vector $\hat{r}$ could be written as

$$\frac{\partial h_i}{\partial t}+u_k\frac{\partial h_i}{\partial x_k}-h_k\frac{\partial u_i}{\partial x_k}=\frac{\nu}{P_M}\frac{\partial^2 h_i}{\partial x_k\partial x_k} \qquad \text{----------- (5.15.1)}$$

and $$\frac{\partial h_j'}{\partial t'}+u_k'\frac{\partial h_j'}{\partial x_k'}-h_k'\frac{\partial u_j'}{\partial x_k'}=\frac{\nu}{P_M}\frac{\partial^2 h_j'}{\partial x_k'\,\partial x_k'}\ . \qquad \text{------------ (5.15.2)}$$

Multiplying equation (5.15.1) by h_j' and equation (5.15.2) by h_i and taking ensemble average, we get

$$\frac{\partial\langle h_ih_j'\ \rangle}{\partial t}+\frac{\partial}{\partial x_k}\left[\langle u_kh_ih_j'\ \rangle-\langle u_ih_kh_j'\ \rangle\ \right]=\frac{\nu}{P_M}\frac{\partial^2\langle h_ih_j'\ \rangle}{\partial x_k\partial x_k} \qquad \text{------------ (5.15.3)}$$

and $$\frac{\partial\langle h_ih_j'\ \rangle}{\partial t'}+\frac{\partial}{\partial x_k'}\left[\langle u_k'h_ih_j'\ \rangle-\langle u_j'h_ih_k'\ \rangle\ \right]=\frac{\nu}{P_M}\frac{\partial^2\langle h_ih_j'\ \rangle}{\partial x_k'\partial x_k'}\ . \qquad \text{------------ (5.15.4)}$$

Angular bracket $\langle - - - - - \rangle$ is used to denote an ensemble average.

Using the transformations

$$\frac{\partial}{\partial x_k}=-\frac{\partial}{\partial r_k},\frac{\partial}{\partial x'_k}=\frac{\partial}{\partial r_k},\left(\frac{\partial}{\partial t}\right)t'=\left(\frac{\partial}{\partial t}\right)\Delta t-\frac{\partial}{\partial \Delta t},\frac{\partial}{\partial t'}=\frac{\partial}{\partial \Delta t} \qquad \text{------------ (5.15.5)}$$

Equations (5.15.3) and (5.15.4) can be written as

$$\frac{\partial\langle h_i h'_j\rangle}{\partial t}+\frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle-\langle u'_j h_i h'_k\rangle\right](\hat{r},\Delta t,t)-\frac{\partial}{\partial r_k}\left[\langle u_k h_i h'_j\rangle-\langle u_i h_k h'_j\rangle\right](\hat{r},\Delta t,t)$$

$$=\frac{2\nu}{P_M}\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k \partial r_k} \qquad \text{---------- (5.15.6)}$$

and $\dfrac{\partial\langle h_i h'_j\rangle}{\partial \Delta t}+\dfrac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle-\langle u'_j h_i h'_k\rangle\right](\hat{r},\Delta t,t)=\dfrac{\nu}{P_M}\dfrac{\partial^2\langle h_i h'_j\rangle}{\partial r_k \partial r_k}$. ---------- (5.15.7)

Using the relations of Chandrasekhar [19]

$$\langle u_k h_i h'_j\rangle=-\langle u'_k h_i h'_j\rangle,\quad \langle u'_j h_i h'_k\rangle=\langle u_i h_k h'_j\rangle.$$

Equations (5.15.6) and (5.15.7) become

$$\frac{\partial\langle h_i h'_j\rangle}{\partial t}+2\frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle-\langle u_i h_k h'_j\rangle\right]=\frac{2\nu}{P_M}\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k \partial r_k} \qquad \text{----------- (5.15.8)}$$

and $\dfrac{\partial\langle h_i h'_j\rangle}{\partial t}+\dfrac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle-\langle u_i h_k h'_j\rangle\right]=\dfrac{\nu}{P_M}\dfrac{\partial^2\langle h_i h'_j\rangle}{\partial r_k \partial r_k}$. ----------- (5.15.9)

Now we write equations (5.15.8) and (5.15.9) in spectral form in order to reduce it to an ordinary differential equation by use of the following three-dimensional Fourier transforms:

$$\langle h_i h'_j\rangle(\hat{r},\Delta t,t)=\int_{-\infty}^{\infty}\langle \psi_i \psi'_j\rangle(\hat{K},\Delta t,t)\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K}\ , \qquad \text{---------- (5.15.10)}$$

$$\langle u_i h_k h'_j\rangle(\hat{r},\Delta t,t)=\int_{-\infty}^{\infty}\langle \alpha_i \psi_k \psi'_j\rangle(\hat{K},\Delta t,t)\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K}\ . \qquad \text{---------- (5.15.11)}$$

Interchanging the subscripts i and j then interchanging the points p and p′ gives

$$\langle u'_k h_i h'_j \rangle(\hat{r}, \Delta t, t) = \langle u_k h_i h'_j \rangle(-\hat{r}, -\Delta t, t + \Delta t)$$

$$= \int_{-\infty}^{\infty} \langle \alpha_i \psi_i \psi'_j \rangle(-\hat{K}, -\Delta t, t + \Delta t) \exp[\hat{i}(\hat{K}.\hat{r})] d\hat{K} \qquad \text{---------- (5.15.12)}$$

where $\hat{K}$ is known as a wave number vector and $d\hat{K}$ =dK_1 dK_2 dK_3. The magnitude of $\hat{K}$ has the dimension 1/length and can be considered to be the reciprocal of an eddy size. Substituting of equation (5.15.10) to (5.15.12) in to equations (5.15.8) and (5.15.9) leads to the spectral equations

$$\frac{\partial \langle \psi_i \psi'_j \rangle}{\partial t} + \frac{2\nu k^2}{P_M} \langle \psi_i \psi'_j \rangle = 2ik_k \left[\langle \alpha_i \psi_k \psi'_j \rangle(\hat{K}, \Delta t, t) - \langle \alpha_k \psi_i \psi'_j \rangle(-\hat{K}, -\Delta t, t + \Delta t) \right] \text{ -(5.15.13)}$$

and

$$\frac{\partial \langle \psi_i \psi'_j \rangle}{\partial \Delta t} + \frac{\nu k^2}{P_M} \langle \psi_i \psi'_j \rangle = ik_k \left[\langle \alpha_i \psi_k \psi'_j \rangle(\hat{K}, \Delta t, t) - \langle \alpha_k \psi_i \psi'_j \rangle(-\hat{K}, -\Delta t, t + \Delta t) \right] \text{ .-(5.15.14)}$$

The tensor equations (5.15.13) and (5.15.14) becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial \langle \psi_i \psi'_i \rangle}{\partial t} + \frac{2\nu k^2}{P_M} \langle \psi_i \psi'_i \rangle = 2ik_k \left[\langle \alpha_i \psi_k \psi'_i \rangle(\hat{K}, \Delta t, t) - \langle \alpha_k \psi_i \psi'_i \rangle(-\hat{K}, -\Delta t, t + \Delta t) \right] \text{ -(5.15.15)}$$

and

$$\frac{\partial \langle \psi_i \psi'_i \rangle}{\partial \Delta t} + \frac{\nu k^2}{P_M} \langle \psi_i \psi'_i \rangle = ik_k \left[\langle \alpha_i \psi_k \psi'_i \rangle(\hat{K}, \Delta t, t) - \langle \alpha_k \psi_i \psi'_i \rangle(-\hat{K}, -\Delta t, t + \Delta t) \right]. \text{ ---(5.15.16)}$$

The terms on the right side of equations (5.15.15) and (5.15.16) are collectively proportional to what is known as the magnetic energy transfer terms.

5.16 Three-Point, Three-Time Correlation and Spectral Equations:

Similar procedure can be used to find the three-point correlation equations. For this purpose we take the momentum equation of dusty fluid MHD turbulence at the point P, the induction equation of magnetic field fluctuations at p′ and p″ separated by the vector $\hat{r}$ and $\hat{r}'$ as

$$\frac{\partial u_l}{\partial t}+u_k\frac{\partial u_l}{\partial x_k}-h_k\frac{\partial h_l}{\partial x_k}=-\frac{\partial w}{\partial x_l}+\nu\frac{\partial^2 u_l}{\partial x_k\partial x_k}-2\in_{mkl}\Omega_m u_l+f(u_l-v_l)\,, \qquad \text{-------- (5.16.1)}$$

$$\frac{\partial h_i'}{\partial t'}+u_k'\frac{\partial h_i'}{\partial x_k'}-h_k'\frac{\partial u_i'}{\partial x_k'}=\frac{\nu}{P_M}\frac{\partial^2 h_i'}{\partial x_k'\partial x_k'}\,, \qquad \text{-------- (5.16.2)}$$

$$\frac{\partial h_j''}{\partial t''}+u_k''\frac{\partial h_j''}{\partial x_k''}-h_k''\frac{\partial u_j''}{\partial x_k''}=\frac{\nu}{P_M}\frac{\partial^2 h_j''}{\partial x_k''\partial x_k''}\,. \qquad \text{-------- (5.16.3)}$$

Multiplying equation (5.16.1) by $h_i' h_j''$, equation (5.16.2) by $u_l h_j''$ and equation (5.16.3) by $u_l h_i'$, taking ensemble average, one obtains

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t}+\frac{\partial}{\partial x_k}\left[\langle u_k u_l h_i' h_j''\rangle-\langle h_k h_l h_i' h_j''\rangle\right]=\frac{\partial\langle w h_i' h_j''\rangle}{\partial x_l}$$

$$+\nu\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k\partial x_k}-2\in_{mkl}\Omega_m\langle u_l h_i' h_j''\rangle+f(\langle u_l h_i' h_j''\rangle-\langle v_l h_i' h_j''\rangle)\,, \qquad \text{-------- (5.16.4)}$$

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t'}+\frac{\partial}{\partial x_k'}\left[\langle u_l u_k' h_i' h_j''\rangle-\langle u_l u_i' h_k' h_j''\rangle\right]=\frac{\nu}{P_M}\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k'\partial x_k'}\,, \qquad \text{--------- (5.16.5)}$$

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t''}+\frac{\partial}{\partial x_k''}\left[\langle u_l u_k'' h_i' h_j''\rangle-\langle u_l u_j'' h_i' h_k''\rangle\right]=\frac{\nu}{P_M}\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k''\partial x_k''}\,. \qquad \text{--------- (5.16.6)}$$

Using the transformations

$$\frac{\partial}{\partial x_k}=-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right),\ \frac{\partial}{\partial x_k'}=\frac{\partial}{\partial r_k},\ \frac{\partial}{\partial x_k''}=\frac{\partial}{\partial r_k'}\,,$$

$$\left(\frac{\partial}{\partial t}\right)t',t''=\left(\frac{\partial}{\partial t}\right)\Delta t,\Delta t'-\frac{\partial}{\partial\Delta t}-\frac{\partial}{\partial\Delta t'}\,,$$

$$\frac{\partial}{\partial t'}=\frac{\partial}{\partial\Delta t},\ \frac{\partial}{\partial t''}=\frac{\partial}{\partial\Delta t'}$$

into equations (5.16.4) to (5.16.6), we have

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t}-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right)\left[\langle u_k u_l h_i' h_j''\rangle-\langle h_k h_l h_i' h_j''\rangle\right]+\frac{\partial}{\partial r_k}\left[\langle u_l u_k' h_i' h_j''\rangle-\langle u_l u_i' h_k' h_j''\rangle\right]$$

$$+\frac{\partial}{\partial r_k'}\left[\langle u_l u_k'' h_i' h_j''\rangle-\langle u_l u_j'' h_i' h_k''\rangle\right]=-\left(\frac{\partial}{\partial r_l}+\frac{\partial}{\partial r_l'}\right)\langle w h_i' h_j''\rangle+\nu\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right)^2\langle u_l h_i' h_j''\rangle$$

$$+\frac{\nu}{P_M}\left[\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k\partial r_k}+\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k'\partial r_k'}\right]-2\in_{mkl}\Omega_m\langle u_l h_i' h_j''\rangle+f\left(\langle u_l h_i' h_j''\rangle-\langle v_l h_i' h_j''\rangle\right), \quad \text{---- (5.16.7)}$$

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial \Delta t}+\frac{\partial}{\partial r_k}\left[\langle u_l u_k' h_i' h_j''\rangle-\langle u_l u_i' h_k' h_j''\rangle\right]=\frac{\nu}{P_M}\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k\partial r_k} \quad \text{------ (5.16.8)}$$

and $$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial \Delta t'}+\frac{\partial}{\partial r_k'}\left[\langle u_l u_k'' h_i' h_j''\rangle-\langle u_l u_j'' h_i' h_k''\rangle\right]=\frac{\nu}{P_M}\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k'\partial r_k'}\,. \quad \text{------ (5.16.9)}$$

In order to convert equations (5.16.7)–(5.16.9) to spectral form, we can define the following six dimensional Fourier transforms:

$$\langle u_l h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}', \text{-(5.16.10)}$$

$$\langle u_l u_k' h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \phi_K' \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}', \text{(5.16.11)}$$

$$\langle w h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \gamma \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}', \text{ - (5.16.12)}$$

$$\langle u_k u_l h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_K \phi_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}', \text{-(5.16.13)}$$

$$\langle h_k h_l h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \beta_K \beta_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}', \text{(5.16.14)}$$

$$\langle u_l u_i' h_K' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \phi_i' \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}', \text{(5.16.15)}$$

$$\langle v_l h_i' h_j'' \rangle (\hat{r}, \hat{r}, \Delta t, \Delta t', t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \mu_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) \exp[\hat{i}(\hat{K}.\hat{r} + \hat{K}'.\hat{r}')] d\hat{K} d\hat{K}' \quad \text{-(5.16.16)}$$

Interchanging the points P′ and P″ along with the indices i and j result in the relations

$$\langle u_l u_k'' h_i' h_j'' \rangle = \langle u_l u_k' h_i' h_j'' \rangle .$$

By use of these facts and the equations (5.16.10)-(5.16.16), we can write equations (5.16.7)-(5.16.9) in the form

$$\frac{\partial}{\partial t} \langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \frac{\nu}{P_M} \left[(1 + P_M)(k^2 + k'^2) + 2P_M kk' + \frac{P_M}{\nu}(2 \in_{mkl} \Omega_m - f) \right]$$

$$\langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = [i(k_k + k_k') \langle \phi_k \phi_l \beta_i' \beta'' \rangle - i(k_k + k_k') \langle \beta_k \beta_l \beta_i' \beta_j'' \rangle$$

$$- i(k_k + k_k') \langle \phi_l \phi_k' \beta_i' \beta_j'' \rangle + i(k_k + k_k') \langle \phi_l \phi_i' \beta_k' \beta_j'' \rangle - i(k_l + k_l') \langle \gamma \beta_i' \beta_j'' \rangle$$

$$- f \langle \mu_l \beta_i' \beta_j'' \rangle] (\hat{K}, \hat{K}', \Delta t, \Delta t', t) , \quad \text{---------- (5.16.17)}$$

$$\frac{\partial}{\partial \Delta t} \langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \frac{\nu k^2}{P_M} \langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)$$

$$= -ik_k \langle \phi_l \phi_k' \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}' \Delta t, \Delta t', t) + ik_k \langle \phi_l \phi_i' \beta_k' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) \quad \text{---------- (5.16.18)}$$

and $\frac{\partial}{\partial \Delta t'} \langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \frac{\nu k'^2}{P_M} \langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)$

$$= -ik_k' \langle \phi_l \phi_k' \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}' \Delta t, \Delta t', t) + ik_k' \langle \phi_l \phi_i' \beta_k' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) . \quad \text{---------- (5.16.19)}$$

If the derivative with respect to x_l is taken of the momentum equation (5.16.1) for the point P, the equation multiplied by $h_i' h_j''$ and time average taken, the resulting equation

$$-\frac{\partial^2 \langle w h_i' h_j'' \rangle}{\partial x_l \partial x_l} = \frac{\partial^2}{\partial x_l \partial x_k} \left(\langle u_l u_k h_i' h_j'' \rangle - \langle h_l h_k h_i' h_j'' \rangle \right) . \quad \text{---------- (5.16.20)}$$

Writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$-\left[\frac{\partial^2}{\partial r_l \partial r_l} + 2\frac{\partial^2}{\partial r_l \partial r_l'} + \frac{\partial^2}{\partial r_l' \partial r_l'} \right] \langle w h_i' h_j'' \rangle = \left[\frac{\partial^2}{\partial r_l \partial r_k} + \frac{\partial^2}{\partial r_l' \partial r_k} + \frac{\partial^2}{\partial r_l \partial r_k'} + \frac{\partial^2}{\partial r_l' \partial r_k'} \right] \times$$

$$\left(\langle u_l u_k h'_i h''_j\rangle - \langle h_l h_k h'_i h''_j\rangle\right). \qquad \text{---------} (5.16.21)$$

Taking the Fourier transforms of equation (5.16.21)

$$-\langle \gamma\beta'_i\beta''_j\rangle = \frac{(k_l k_k + k'_l k_k + k_l k'_k + k'_l k'_k)\left(\langle \phi_l\phi_k\beta'_i\beta''_j\rangle - \langle \beta_l\beta_k\beta'_i\beta''_j\rangle\right)}{k_l k_l + 2k_l k'_l + k'_l k'_l}. \qquad \text{----------} (5.16.22)$$

Equation (5.16.22) can be used to eliminate $\langle \gamma\beta'_i\beta''_j\rangle$ from equation (5.16.17)

The tensor equations (5.16.17) to (5.16.19) can be converted to scalar equation by contraction of the indices i and j

$$\frac{\partial}{\partial t}\langle \phi_l\beta'_i\beta''_i\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + \frac{\nu}{P_M}\left[(1+P_M)(k^2+k'^2)+2P_M kk'\right] +$$

$$\frac{P_M}{\nu}(2\in_{mkl}\Omega_m - f)\langle \phi_l\beta''_i\beta''_i\rangle(\hat{K},\hat{K},\Delta t,\Delta t',t) = i(k_k+k'_k)\langle \phi_k\phi_l\beta'_i\beta''_j\rangle$$

$$(\hat{K},\hat{K}',\Delta t,\Delta t',t) - i(k_k+k'_k)\langle \beta_K\beta_l\beta'_i\beta''_i\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) - i(k_k+k'_k)$$

$$\langle \phi_l\phi'_k\beta'_i\beta''_i\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + i(k_k+k'_k)\langle \phi_l\phi'_i\beta'_k\beta''_i\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) - i(k_l+k'_l)$$

$$\langle \gamma\beta'_i\beta''_i\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) - f\langle \mu_l\beta'_i\beta''_i\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t), \qquad \text{---------} (5.16.17a)$$

$$\frac{\partial}{\partial \Delta t}\langle \phi_l\beta'_i\beta''_i\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + \frac{\nu k^2}{P_M}\langle \phi_l\beta'_i\beta''_i\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)$$

$$= -ik_k\langle \phi_l\phi'_k\beta'_i\beta''_i\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + ik_k\langle \phi_l\phi'_i\beta'_k\beta''_i\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) \qquad \text{----------} (5.16.18a)$$

and $\frac{\partial}{\partial \Delta t'}\langle \phi_l\beta'_i\beta''_i\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + \frac{\nu k'^2}{P_M}\langle \phi_l\beta'_i\beta''_i\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)$

$$= -ik'_k\langle \phi_l\phi'_k\beta'_i\beta''_i\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + ik'_k\langle \phi_l\phi'_i\beta'_i\beta''_i\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t). \qquad \text{----------} (5.16.19a)$$

5.17 Solution for Times Before the Final Period:

It is known that the equation for final period of decay is obtained by considering the two-point correlations after neglecting third-order correlation terms. To study the decay for times before the final period, the three-point correlations are considered and the quadruple correlation terms are neglected because the quadruple correlation terms decays faster than the lower-order correlation terms. Equation (5.16.22) represents that the term $\langle \gamma\beta_i'\beta_j''\rangle$ associated with the pressure fluctuations should also be neglected. Thus neglecting all the terms on the right hand side of equations (5.16.17a) to (5.16.19a)

$$\frac{\partial}{\partial t}\langle \phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)+\frac{\nu}{P_M}\Big[(1+P_M)(k^2+k'^2)+2P_M kk'+$$

$$\frac{P_M}{\nu}(2\in_{mkl}\Omega_m - fs)\Big]\langle \phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=0\ , \qquad \text{----------} (5.17.1)$$

$$\frac{\partial}{\partial \Delta t}\langle \phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)+\frac{\nu k^2}{P_M}\langle \phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=0 \qquad \text{----------} (5.17.2)$$

and

$$\frac{\partial}{\partial \Delta t'}\langle \phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)+\frac{\nu k'^2}{P_M}\langle \phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=0 \qquad \text{-----------} (5.17.3)$$

where $\langle \mu_l\beta_i'\beta_i''\rangle = R\langle \phi_l\beta_i'\beta_i''\rangle$ and 1-R=S, here R and S are arbitrary constant.

Integrating equations (5.17.1) to (5.17.3) between t_o and t, we obtain

$$k_l\langle \phi_l\beta_i'\beta_i''\rangle = f_l\exp\Big\{-\frac{\nu}{P_M}\Big[(1+P_M)(k^2+k'^2)+2P_M kk'\cos\theta+$$

$$\frac{P_M}{\nu}(2\in_{mkl}\Omega_m - fs)\Big](t-t_o)\Big\},$$

$$k_l\langle \phi_l\beta_i'\beta_i''\rangle = g_l\exp\left[-\frac{\nu k^2}{P_M}\Delta t\right]$$

and $k_l\langle \phi_l\beta_i'\beta_i''\rangle = q_l\exp\left[-\frac{\nu k'^2}{P_M}\Delta t'\right]$.

For these relations to be consistent, we have

$$k_l\langle\phi_i\beta_i'\beta_i''\rangle = k_l\langle\phi_i\beta_i'\beta_i''\rangle_o \exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)+k^2\Delta t+k'^2\Delta t'$$

$$+2P_M kk'\cos\theta(t-t_o)+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)(t-t_o)\;\} \qquad \text{-------------- (5.17.4)}$$

where θ is the angle between $\hat{K}$ and $\hat{K}'$ and $\langle\phi_i\beta_i'\beta_i''\rangle_o$ is the value of $\langle\phi_i\beta_i'\beta_i''\rangle$ at $t = t_o$, $\Delta t = \Delta t' = 0$, $\lambda = \frac{\nu}{P_M}$

By letting $\hat{r}' = 0$, $\Delta t' = 0$ in the equation (5.16.10) and comparing with equations (5.15.11) and (5.15.12) we get

$$\langle\alpha_i\psi_k\psi_i'\rangle(\hat{K},\Delta t,t)=\int_{-\infty}^{\infty}\langle\phi_i\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,o,t)d\hat{K}' \qquad \text{----------- (5.17.5)}$$

and $\langle\alpha_i\psi_k\psi_i'\rangle(-\hat{K},-\Delta t,t+\Delta t)=\int_{-\infty}^{\infty}\langle\phi_i\beta_i'\beta_i''\rangle(-\hat{K},\hat{K}',\Delta t,o,t)d\hat{K}'$. ----------- (5.17.6)

Substituting equation (5.17.4) to (5.17.6) into equation (5.15.15), one obtains

$$\frac{\partial}{\partial t}\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)+2\lambda k^2\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)=\int_{-\infty}^{\infty}2ik_l[\langle\phi_i\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,0,t)$$

$$-\langle\phi_i\beta_i'\beta_i''\rangle(-\hat{K},-\hat{K}',\Delta t,0,t)\;]_o\exp[-\lambda\{(1+P_M)(k^2+k'^2)(t-t_o)$$

$$+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)(t-t_o)\;\}\;]. \qquad \text{---------- (5.17.7)}$$

Now, $d\hat{K}'$ can be expressed in terms of k′ and θ as $-2\pi k' d(\cos\theta)dk'$ (cf. Deissler [28])

i.e. $d\hat{K}' = -2\pi k' d(\cos\theta)dk'$ --------- (5.17.7a)

Substituting of equation (5.17.7a) in equation (5.17.7) yields

$$\frac{\partial}{\partial t}\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)+2\lambda k^2\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)=2\int_0^{\infty}2\pi ik_l[\langle\phi_i\beta_i'\beta_i''\rangle(\hat{K},\hat{K}')$$

$$-\langle\phi_i\beta_i'\beta_i''\rangle(-\hat{K},-\hat{K}')\;]_o k'^2\left[\int_{-1}^{1}\exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)\right.$$

$$+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)(t-t_o)\;]\;\}d(\cos\theta)\;]dk'\;. \text{---- (5.17.8)}$$

The quantity $[\langle\phi_1\beta_i'\beta_i''\rangle(\hat{K},\hat{K}')-\langle\phi_1\beta_i'\beta_i''\rangle(-\hat{K},-\hat{K}')]_0$ depends on the initial conditions of the turbulence.

In order to find the solution completely and following Loeffler and Deissler [72] we assume that

$$ik_1\left[\langle\phi_1\beta_i'\beta_i''\rangle(\hat{K},\hat{K}')-\langle\phi_1\beta_i'\beta_i''\rangle(-\hat{K},-\hat{K}')\right]_0=\frac{-\delta_0}{(2\pi)^2}\left(k^2k'^4-k^4k'^2\right) \qquad \text{---------- (5.17.9)}$$

where δ_0 is a constant determined by the initial conditions. The negative sign is placed in front of δ_0 in order to make the transfer of energy from small to large wave numbers for positive value of δ_0.

Substituting equation (5.17.9) into equation (5.17.8) we get

$$\frac{\partial}{\partial t}2\pi\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)+2\lambda k^2 2\pi\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)=-2\delta_0\int_0^\infty\left(k^2k'^4-k^4k'^2\right)k'^2$$

$$\left[\int_{-1}^{1}\exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta\right.$$

$$\left.+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)(t-t_o)\;]\;\}d(\cos\theta)\;\right]d\hat{k}'\;. \qquad \text{--------- (5.17.10)}$$

Multiplying both sides of equation (5.17.10) by k^2, we get

$$\frac{\partial E}{\partial t}+2\lambda k^2E=F \qquad \text{---------- (5.17.11)}$$

where, $E=2\pi k^2\langle\psi_i\psi_i'\rangle$, E is the magnetic energy spectrum function and F is the magnetic energy transfer term and is given by

$$F=-2\delta_0\int_0^\infty\left(k^2k'^4-k^4k'^2\right)k^2k'^2\times\left[\int_{-1}^{1}\exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)\right.$$

$$+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)(t-t_o)\ ]\ \}d(\cos\theta)\ \Big]dk' \quad .\text{---}\ (5.17.12)$$

Integrating equation (5.17.12) with respect to cosθ, we have

$$F=-\frac{\delta_0}{\nu(t-t_o)}\int_0^\infty\left(k^3k'^5-k^5k'^3\right)\Big[\ \exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)+k^2\Delta t$$

$$-2P_Mkk'(t-t_o)+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)(t-t_o)\ ]\ \}\ \Big]dk'$$

$$+\frac{\delta_0}{\nu(t-t_o)}\int_0^\infty\left(k^3k'^5-k^5k'^3\right)\Big[\ \exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)+k^2\Delta t$$

$$+2P_Mkk'(t-t_o)+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)(t-t_o)\ ]\ \}\ \Big]dk' \quad .\qquad \text{----------}\ (5.17.13)$$

Again integrating equation (5.17.13) with respect to k′, we have

$$F=-\frac{\delta_oP_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_o)^{3/2}(1+P_M)^{5/2}}\exp\left\{-\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)(t-t_o)\right\}\times$$

$$\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{1+P_M}{1+2P_M}\Delta t\right)\right]\times\left[\frac{15P_Mk^4}{4\nu^2(t-t_o)^2(1+P_M)}\right.$$

$$\left.+\left\{\frac{5P_M^2}{(1+P_M)^2}-\frac{3}{2}\right\}\frac{k^6}{\nu(t-t_o)}+\left\{\frac{P_M^3}{(1+P_M)^3}-\frac{P_M}{1+P_M}\right\}k^8\right]$$

$$-\frac{\delta_oP_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_o+\Delta t)^{3/2}(1+P_M)^{5/2}}\exp\left\{-\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)(t-t_o)\right\}\times$$

$$\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{P_M}{1+P_M}\Delta t\right)\right]\times\left[\frac{15P_Mk^4}{4\nu^2(t-t_o+\Delta t)^2(1+P_M)}\right.$$

$$\left.+\left\{\frac{5P_M^2}{(1+P_M)^2}-\frac{3}{2}\right\}\frac{k^6}{\nu(t-t_o+\Delta t)}+\left\{\frac{P_M^3}{(1+P_M)^3}-\frac{P_M}{1+P_M}\right\}k^8\ \right] \quad .\qquad \text{----------}\ (5.17.14)$$

The series of equation (5.17.14) contains only even power of k and start with k^4 and the equation represents the transfer function arising owing to consideration of magnetic field at three-point and three-times.

If we integrate equation (5.17.14) for Δt=0 over all wave numbers, we find that

$$\int_0^\infty F dk = 0 \qquad \text{---------- (5.17.15)}$$

which indicates that the expression for F satisfies the condition of continuity and homogeneity. Physically it was to be expected as F is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero.

The linear equation (5.17.11) can be solved to give

$$E = \exp\left[-2\lambda k^2 (t - t_o + \Delta t/2)\int F \exp\left[2\lambda k^2 (t - t_o + \Delta t/2)\right]dt\right.$$
$$\left. + J(k)\exp\left[-2\lambda k^2 (t - t_o + \Delta t/2)\right]\right], \qquad \text{------------ (5.17.16)}$$

where $J(k) = \frac{N_o k^2}{\pi}$ is a constant of integration. Substituting the values of F from equation (5.17.14) into equation (5.17.16) gives the equation

$$E = \frac{N_o k^2}{\pi}\exp\left[\ -2\lambda k^2 (t - t_o + \Delta t/2)\ \right] + \frac{\delta_o P_M \sqrt{\pi}}{4\lambda^{3/2}(1 + P_M)^{7/2}} \times$$

$$\exp\left[-(2 \in_{mkl} \Omega_m - fs)(t - t_o)\right]\exp\left[\frac{-k^2\lambda(1 + 2P_M)}{1 + P_M}\left(t - t_o + \frac{1 + P_M}{1 + 2P_M}\Delta t\right)\right]$$

$$\left[\ \frac{3k^4}{2P_M \lambda^2 (t - t_0)^{5/2}} + \frac{(7P_M - 6)k^6}{3\lambda(1 + P_M)(t - t_0)^{3/2}} - \frac{4(3P_M^2 - 2P_M + 3)k^8}{3(1 + P_M)^2 (t - t_0)^{1/2}}\right.$$

$$\left. + \frac{8\sqrt{\lambda}(3P_M^2 - 2P_M + 3)k^9}{3(1 + P_M)^{5/2}} F(\omega)\ \right] + \frac{\delta_o P_M \sqrt{\pi}}{4\lambda^{3/2}(1 + P_M)^{7/2}}\ \exp\left[-(2 \in_{mkl} \Omega_m - fs)(t - t_o)\right]$$

$$\exp\left[\frac{-k^2\lambda(1 + 2P_M)}{1 + P_M}\left(t - t_o + \frac{P_M}{1 + P_M}\Delta t\right)\right]\left[\ \frac{3k^4}{2P_M \lambda^2 (t - t_0 + \Delta t)^{5/2}} + \frac{(7P_M - 6)k^6}{3\lambda(1 + P_M)(t - t_0 + \Delta t)^{3/2}}\right.$$

$$-\frac{4\left(3P_M^2-2P_M+3\right)k^8}{3(1+P_M)^2(t-t_0+\Delta t)^{1/2}}+\frac{8\sqrt{\lambda}\left(3P_M^2-2P_M+3\right)k^9F(\omega)}{(1+P_M)^{5/2}P_M^{1/2}}\Bigg], \qquad \text{---------- (5.17.17)}$$

where $F(\omega)=e^{-\omega^2}\int_0^{\omega}e^{x^2}dx$,

$$\omega=k\sqrt{\frac{\lambda(t-t_0)}{1+P_M}} \text{ or } k\sqrt{\frac{\lambda(t-t_0+\Delta t)}{1+P_M}}.$$

By setting $\hat{r}=0$, j=i, $d\hat{k}=-2\pi k^2 d(\cos\theta)d\hat{k}$ and $E=2\pi k^2\langle\psi_i\psi_j'\rangle$ in equation (5.15.10) we get the expression for magnetic energy decay law as

$$\frac{\langle h_i h_i'\rangle}{2}=\int_0^{\infty}Edk. \qquad \text{---------- (5.17.18)}$$

Substituting equation (5.17.17) into equation (5.17.18) and after integration, we get

$$\frac{\langle h_i h_i'\rangle}{2}=\frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}(T+\Delta T/2)^{3/2}}+\frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}}\exp[-(2\in_{mkl}\Omega_m-fs)]$$

$$\times\Bigg[\frac{9}{16T^{5/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta t\right)^{3/2}}+\frac{9}{16(T+\Delta T)^{5/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{5/2}}$$

$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)T^{3/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{7/2}}+\frac{5P_M(7P_M-6)}{16(1+2P_M)(T+\Delta T)^{3/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{7/2}}$$

$$+\frac{35P_M\left(3P_M^2-2P_M+3\right)}{8(1+2P_M)T^{1/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{9/2}}+\frac{35P_M\left(3P_M^2-2P_M+3\right)}{8(1+2P_M)(T+\Delta T)^{1/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{9/2}}$$

$$+\frac{8P_M\left(3P_M^2-2P_M+3\right)(1+2P_M)^{5/2}}{3.2^{23/2}(1+P_M)^{11/2}}\sum_{n=0}^{\infty}\frac{1.3.5............(2n+9)}{n!(2n+1)2^{2n}(1+P_M)^n}\times$$

$$\left\{\frac{T^{(2n+1)/2}}{\left(T+\Delta T/2\right)^{(2n+1)/2}}+\frac{(T+\Delta T)^{(2n+1)/2}}{\left(T+\Delta T/2\right)^{(2n+1)/2}}\right\}\Bigg], \qquad \text{---------- (5.17.19)}$$

where T=t-t_0 .

For $T_m = T + \Delta T/2$, equation (5.17.19) takes the form

$$\frac{\langle h_i h_i' \rangle}{2} = \frac{N_0}{8\sqrt{2\pi}\lambda^{3/2} T_m^{3/2}} + \frac{\pi\delta_0}{4\lambda^6 (1+P_M)(1+2P_M)^{5/2}} \exp[-(2\in_{mkl}\Omega_m - fs)]$$

$$\times\left[\frac{9}{16\left(T_m - \Delta T/2\right)^{5/2}\left(T_m + \frac{\Delta T}{1+2P_M}\right)^{5/2}} + \frac{9}{16\left(T_m + \Delta T/2\right)^{5/2}\left(T_m - \frac{\Delta T}{2(1+2P_M)}\right)^{5/2}}\right.$$

$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m - \frac{\Delta T}{2}\right)^{3/2}\left(T_m + \frac{\Delta T}{2(1+2P_M)}\right)^{7/2}}$$

$$\left. + \frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m + \frac{\Delta T}{2}\right)^{3/2}\left(T_m - \frac{\Delta T}{2(1+2P_M)}\right)^{7/2}} + --- \right]. \qquad \text{-----------(5.17.20)}$$

This is the decay law of magnetic energy fluctuations of dusty fluid MHD turbulence in a rotating system before the final period for the case of multi-point and multi-time.

5.18 Concluding Remarks:

In equation (5.17.20) we obtained the decay law of magnetic energy fluctuations in MHD turbulence before the final period in a rotating system in presence of dust particle considering three-point correlation terms for the case of multi-point and multi-time. If the fluid is clean and the system is non-rotating then $f = 0, \Omega_m = 0$, the equation (5.17.20) becomes

$$\frac{\langle h_i h_i' \rangle}{2} = \frac{N_0}{8\sqrt{2\pi}\lambda^{3/2} T_m^{3/2}} + \frac{\pi\delta_0}{4\lambda^6 (1+P_M)(1+2P_M)^{5/2}}$$

$$\times\left[\frac{9}{16\left(T_m - \Delta T/2\right)^{5/2}\left(T_m + \frac{\Delta T}{1+2P_M}\right)^{5/2}} + \frac{9}{16\left(T_m + \Delta T/2\right)^{5/2}\left(T_m - \frac{\Delta T}{2(1+2P_M)}\right)^{5/2}}\right.$$

$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m-\frac{\Delta T}{2}\right)^{3/2}\left(T_m+\frac{\Delta T}{2(1+2P_M)}\right)^{7/2}}+$$

$$\left.+\frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m+\frac{\Delta T}{2}\right)^{3/2}\left(T_m-\frac{\Delta T}{2(1+2P_M)}\right)^{7/2}}+---\right] \qquad \text{--------- (5.18.1)}$$

which was obtained earlier by Sarker and Islam [115].

If we put $\Delta T=0$, we can easily find out

$$\frac{\langle h^2\rangle}{2}=\frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}}T^{-3/2}+\frac{\pi\delta_0}{2\lambda^6(1+P_M)(1+2P_M)^{5/2}}T^{-5}\times\left\{\frac{9}{16}+\frac{5}{16}\frac{P_M(7P_M-6)}{1+2P_M}+---\right\}$$

$$=AT^{-3/2}+BT^{-5}\ . \qquad \text{---------- (5.18.2)}$$

where

$$A=\frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}}\ ,$$

$$B=\frac{\pi\delta_0}{2\lambda^6(1+P_M)(1+2P_M)^{5/2}}\times\left\{\frac{9}{16}+\frac{5}{16}\frac{P_M(7P_M-6)}{1+2P_M}+---\right\}$$

which is same as obtained earlier by Sarker and Kishore [108].

This study shows that due to the effect of rotation of fluid in presence of dust particles in the magnetic field, the turbulent energy decays more rapidly than the energy for non-rotating clean fluid. From the assumption we conclude that the higher-order correlation terms may be neglected in comparison with lower-order correlation terms. By neglecting the quadruple correlation terms in three-point, three-time correlation equation the result (5.17.20) applicable to the dusty fluid MHD turbulence in a rotating system before the final period of decay were obtained. If higher-order correlation equations are considered in the analysis, it appears that more terms of higher power of time would be added to the equation (5.17.20). For large times the last term in the equation (5.17.20) becomes negligible, leaving the -3/2power decay law for the final period.

CHAPTER-VI

A REVIEW OF THE WORKS WITH CONCLUSIONS

The thesis entitled "Some Theoretical Studies on Turbulence and Magneto-hydrodynamic turbulence" has been divided into five chapters.

The first chapter gives the general idea of turbulence and Magneto-hydrodynamic turbulence and its principal concepts. A brief review of the past researches related to this thesis has also been given.

The second chapter consists of three parts :

In Part-A, the decay of temperature fluctuations in homogeneous turbulence before the final period for the case multi-point and multi-time in a rotating system are studied. In this part we have considered correlations between fluctuating quantities at two-and three-point in a rotating system. We obtained correlations equations in a rotating system and these equations are converted to spectral form by fourier transforms. Lastly, we obtained the energy decay law of temperature fluctuations in homogeneous turbulence at times before the final period for the case of multi-point and multi-time in a rotating system. Equation (2.4.21) denotes this decay law of temperature energy and it expresses that due to the rotation, the temperature energy decays faster than the energy for non-rotating fluid for times before the final period. If $\Omega_m = 0$, the equation (2.4.21) becomes the equation (2.5.1) which was obtained earlier by Sarker and Islam [116]. If we put $\Delta T_0 = 0$ in equation (2.5.1), we can easily find out the equation (2.5.2) which was obtained by Loeffler and Deissler [72].

The first term of the right side of equation (2.4.21) corresponds to the temperature energy for two-point correlation and the second term represents temperature energy for three point correlation. For large times, the second term in this equation becomes negligible leaving the –3/2 power decay law for the final period.

In Part-B, the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in presence of dust particles are studied.

In this part, the same procedure is followed as in part II-A. We obtained the energy decay law of temperature fluctuations in presence of dust particles. The equation (2.9.21) points out the fact that the energy of temperature fluctuations in presence of dust particles decays more rapidly than the energy for clean fluid for times before the final period. If the fluid is clean, then $f = 0$, the equation (2.9.21) becomes the equation (2.10.1), which was obtained earlier by Sarker and Islam [116]. If we put $\Delta T_0 = 0$ in equation (2.10.1), we can find out the equation (2.10.2), which was obtained earlier by Loeffler and Deissler [72].

In Part-C, we have studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time under the effect of rotation with an angular velocity Ω_m in presence of dust particles and we obtained the equation (2.14.21). This equation indicates that the energy of temperature fluctuations in presence of dust particles under the effect of coriolis force decays more rapidly. If the fluid is clean and the system is non-rotating then $f = 0$ and $\Omega_m = 0$, the equation (2.14.21) becomes equation (2.15.1), which was obtained earlier by Sarker and Islam [116].

If we considered the higher order correlation equations in the analysis it appears that more terms of higher power of time would be added to the equation (2.4.21), (2.9.21) and (2.14.21).

In the third chapter, we have been studied a hierarchy of distribution functions in the statistical theory for simultaneous velocity, magnetic field, concentration and temperature fluctuations in MHD turbulent flow in a rotating system in presence of dust particles. We have derived the transport equations (3.6.17) and (3.6.18) for evolution of one point distribution function $F_1^{(1)}$ and two point distribution function $F_2^{(1,2)}$ in dusty fluid MHD turbulent flow under the effect of coriolis force. We can also derive the equations for evolution of $F_3^{(1,2,3)}$,$F_4^{(1,2,3,4)}$ and so on. It is possible to have an equation for every F_n (n is an integer) but the system of equations so obtained is not closed.

But it is a great difficulty that the N-point distribution function depends upon the N+1-point distribution function and thus result is an unclosed system. This is so-called "closer problem". In this chapter, the closure difficulty is to be removed as in the case of ordinary turbulence and some properties of distribution functions have been discussed.

In Part-A of the fourth chapter, we have studied the decay of temperature fluctuations in MHD turbulence before the final period in a rotating system. Here we have considered correlations between fluctuating quantities at two and three point and fourth-order correlation terms have neglected in comparison to the second- and third-order correlation terms. Finally, we obtained the equation (4.5.19), which represents the energy decay law of temperature fluctuations in MHD turbulence before the final period in a rotating system. The first term of the right hand side of equation (4.5.19) corresponds to the temperature energy for two-point correlation and second term represents the temperature energy for three-point correlation. The result (4.5.19) shows that due to the effect of rotation in the flow field the turbulent energy decays faster than the energy for non-rotating fluid.

If the system is non-rotating then $\Omega_m = 0$, the equation (4.5.19) become the equation (4.6.1), which was obtained earlier by Sarker and Rahman [113].

In Part-B of the fourth chapter, we have studied the decay of temperature fluctuations in dusty fluid MHD turbulence before the final period in a rotating system. In equation (4.11.19) we obtained the energy decay law of temperature fluctuations of dusty fluid MHD turbulence in a rotating system before the final period. The equation (4.11.19) shows that under the effect of rotation in presence of dust particles in the flow field, the turbulent energy decays more rapidly than the energy for non-rotating clean fluid. In the absence of a magnetic field, magnetic Prandtl number coincides with the Prandtl number (i.e. $P_r = P_M$) and the system is non-rotating with clean fluid the equation (4.11.19) becomes

$$\frac{\langle T^2\rangle}{2} = \frac{N_0 P_r^{3/2}}{8\sqrt{2\pi}\nu^{3/2}(t-t_0)^{3/2}} + \frac{\beta_0 z}{\nu^6 (t-t_0)^5}$$

which was obtained earlier by Loeffler and Deissler [72].

If the equation (4.11.10) is integrated with respect to k from zero to infinity and use is made of equations (4.11.14) and (4.11.17) the resulting equation is

$$-\frac{\partial}{\partial t}\frac{\langle T^2\rangle}{2} = \nu\left(\frac{1}{P_M} + \frac{1}{P_r}\right)\int_0^\infty K^2 Q dK$$

This equation points out the interesting fact that for a given viscosity and temperature fluctuation spectrum the decay rate is inversely proportional to the Prandtl number.

Chapter V divided into three parts :

In Part-A, we have studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time in a rotating system. By neglecting the quadruple correlation terms in the three-point three-time correlation equation in a rotating system, the result (5.5.20) applicable to the MHD turbulence before the final period of decay has been obtained. If higher order correlation equations were considered in the analysis i.e. if the quadruple correlations were not neglected, it appears that more terms in higher power of $(t-t_0)$ would be added to equation (5.5.20).

If the system is non-rotating the equation (5.5.20) becomes (5.6.1), which was obtained earlier by Sarker and Islam [115]. If we put $\Delta T = 0$ in equation (5.6.1), we can easily find out equation (5.6.2) which is same as obtained earlier by Sarker and Kishore [108].

In Part-B, the same procedure is applied as in Part-A. In equation (5.11.20) we obtained the decay law of magnetic energy fluctuations in MHD turbulence before the final period in presence of dust particles considering three-point correlation terms for the case of multi-point and multi-time. The result (equation 5.11.20) shows that in presence of dust particles, the magnetic energy of MHD turbulence decays faster than the energy for clean fluid. For large times the last term in the equation (5.11.20) becomes negligible, leaving the $-3/2$power decay law for the final period.

For clean fluid, i.e. in absence of dust particles, we put $f = 0$, the equation (5.11.20) becomes (5.11.1) which was obtained earlier by Sarker and Islam [115]. Again if we put $\Delta T = 0$, in equation (5.12.1) we get the equation (5.12.2), which was obtained earlier by Sarker and Kishore [108].

In Part-C, we have studied the decay of dusty fluid MHD turbulence before the final period in a rotating system for the case of multi-point and multi-time. It is the extension work of Part-A and Part-B of this chapter. In Part-A, we have considered the rotating system and in part-B, have considered the dust particles. But in this part, we have considered both the rotating system and dust particles. Here two and three point correlation terms have been considered and the fourth order correlation terms are neglected in comparison to the second-and third-order correlation terms. In equation (5.17.20) we obtained the decay law of magnetic energy fluctuations in MHD turbulence before the final period in rotating system in presence of dust particles for the case of multi-point and multi-time. If the fluid is clean and the system is non-rotating then $f = 0$, $\Omega_m = 0$, the equation (5.17.20) becomes (5.18.1), which was obtained earlier by Sarker and Islam [115].

If we put $\Delta T=0$ in equation (5.18.1) we can easily find out equation (5.18.2), which is same as obtained earlier by Sarker and Kishore [108].

The equation (5.17.20) shows that due to the effect of rotation in presence of dust particles in the magnetic field, the turbulent energy decays more rapidly than the energy for non-rotating clean fluid.

By using the equations (5.17.15) and (5.17.18), if we integrate equation (5.17.11) with respect to k from 0 to ∞, we get

$$\frac{\partial}{\partial t}\left\langle \frac{h_i h_i'}{2} \right\rangle = -2\lambda \int_0^\infty k^2 E dk ,$$

which shows that for a given magnetic field fluctuations, the decay rate is inversely proportional to the magnetic Prandtl number.

If higher-order correlation equations were considered in the analysis i.e. if the quadruple correlations were not neglected, it appears that more terms in higher power of $(t-t_0)$ would be added to the equation (5.18.2).

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