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Modeling of Demographic Parameters of Bangladesh - An Empirical Forecasting

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University of Rajshahi

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A
Dissertation
Submitted to the University of Rajshahi
in Fulfillment of the Requirements for the
Degree of Doctor of Philosophy

By

Md. Rafiqul Islam

University of Rajshahi
October, 2003

Department of Population Science
and Human Resource Development
University of Rajshahi, Bangladesh.

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October, 2003

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Dedicated

to

My Beloved Parents

Certificate

We are pleased to certify that Mr. Md. Rafiqul Islam, Assistant Professor, Department of Population Science and Human Resource Development, Rajshahi University for submission of the Ph.D. thesis entitled "Modeling of Demographic Parameters of Bangladesh – An Empirical Forecasting" to the University of Rajshahi, Bangladesh.

We do hereby certify that the works embodied in this dissertation were carried out by the candidate and to the best of our knowledge he used the secondary data of population censuses published by Bangladesh Bureau of Statistics. His work is original and genuine. No part of this study has been submitted in substance for any higher degree or diploma.

We wish him a colourful future and every success in his life.

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Statement of Originality

This dissertation does not incorporate any part without acknowledgement of any material previously submitted for a higher degree or diploma in any University / Institute and to the best of my knowledge and belief, does not contain any material previously published or written by another person except where due reference is made in the text.

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Md. Rafiqul Islam

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Chapter 1

Introduction

1.1 Background and Importance of the Study

Bangladesh is a developing country with an accelerated population growth. The government of Bangladesh has long been trying to control its population and has introduced various programs after its independence. But, the government has already failed to control its population for the lack of correct information about the national based demographic parameters. Many researches have been done to provide the information about the demographic parameters of Bangladesh. But, only a few of them are on national based. Some of their work has been concentrated in the area of fertility, some in the mortality etc. But very few of those covered all the demographic parameters of Bangladesh. Thus, the national planners are not in a position to chalk out the development plan for the shortage of reliable national based demographic parameters. Someone should make an attempt to provide these parameters using national based data as early as possible.

Keeping these points in view an attempt has been made in this study to provide various demographic parameters of Bangladesh like mortality, fertility and reproductivity. As there is no national based vital registration data in Bangladesh, any direct estimation of mortality, fertility and reproductivity is almost impossible. So, the census data is the only source to provide the information of national based demographic parameters. But like many countries, the census data of Bangladesh is still thought to be incomplete and not perfectly accurate so that indirect techniques are very important to estimate the parameters. Thus, those parameters have been estimated applying indirect techniques using census data of 1961, 1974, 1981 and 1991 of Bangladesh.

It is also a matter of growing interest of the recent research workers and the academicians to study the shape and size and characteristics of the measures of mortality, fertility and reproductivity and even if, age distribution of population by sex in mathematical modeling. Traditionally, one can draw some graphs of the demographic parameters. But, very few of us know what types of functional form are appropriate for the estimates of fertility, mortality and also growth rate of population and age-sex distribution of population in the context of Bangladesh. To provide the above information an attempt has been made here to find out the mathematical models of demographic parameters of Bangladesh. These mathematical models have been developed using some observed data as well as estimated parameters. An attempt has also been made to forecast the demographic parameters. Some sophisticated statistical devices and techniques are used to find out the mathematical justification and model validation in the modeling of the demographic parameters of Bangladesh.

It is hoped that this work will be useful to the researchers and academicians as well as the government planners to provide sound basis for comprehending any future plan of action for the socio-economic development and health care programs in the country.

1.2 Review of Literature

Many research works have been done on fertility using survey information. But, census based research of fertility and indirect estimates of fertility measures have been done on a limited scale. Only a few studies have analyzed adult mortality and estimation of demographic parameters of Bangladesh. A brief account of the earlier nationally and internationally relevant literature in the context of the present study is given below:

Bangladesh Retrospective Survey of Fertility and Mortality (BRSFM, 1974) have estimated CBR=20 and IMR= 53 per thousand. The expectation of life at birth was found to be 45.8 years and 46.6 years for male and female respectively.

Bangladesh Fertility Survey 1975 (1978) as a part of World Fertility Survey (WFS) was conducted during 1975-1976 which yields an estimated TFR of 6.34, an average for the period 1971-1975

Hill, K. (1977) developed indirect techniques for mortality estimation in societies lacking adequate vital registration records. Information on orphanhood data has been widely used as an estimator of adult mortality with generally plausible results. Though droughts have remained, however, about potential biases, and the method is less satisfactory for the estimation of male mortality. Information on widowhood, or more strictly the survival of first spouse, has several possible advantages over information on orphanhood. Model first marriage functions and model life tables are used to calculate proportions widowed of first spouse. for both male and female by marital duration and by age. These proportions widowed are then related to life table survivorship probabilities to provide weighting factors for the conversion of observed proportions widowed into estimates of survivorship probabilities. The application of the method is illustrated with data collected by the 1974 post-enumeration survey of Bangladesh, with apparently encouraging results.

Islam, M. N. (1977) reviewed an analysis of average fertility and mortality experiences of Bangladesh for the 1961-1974 period from the census data. Model life tables prepared by Coale and Demeny have been used to estimate the average crude death rate for the period which in conjunction with the use of rate of intercensal growth makes it possible to give an estimate of crude birth rate. The other measures of fertility were obtained through the application of some linear models developed by Bogue and Palmore.

Sivamurthy, M. and K. S. Seetharam (1980) have written a book entitled "Handbook of Indirect Methods for Mortality Estimation" in which adult mortality for male from the 1966 census of Algeria have been estimated applying Widowhood Method on female widowhood data assuming the assumptions: (i) fertility and mortality have been constant during the recent past and (ii) death of the spouse does not affect the mortality of the surviving spouse. In this study they also reviewed female adult mortality using male widowed data from the same source mentioned above by Widowhood Method first introduced by Hill. Then, in both cases they have also smoothened the linkage of adult mortality as well as child mortality to estimate the complete picture of life table survival function (l_x) at an exact age x at the same time to construct a life table.

Islam, M. N., M. S. Rahman and S. A. Mallick (1983) examined the mortality levels in Dhaka City from the data collected in a household survey conducted in 1979. The level of child mortality as implied by child survival data appeared to be low as compared to the other available estimates in Bangladesh. The infant mortality rate stands in the neighbourhood of 50 per thousand live births. In this article, the authors attempted to estimate adult mortality for male (female) using widowed data of marital status composition of female (male) by Widowhood Method. The adult mortality estimates in conjunction with the child mortality estimates indicated an expectancy of life at birth of about 50 years for male and 52 years for female respectively.

United Nations (1983) published Manual-X entitled "Indirect Techniques for Demographic Estimation" in which adult mortality for Bolivia (1975) had been estimated using widowhood information from marital status composition by Widowhood Method.

Islam, M. N. (1984) reviewed the levels, patterns and trends of mortality in two rural communities namely Pabna and Shibpur in Bangladesh. The data have been used for this study collected by Cairo Demographic Centre as a part of its 1977 research programme on "Rural Population and Socio-Economic Changes in Some African and Asian Countries" in which Bangladesh also participated. In this study Widowhood method have been used to estimate adult mortality for male and female for Pabna and Shibpur. To calculate l_x values for male and female, child mortality has been linked with adult mortality. Then abridged life tables have been constructed from the estimated l_x values. The male and female life expectancies from parental and spousal survival data were found to be 46.9 and 51.7 years for Pabna, and 44.2 and 47.2 years for Shibpur respectively. The indirect estimate indicated a crude death rate of 16.8 for Pabna and 20.0 for Shibpur.

Kabir, M. and M. Moslch Uddin (1984) estimated birth rate for the intercensal period 1974-1981 to be in the neighbourhood of 45 per thousand. They also estimated expectancy at age 5 of 53.3 for the intercensal period 1974-1981 by the method of Preston and Coale.

Kabir, M. and Moslch Uddin (1987) found that ASFR at various periods between 1953 and 1986 of Bangladesh was increasing during 1953-1974 and the peaks of the curves were gradually increasing and then, started to decrease up to 1986, that is, the peaks of the curves were gradually being flatter. They also found that total fertility rate (TFR) of Bangladesh was also increasing up to 1974 and then, began to reduce up to 1986.

Kabir, M. and M. Moslch Uddin (1989) have written a book entitled "Techniques of Demographic Analysis" in which a chapter is entitled "Estimation of Adult Mortality from Widowhood Information". In this chapter they have shown how to estimate adult mortality from marital status composition. They applied a sophisticated and well known indirect method Widowhood to estimate male adult mortality on female marital status composition of 1974 census. In this study, l_x values have been estimated by linking child mortality (l_2) as well as adult mortality and constructed Brass two parameters abridged life table in which life expectancy at birth (e_0^0) was estimated and it was found to be 46.2 years for male in 1974.

Islam, M. N. (1989) reviewed to identify the levels and age patterns of mortality in one rural area of Bangladesh. Some of the most recently developed nontraditional methods were applied to the relevant data set for obtaining estimates of infant and child mortality as well as adult mortality. Adult mortality for male (female) using widowhood data of the marital status composition of female (male) have been computed by Widowhood method. The validity of estimates was assessed by making a comparison with a few indicators of mortality available from various sources. Although the basic data from local studies might have been deficient, the application of nontraditional methods to such information on mortality could still provide consistent and plausible estimates both at regional and national levels.

Ali, M. K. (1990) estimated adult mortality for male (female) applying Widowhood Method on marital status data for female (male) of 1981 census of Bangladesh. Before estimating adult mortality, marital status composition and age distribution data for male and female have also been adjusted by freehand curve method for avoiding the error of data. The estimated adult mortality was linked as child mortality (l_2) to estimate l_x values, the life table survival function at an exact age x. Then two abridged life tables, one for male and other for female have been constructed from the estimated l_x values. Estimated life expectancies at birth for male and female have been used to calculate life expectancy at birth for both sexes. These life expectancies were found to be reliable in comparison with the other estimates in contemporary time in Bangladesh. It is to be observed that the life expectancy at birth for males is less than that of females. Age specific death rates (ASDR) for male, female and both sexes have also been calculated from the constructed abridged life tables which was showed the tradition U-shape pattern. In this study crude death rate (CDR) has also been calculated using age structure and ASDR. Intercensal growth rates between 1981 census and 1991 census have been enumerated by the exponential growth rate method. Then he estimated crude birth rate (CBR) applying balancing equation. For estimation of age specific fertility rate (ASFR), proportion of married women from 1981 census and standard age pattern of female marital fertility had been used. Standard fertility schedule has been interpolated from Coale-Demeny Model Life Table for corresponding mean age at fertility schedule. It was found that ASFR was the highest in the age group 25 to 29 and least in the age interval 45 to 49 and exhibited the reciprocal of V-shape pattern. Lastly, total fertility rate (TFR), gross reproduction rate (GRR) and net reproduction rate (NRR) had also been estimated from the estimated ASFR.

Kabir, M. and A.K.U. Rab (1990) reviewed that fertility in Bangladesh continued to remain high despite some decline in recent years. Depending on various estimates, total fertility rate (TFR) of the country appears to be of 5 children per woman or high, crude birth rate (CBR) per thousand seems to be in the upper 30.

Ali, M. K. (1994) reviewed the relationship of total separation rates and separation rates due to death with their age variable and found a semi log function of the type $\log_e y = \alpha + \beta x$.

Islam, M. N. (1996) submitted a Ph. D. thesis at Rajshahi University in which he found that ASFRs and TFR were showing increasing trend in the interval between 1961 and 1974 and then, started to decrease up to 1991 for which ASFRs were obtained from secondary sources.

Bairagi, R. and Ashish kumar Datta (2001) found that total fertility rate (TFR) of Bangladesh was decreasing up to1966 and then, started to increase up to 1974. They also found that after 1974, TFR began to reduce up to 1998.

Mondal, N. I. (2001) submitted a Ph.D. thesis at Rajshahi University in which he observed that age specific accession rates (ASAR) follow simple linear model and age specific separation rates (ASSR) follow exponential model in the SAARC countries.

Reviewing the above literature, it is seen that there are some studies which dealt with the estimation of demographic parameters using indirect method with single census data and there are some more studies which dealt with the fitting of models for ASAR and ASSR of labor force of Bangladesh and SAARC countries. But, none of them made an attempt to estimate the demographic parameters for successive censuses. So, this study has the opportunity to estimate those parameters and to exhibit trends, patterns and fitting some mathematical models.

1.3 Country Profile

Bangladesh is located in southern Asia, borders the Bay of Bengal, between Burma and India and occupies a total of 144,000 square kilometers. It is situated latitudinally between $21^{0}5'$ and $26^{0}4'$ North and longitudinally between $88^{0}5'$ and $92^{0}5'$ East. The country has a tropical climate with cool, dry weather conditions from October to March. Summers are hot and humid. The monsoon season is characterized by heavy rainfall from June to October. It is frequently affected by natural calamities such as floods, cyclones, and droughts. Bangladesh is hilly in the Southeast consisting mostly of flat alluvial plains. About 73 percent of the land is arable. The landscape has an extensive network of rivers that are very important for the socioeconomic condition of the nation. Among them are the Ganges-Padma, Brahmaputra-Jamuna, and the Megna. Bangladesh is divided into six administrative divisions: Dhaka, Khulna, Rajshahi, Chittagong, Barisal and Sylhet. Dhaka is its capital. Electricity has only been supplied to only 32 percent of households in Bangladesh. Ninety-percent households obtain their drinking water from tubewells making them the major source of drinking water. Only 43 percent of homes in

Bangladesh have hygienic toilets while 26 percent have no facility at all. Tin is the most common roofing material and ninety-percent households live in residencies with floors made of earth (Mitra and Associates. 1997, 2001).

The population of Bangladesh has increased from 42 million in 1941 to 129 million in 2001 (Census. 2001). It is also important to note that Bangladesh is one of the most densely populated countries in the world containing 12,92,47,233 citizens of which males are 6,58,41,419 and females are 6,34,05,814 (BBS. 2001). In fact, it is the ninth most populous country in the world where about 834 persons lived per square k.m. Population growth is at a rate of 1.47 percent a year. The national language is Bangla, Muslims make up 85 percent of the population and Hindus and others are 15 percent. Bangladesh has a larger proportion of its population in the younger age groups than in older age groups. The largest proportion of people in Bangladesh are between the ages of 10-14. According to 1999 data, 43 percent of the population is below 15 years of age and 52 percent are between 15 and 64 years. Only five percent of the population are 65 and over. Urban areas have fewer people under age 15 than rural areas, 36 vs. 42 percent. The average household size in Bangladesh is 5.3 persons, with no difference between rural and urban areas. The median age of a woman at her first birth is between 17 and 18. Studies show that the age at which women in Bangladesh have their first child has increased steadily over time, along with increases in age at marriage (Mitra and Associates, 1997, 2001).

More than 90 percent of households are headed by male. Education has become more widespread in Bangladesh over time. Seventeen percent of males aged 10-14 have never attended school compared to 52 percent of males 65 years and older. Seventeen percent of females aged 10-14 have never attended school compared to 87 percent of females age 65 and older (Mitra and Associates, 1997). Despite improvements in the spread of education, levels of educational attainment still remain low in Bangladesh and there is a strong differential that persists between male and female and between urban and rural residents. One-third of men and 44 percent of women age 6 and over have never been formally educated and the median number of years of schooling for men is 1.7 years compared to less than one full year for women (Mitra and Associates, 1997, 2001).

Bangladesh has undergone a major demographic transition because of declining fertility over the last twenty years. In 1999, there were five million more children between the ages of 10 and 14 than between the ages of five and nine. The total fertility rate has declined from about 6.3 births per women in the early 1970s to 3.3 births in the mid 1990s. Fertility is approximately 60 percent higher in rural than in urban areas. Studies have found that fertility is closely related to women's education. Analysis of 1996 survey data shows that women with no formal education gave birth to an average of 3.9 children in their lifetime, compared to 2.1 children for women with at least some secondary education. The majority, 54 percent of ever-married women are between the ages of 15 and 29. Overall, 49 percent of married women in Bangladesh are using contraception, with 42 percent using a modern method. The pill accounts for 42 percent of modern contraceptive use, making it the most popular method. Female sterilization is reported for eight percent of married women. Other common forms of contraception include periodic abstinence and use of condoms.

Although infant and child mortality rates are declining, they are still high. Between 1982 and 1996, the infant mortality rate declined by 30 percent. In 1982, the infant mortality rate was 116 deaths per 1,000 live births and this fell to 82 deaths per 1,000 live births in 1996. Between 1982 and 1996, the child mortality rate declined by 42 percent from 63 deaths per 1,000 births to 37 per 1,000. The rate of decline was faster for child mortality than for infant mortality. Life expectancy at birth for male is 60.0 years and 60.5 years for female (Mitra and Associates, 1997, 2001).

To improve the economic and demographic prospects, Bangladesh remains one of the world's poorest and least developed nations despite sustained domestic and international efforts. Approximately 36 percent of the population is living below the poverty line and a similar percent of the population is unemployed (Mitra and Associates, 1996). There is a seven percent inflation rate in consumer prices. In 1999, the gross domestic product was $ 175.5 billion with a real growth rate of four percent. The per capita income was $ 360, one of the lowest in South Asia. The country's labor force consists of 56 million. Agriculture is the most important sector of the nation's economy, providing employment for 64 percent of the work force (Mitra and Associates, 2001). To highlight the demographic and socio-economic characteristics of Bangladesh the information has been briefly presented in Table 1.1.

Table 1.1 Some Selected Demographic and Socio-economic Characteristics of Bangladesh

Population	
Total	12,92,47,233**
Male	6,58,41,419**
Female	6,34,05,814**
Area	144,000 square k.m.**
Growth rate	1.47%**
Population density	834 per square k.m.**
Sex ratio	1.038**
Average household size	4.8 persons**
Population in the age group 0-14	43%*
Population in the age group 15-64	52%*
Population in the age group 65+	5%*

Fertility	
Age group	**ASFR (1999-2000) per woman***
15-19	0.144
20-24	0.188
25-29	0.165
30-34	0.099
35-39	0.044
40-44	0.018
45-49	0.003
TFR	3.31 per woman*
GRR	1.57 per woman*
NRR	1.41 per woman*
CBR	30.1 per thousand***
CPR (all methods)	53.8%***
CPR (modern methods)	43.4%***

Mortality	
IMR	68 per thousand***
CDR	8.8per thousand***
e^0_0 (m)	60 years***
e^0_0 (f)	61 years***
Mortality under age 5	94 per thousand***

Religion	
Muslim	85%*
Others	15%*

Education	
Literacy (male)	71.9%*
Literacy (female)	62.2%*
Illiteracy (male)	28.1%*
Illiteracy (female)	37.8%*

Economic conditions	
GDP	$175.5*
GNP per capita	$360*
Labor force	56 million*
Work force	64%*
Electricity supplied	32% households*
Drinkingwater supplied from tubewells	90% households*
Hygenic toilets	43% households*
Residencies with floors made of earth	90% households*
Inflation rate	7%

Source: * (Mitra and Associates, 2001)
** (BBS , 2001)
***(UN, 2002)

From the above country profile, it is evident that Bangladesh is a densely populated country with high fertility and high mortality especially in infancy. It is also observed that the literacy level of Bangladesh is very poor with low level of economic conditions. Bangladesh is also a Muslim dominated country which may lead to higher fertility in our country.

1.4 Objectives of the Study

There have been accomplished a good number of research works on fertility as well as reproductivity using survey data. But a few researches on fertility and mortality have been made in our country using census data. The mortality study in our country has been done on a very limited scale. A few researches on mortality have been reviewed on sub region basis. In this study, an attention has been made to study adult mortality using only widowhood information. Then all fertility and mortality measures have been estimated by applying various sophisticated demographic, mathematical and statistical methodologies. Nevertheless, mathematical modeling in demography in our country has rarely been used. The mathematical model building and graphical interpretation are of greater importance in modern era. Therefore, the study and analysis in this thesis has been concentrated on the mathematical modeling to the parameters. The model validation technique has been used to examine these models. In addition the methods which have been used for analysis in this study are different from those of other earlier studies. So, it is expected that this study will provide some new and excellent information about the demographic parameters and mathematical modeling on fertility and mortality.

The specific objectives of this study are thus specified as follows:

i) to estimate demographic parameters of Bangladesh using various census data and thereby to study their trends,

ii) to fit mathematical models for age structure, survival function, age specific death rates and age specific fertility rates, and

iii) to forecast infant mortality rate, crude death rate, life expectancy at birth for male and female, crude birth rate, total fertility rate, gross reproduction rate and net reproduction rate with the help of the fitted time trend models.

1.5 Organization of the Study

In order to furnish a meaningful explanation and presentation of this study, the complete work of this dissertation has been divided into seven chapters. The first chapter is introduction in which brief descriptions about background and importance of the study, review of literature, country profile and objectives of the study have been included. The second chapter entitled data and methodology comprises sources of data, quality of data, evaluation of data and their concerned tables and figures and methodology. The third chapter is estimation of adult mortality by Widowhood Method including how to estimate adult mortality for male and female using marital status data of widowed for female and male respectively. This chapter also describes to link adult mortality to child mortality to obtain complete figure of surviving function for the construction of period life tables. The fourth chapter contains construction of abridged life tables for male and female constructed from surviving function obtained in the third chapter in which a short discussion of trends and patterns of number of survival and life expectancy have also been observed. Estimation of demographic parameters have also been included in fifth chapter. In this chapter time trends and patterns of age specific death rates and crude death rate have been briefly discussed. Moreover, trends, patterns and levels of crude birth rate and age specific fertility rates are also observed here. Nevertheless, a brief discussion on the reproductivity measures, total fertility rate, gross reproduction, net reproduction rate etc. have also been included in this section and also observe their time trends. The sixth chapter contains mathematical modeling. The concept of mathematical modeling, formation of mathematical modeling of age distribution, the survival function and age specific death rates for male and female have been discussed here. Moreover, modeling of age specific fertility rates has also been described in this chapter. The time trend mathematical models for infant mortality rate, crude death rate, crude birth rate, total fertility rate, gross reproduction rate and net reproduction rate have also been fitted here. Model validation technique, cross-validity prediction power is also applied to these mathematical models to check how much these models are valid or not. This chapter also provides forecasted values of the time trend models of the demographic parameters. The seventh chapter is the last chapter of this dissertation in which conclusion is included. This chapter will accommodate findings of the study, policy implications and further study, limitations of the study.

Chapter 2

Data and Methodology

Chapter 2

Data and Methodology

The present chapter is used to give an outline of the sources of data that are used in the present study with a short discussion. Indeed, statistics on vital events, that is, birth, death, migration, marriage, widowhood, divorced and separated are collected from a well maintained vital registration system. The vital registration system in Bangladesh introduced by the British Government in 1873 was incomplete and unreliable. In the absence of a reliable or complete registration of vital events, retrospective questioning is a useful way of obtaining current estimates and historical trends in the major demographic variables (McDonal and Abdurahman, 1974). Vital registration system is inadequate in a developing country like Bangladesh. So, heavy reliance is given to attention on censuses and sample surveys that provide more accurate information on marital status composition, age-sex distribution, fertility, mortality and migration relevance. In the analysis of fertility, mortality, marital status, age-sex distribution, level of fertility and mortality with differentials various methods of analysis have been used in this study. The methods and techniques are mentioned and briefly described in the respective chapters.

2.1 Sources of Data

The present study covers a period of approximately 30 years from 1961 to 1991. In many of the developing countries like Bangladesh, vital registration system have not been started on a national basis. So, we highly depend on the information of demographic data on census and sample survey. Bangladesh has a long experience of census taking and currently quite a few national wide fertility and contraceptive survey have been conducted in the last few decades. They are mainly Bangladesh Fertility Surveys of 1975, 1989 and Bangladesh Demographic Health Survey of 1993-1994, 1996-1997, 1999-2000 and Contraceptive Prevalence Surveys of 1979, 1081, 1983, 1986, 1989, 1991. The data of the present study have been eventually chosen from two sources, namely, Bangladesh Population Census 1961 [East Pakistan], 1974, 1981, 1991 and Bangladesh Fertility Survey of 1975. While the census have provided national data on age-sex distribution, and marital status composition on a complete enumeration basis. Population census is indeed the only source of reliable and comprehensive database in Bangladesh providing its size, spatial distribution and basic characteristics of the population from national level down to village. The Fertility Survey data of 1975 have also been provided remarriage rates on sampling basis but that representing the overall population of the country since the survey have been taken the national wide survey.

The total enumerated population of Bangladesh (former East Pakistan) on the 31st January 1961 as revealed by the census was 50,8,40,235 (excluding Pakistanis) of which 2,24,91,392 were female and 2,63,48,843 were male (Nomani, 1964).

After the independence of Bangladesh, the first census of population of the country scheduled in 1971 was delayed because of the bloodshed war of liberation. It was taken in 1974. After completing the first census in the area now comprising Bangladesh, the enumerated population was 7,14,77,753 of which 3,44,07,027 were female and 3,70,70,726 were male (BBS, 1977).

The population census 1981, the second population census of independent Bangladesh, was conducted on March 6, 7 and 8, 1981. The reference period for the census enumeration was midnight of March 5. Enumeration of persons living in dwelling households and business and industrial units was preceded by the enumeration of transient persons from midnight till down of March 6. The enumerated population of Bangladesh in 1981 census was 87.1 million of which 42.2 million were female and 44.9 million were male (BBS, 1984).

The third population census 1991 of Bangladesh, the thirteenth in the series, was conducted by Bangladesh Bureau of Statistics during 12 to 15th March 1991. The mid-night between 11th and 12th March 1991 or the zero hours of 12th March were treated as the census moment. The population census 1991 has been conducted in three phases. The enumerated population of Bangladesh in 1991 census was 111.45 million of which 57.31 million were male and 54.14 million were female (BBS, 1994).

2.2 Quality of Data

For a meaningful and proper analysis, an adequate assessment of the reliability of demographic data is essential. In a developing country like Bangladesh, it is learnt that population statistics generated from enumeration, registration or sample surveys are usually affected by different types of errors which preclude the adequate measurement of demographic trends, patterns and levels. Thus, the determination of the accuracy of demographic information and the quantification of the magnitude of errors for making compensating adjustments provide a clear understanding of the dynamics of population characteristics.

A more systematic way of the assessment of the accuracy of the census data was initiated in 1961 through the post-enumeration surveys and the internal analysis of age data. The census of 1961 of Bangladesh (Former East Pakistan) marked the first attempt of the government to verify the completeness of enumeration and assess the quality of the data counted through the post-enumeration quality check. The check has been accomplished within a week after the census enumeration. There was a remarkably close correspondence in counts. The differences in urban areas are not as great as in the rural areas. Where the post enumeration survey results exceeded the census enumeration, the difference was only 0.2% for the rural areas and 1.2% for the urban areas for males and 0.8% and 1.0% respectively for females. Moreover, the census count exceeded the post-enumeration survey, the difference was 2.8% in both rural and urban areas for male, but 0.9% for rural and 2.7% for urban female (Nomani, 1964).

The post enumeration check of the population census in 1974 was completed in April 1974 after about one month of the census 1974. The purpose of this survey was to (i) estimate the coverage error due to omission or duplication of households and (ii) count the content error with respect to age, sex and marital status as recorded in the individual census slips. In 1974 census, the under enumeration was 6.5%. For the view of evaluation, the 2.6 for male and 2.4 for female, a sex ratio score of 1.5 and a joint score of 9.5 for a demographically normal population with a highly accurate system of age and sex reporting. The age misreporting was always more pronounced among the female than that of male (BBS, 1974).

To enumerate the extent of under coverage error of the population census 1981, the post-enumeration check program was accomplished two weeks after the completion of the field enumeration work by an independent group of expert persons. The post enumeration check programme was planned and carried out by independent teams of trained supervisors and enumerators who were selected from experienced staff members of the Bangladesh Bureau of Statistics. The results of the analysis of post enumeration check had been shown under coverage error around 3%. There was a recorded attainment compared to under coverage errors of post censuses which were much higher and varied from a low of 8% to a high of 15% (BBS, 1984).

The post enumeration check (PEC) of the population census 1991 was conducted during April 1-7, 1991 after three weeks of the census 1991. The PEC was designed to estimate coverage and content errors for the total population, for divisions and for the rural and the urban components separately. The PEC showed that the net under count was 4.61% of the population of Bangladesh. This net undercount can be attributed to 6.61% for missed rate and 2.00% for erroneously counted (over count) rate. Remarkable differences were observed in the census coverage rates calculated for different areas. Rural area showed 4.0% net undercount while municipal areas showed 8.6% net undercount which was more than twice of rural rate (BBS, 1994).

2.3 Evaluation of Data: Indices of Age Preference

The errors in age reporting were observed in all censuses. Whipple's Index, Myer's Index and the United Nations (UN) Age-Sex Accuracy Index have been applied to estimate the extent of inaccuracy of the incidence of age heaping and digit preferences. The values of the indices for different censuses have been presented in Table 2.1. Since single year age distributions for male and female population were not available for the census year 1961, therefore, Whipple's Index as well as Myer's Index had not been applied for the population of this census. Following criteria to account the extent of the quality of age-sex distribution on the basis of these indices and observations as regards to the amount of the quality of age data used in the analysis are presented below:

Table 2.1 Various Indices to Evaluate Age Preference of Indicated Census Years

Year	Whipple's Index		Myer's Index		UN Accuracy Index
	Male	Female	Male	Female	Both Sexes
1961	-	-	-	-	74.1
1974	337.6	354.2	70.9	77.7	68.2
1981	316.0	335.0	68.0	71.0	68.5
1991	310.79	325.72	36.14	37.0	70.41

Source: Censuses 1961, 1974, 1981 and 1991

From the indices shown above it can be observed that the single year age data seem to suffer from age misreporting. However, these errors do not seem to be of such magnitude as to making the data unusable at the level of quinquennial age groups. The present study is based on the quinquennial age distributions and for the most part, an attempt has been made to adjust the reported age data. Since these data are highly inaccurate and for this reason these data need to be adjusted. In this study age distributions of these census population, widowed and ever married population of marital status have been smoothed by latest Smoothing Method named 4253 H, twice (Velleman, 1980). The procedures of smoothing of the aforesaid data have been accomplished using the package Minitab Release 12.1. The observed and smoothed population and their percentage during 1961-1991 by age have been presented in Table 2.2 to Table 2.5(b) and depicted in Figure 2.1(a) to Figure 2.4(c). Moreover, observed proportion of population not widowed and ever married of the marital status composition at the indicated census years and their smoothed data have been presented in Table 2.6 to Table 2.10 and depicted in Figure 2.5(a) to Figure 2.9(d). Moreover, for the convenience of constructing a time trend model for infant mortality rate (IMR), crude death rate (CDR), life expectancy at birth (e_0) for male and female, crude birth rate (CBR), total fertility rate (TFR), gross reproduction rate (GRR) and net reproduction rate (NRR) for which the secondary data of successive calendar years during 1980-1998 have been taken from Statistical Year Book and presented in Table 2.11. The CDR for different years has been plotted in graph paper shown in Figure 2.10. It is seen that there are some sort of distortions exists in the data. So the data is needed to smooth and the smoothed data for CDR has also shown in Figure 2.10. After constructing the models, those are used for forecasting purpose.

There are various methods of smoothing age data, such as, Moving average method, Graphical method, Logit transformation method, Carrier-Farrag ratio method and 4253H, twice method. In this study, latest smoothing method 4253H, twice has been used to smooth age data. This method has provided us with quite meaningful results.

Table 2.2 Observed and Smoothed Population by Age and Their Percentage Distribution of Census Population of Bangladesh for Male, Female and Both Sexes in 1961

Age Group in Years	Observed Population (000)	Smoothed Population (000)	Observed Percent	Smoothed Percent
Male				
0-4	4580	4580	17.38	17.89
5-9	4869	3922	18.48	15.32
10-14	2610	2928	9.91	11.43
15-19	1922	2127	7.30	8.31
20-24	1825	1833	6.93	7.16
25-29	2002	1770	7.60	6.91
30-34	1693	1672	6.43	6.53
35-39	1559	1485	5.91	5.80
40-44	1254	1245	4.76	4.86
45-49	1016	1056	3.86	4.13
50-54	947	996	3.60	3.89
55-59	611	996	2.32	3.89
60+	1463	996	5.55	3.89
Female				
0-4	4685	4685	19.13	19.30
5-9	4661	3836	19.03	15.80
10-14	2037	2811	8.32	11.58
15-19	1984	2125	8.10	8.75
20-24	1989	1926	8.12	7.93
25-29	1998	1802	8.16	7.42
30-34	1545	1559	6.31	6.42
35-39	1254	1277	5.12	5.26
40-44	1113	1022	4.55	4.21
45-49	802	850	3.27	3.50
50-54	801	795	3.27	3.27
55-59	435	795	1.78	3.27
60+	1190	795	4.86	3.27
Both Sexes				
0-4	9264	9264	18.22	18.56
5-9	9529	7762	18.74	15.55
10-14	4647	5742	9.14	11.50
15-19	3906	4253	7.68	8.52
20-24	3813	3757	7.50	7.53
25-29	4000	3570	7.87	7.15
30-34	3238	3228	6.37	6.47
35-39	2813	2769	5.53	5.55
40-44	2367	2286	4.66	4.58
45-49	1817	1915	3.58	3.84
50-54	1748	1791	3.44	3.59
55-59	1045	1791	2.06	3.59
60+	2653	1791	5.22	3.59

Source: Population Census 1961

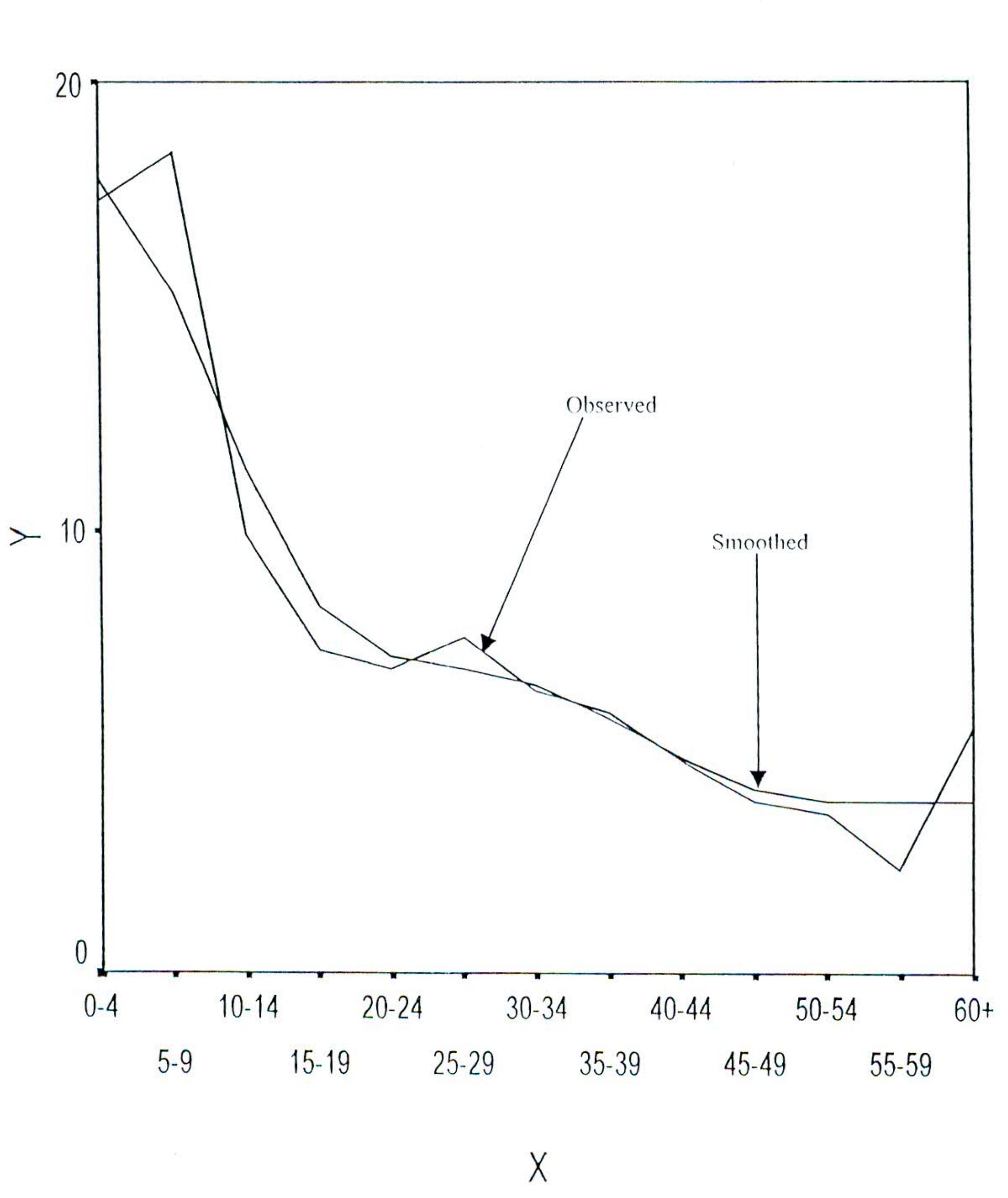


Figure 2.1(a) Observed and Smoothed Percentage Distribution of Population by Age for Male of Bangladesh in 1961. X: Age Group in Years and Y: Percent Population.

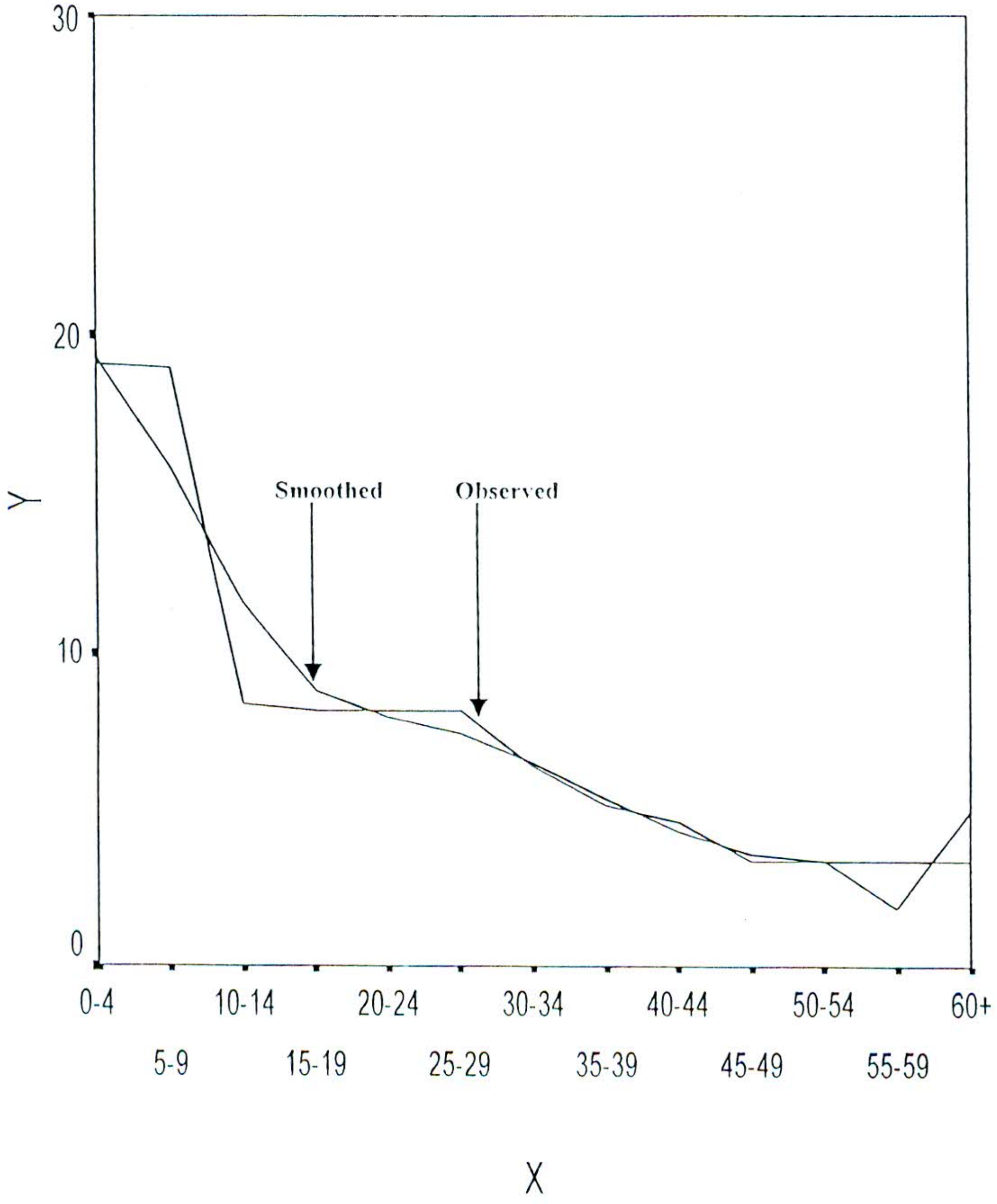


Figure 2.1(b) Observed and Smoothed Percentage Distribution of Population by Age for Female of Bangladesh in 1961. X: Age Group in Years and Y: Percent Population.

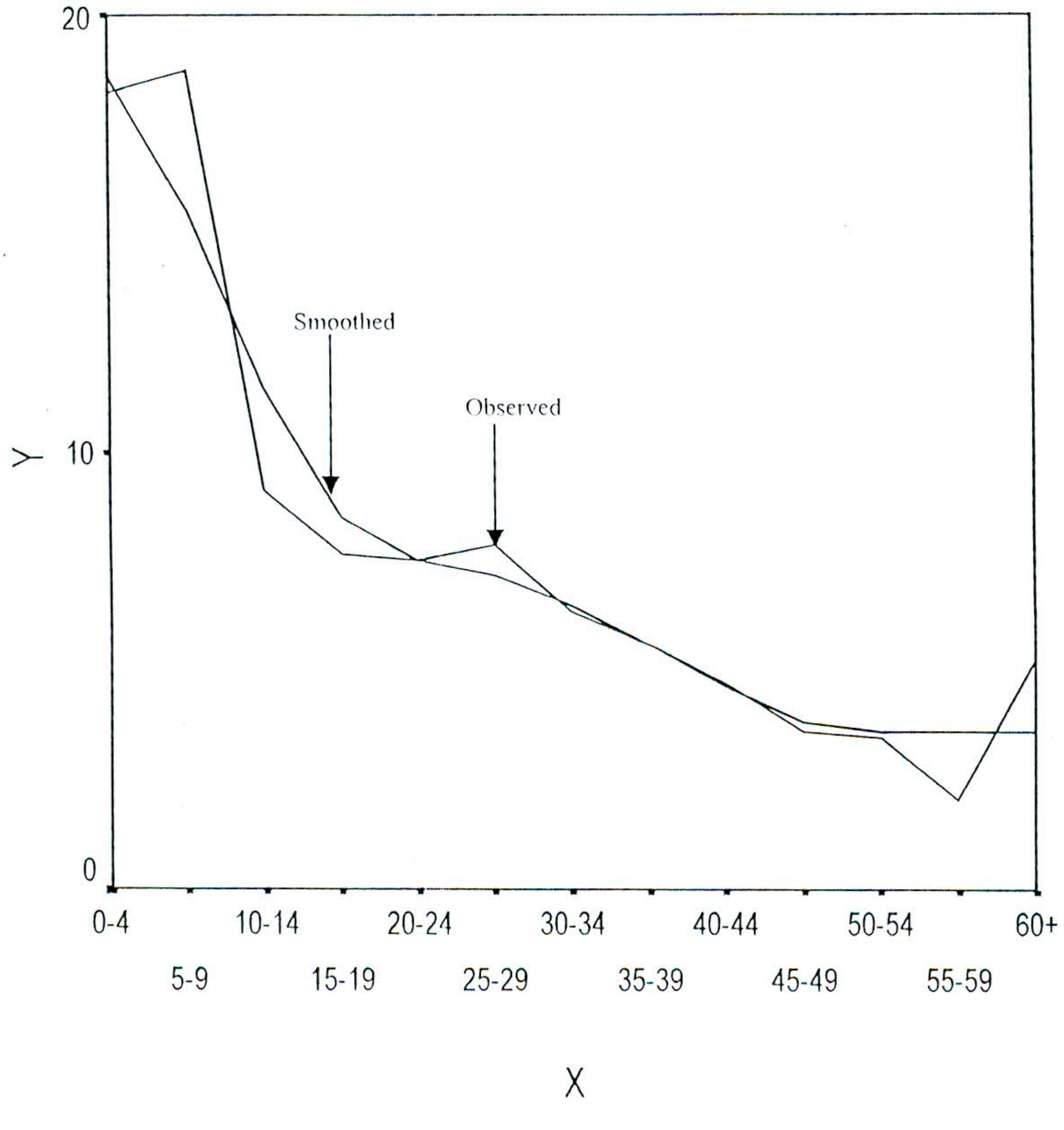


Figure 2.1(c) Observed and Smoothed Percentage Distribution of Population by Age for Both Sexes of Bangladesh in 1961. X: Age Group in Years and Y: Percent Population.

Table 2.3 Observed and Smoothed Population by Age and Their Percentage Distribution of Census Population of Bangladesh for Male, Female and Both Sexes in 1974

Age Group in Years	Observed Population (000)	Smoothed Population (000)	Observed Percent	Smoothed Percent
Male				
0-4	6015	6015	16.23	16.53
5-9	6600	5652	17.8	15.53
10-14	4987	4706	13.45	12.93
15-19	3154	3497	8.51	9.61
20-24	2416	2643	6.52	7.26
25-29	2353	2261	6.35	6.22
30-34	2036	2103	5.49	5.78
35-39	2035	1945	5.49	5.34
40-44	1745	1718	4.71	4.72
45-49	1379	1458	3.72	4.01
50-54	1284	1201	3.46	3.3
55-59	776	956	2.09	2.63
60-64	919	730	2.48	2.01
65-69	430	562	1.16	1.55
70-74	489	482	1.32	1.32
75+	455	455	1.23	1.25
Female				
0-4	6058	6058	17.61	18.00
5-9	6519	5416	18.95	16.09
10-14	4194	4259	12.19	12.65
15-19	2765	3136	8.03	9.32
20-24	2496	2546	7.25	7.56
25-29	2512	2294	7.30	6.81
30-34	2027	2061	5.89	6.12
35-39	1780	1773	5.17	5.27
40-44	1514	1476	4.40	4.39
45-49	1097	1213	3.19	3.60
50-54	1105	989	3.21	2.94
55-59	576	770	1.67	2.29
60-64	763	561	2.22	1.67
65-69	305	419	0.89	1.24
70-74	365	355	1.06	1.05
75+	331	331	0.96	0.98
Both Sexes				
0-4	12073	12073	16.89	17.19
5-9	13118	11082	18.35	15.78
10-14	9181	8999	12.84	12.82
15-19	5918	6667	8.28	9.49
20-24	4912	5205	6.87	7.41
25-29	4866	4561	6.81	6.50
30-34	4063	4178	5.68	5.95
35-39	3814	3738	5.34	5.32
40-44	3259	3213	4.56	4.58
45-49	2477	2680	3.47	3.82
50-54	2389	2193	3.34	3.12
55-59	1352	1729	1.89	2.46
60-64	1683	1294	2.35	1.84
65-69	735	982	1.03	1.40
70-74	854	837	1.19	1.19
75+	786	786	1.10	1.12

Source: Population Census 1974

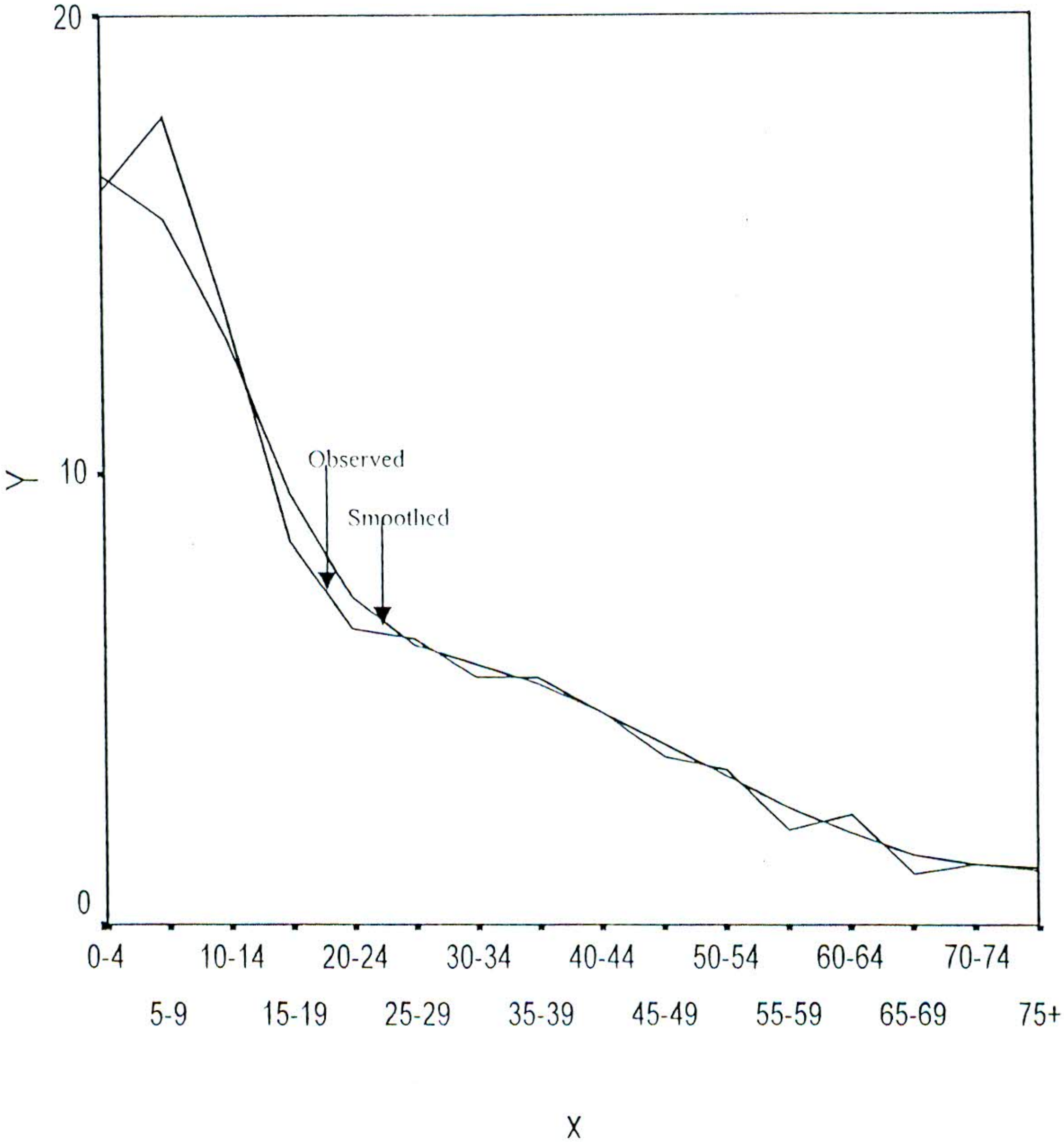


Figure 2.2(a) Observed and Smoothed Percentage Distribution of Population by Age for Male of Bangladesh in 1974. X: Age Group in Years and Y: Percent Population.

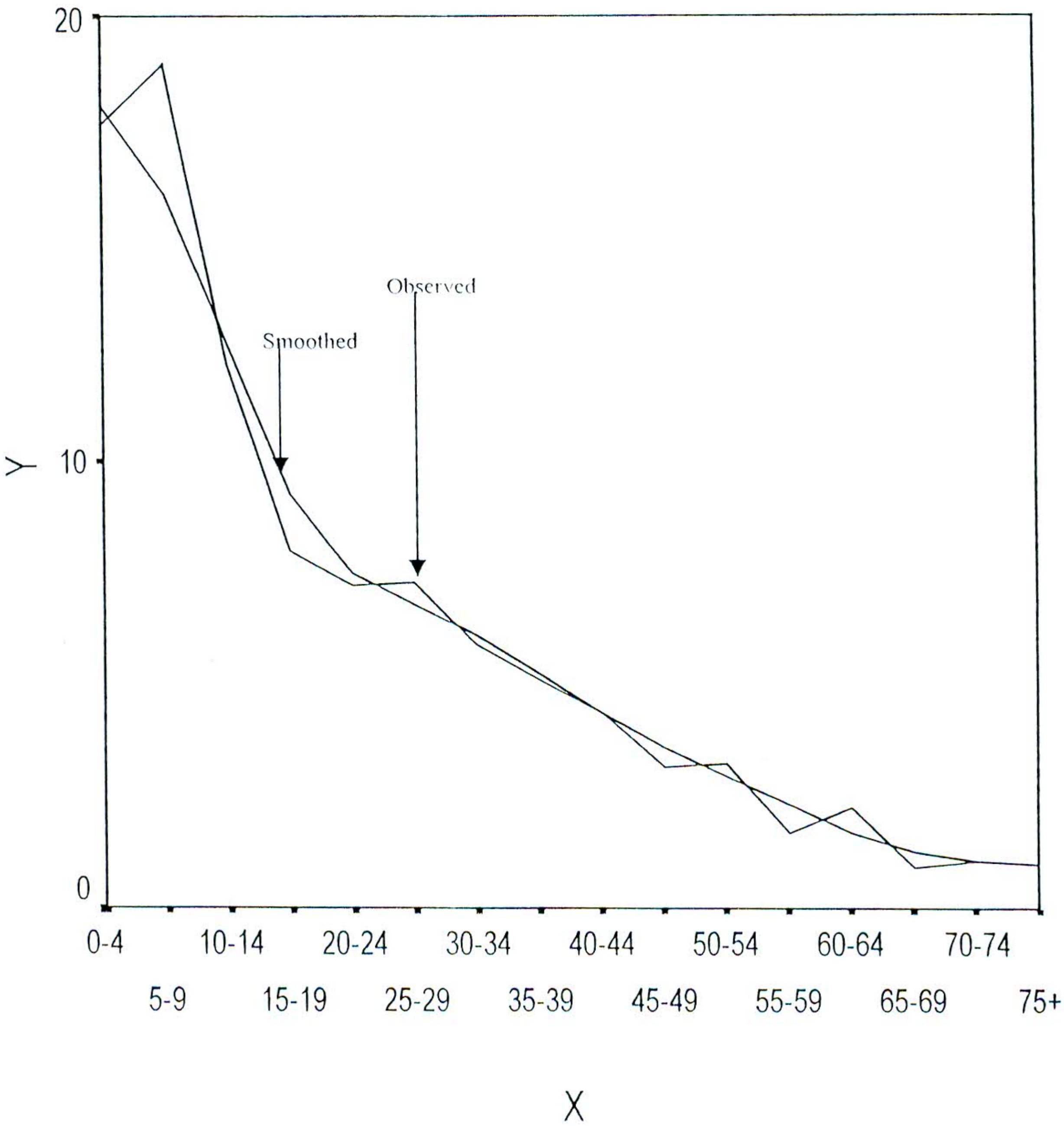


Figure 2.2(b) Observed and Smoothed Percentage Distribution of Population by Age for Female of Bangladesh in 1974. X: Age Group in Years and Y: Percent Population.

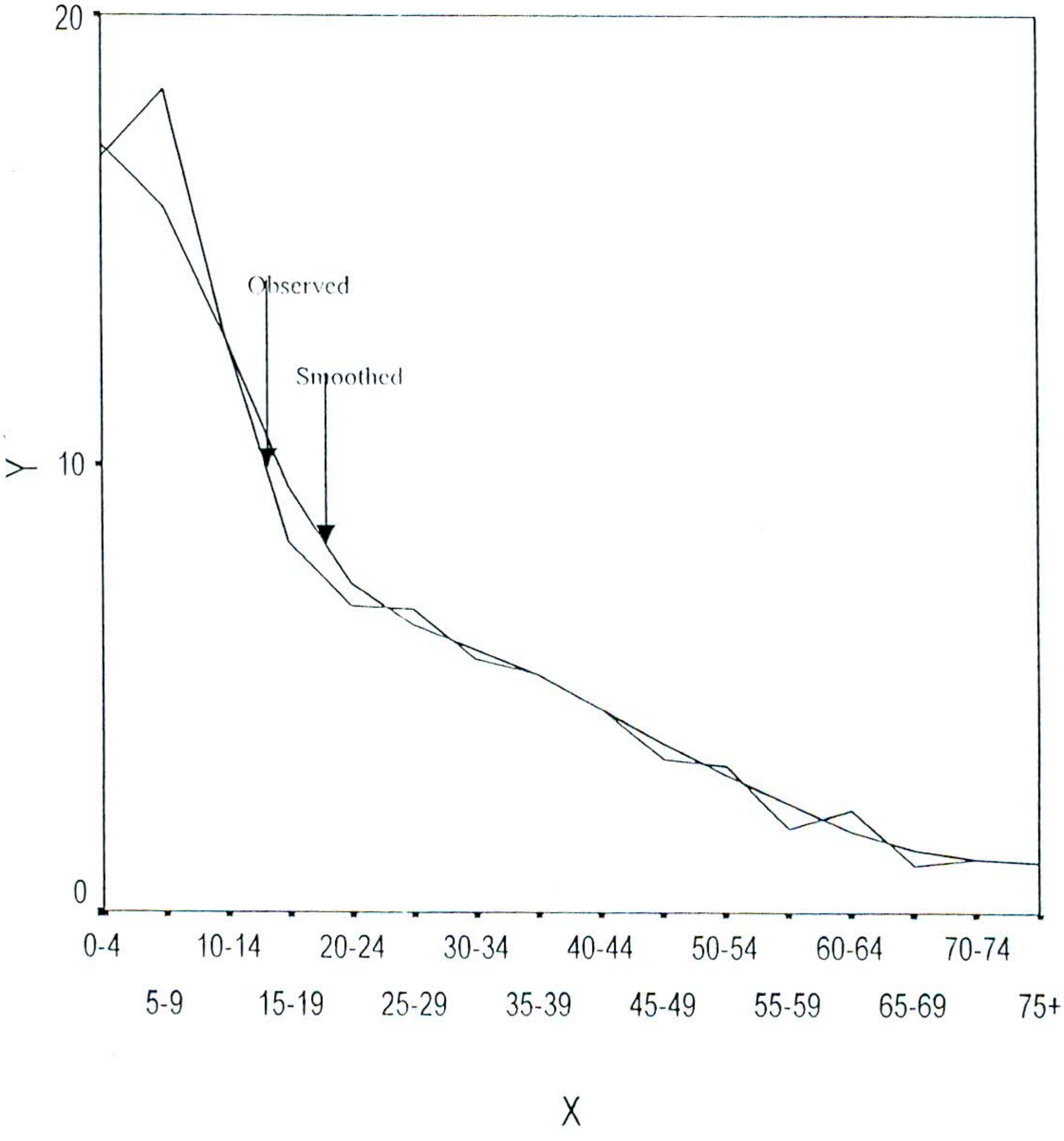


Figure 2.2(c) Observed and Smoothed Percentage Distribution of Population by Age for Both Sexes of Bangladesh in 1974. X: Age Group in Years and Y: Percent Population.

Table 2.4 Observed and Smoothed Population by Age and Their Percentage Distribution of Census Population of Bangladesh for Male, Female and Both Sexes in 1981

Age Group in Years	Observed Population (000)	Smoothed Population (000)	Observed Percent	Smoothed Percent
		Male		
0-4	7718	7718	16.67	16.7
5-9	7363	7074	15.9	15.31
10-14	6442	6006	13.92	12.99
15-19	4290	4683	9.27	10.13
20-24	3359	3673	7.26	7.95
25-29	3369	3103	7.28	6.71
30-34	2551	2697	5.51	5.84
35-39	2426	2322	5.24	5.02
40-44	1982	1971	4.28	4.26
45-49	1613	1636	3.48	3.54
50-54	1440	1393	3.11	3.01
55-59	932	1314	2.01	2.84
60-64	1074	1314	2.32	2.84
65+	1736	1314	3.75	2.84
		Female		
0-4	7615	7615	17.46	17.45
5-9	7181	6749	16.46	15.47
10-14	5595	5604	12.83	12.84
15-19	4223	4469	9.68	10.24
20-24	3635	3660	8.33	8.39
25-29	3261	3087	7.48	7.07
30-34	2520	2582	5.78	5.92
35-39	2128	2127	4.88	4.87
40-44	1811	1740	4.15	3.99
45-49	1304	1414	2.99	3.24
50-54	1321	1198	3.03	2.75
55-59	721	1131	1.65	2.59
60-64	956	1131	2.19	2.59
65+	1346	1131	3.09	2.59
		Both Sexes		
0-4	15333	15333	17.05	15333
5-9	14544	13836	16.18	13836
10-14	12037	11655	13.39	11655
15-19	8513	9219	9.47	9219
20-24	6994	7404	7.78	7404
25-29	6630	6250	7.37	6250
30-34	5071	5315	5.64	5315
35-39	4554	4474	5.06	4474
40-44	3793	3736	4.22	3736
45-49	2917	3068	3.24	3068
50-54	2761	2598	3.07	2598
55-59	1653	2446	1.84	2446
60-64	2030	2446	2.26	2446
65+	3082	2446	3.43	2446

Source: Population Census 1981

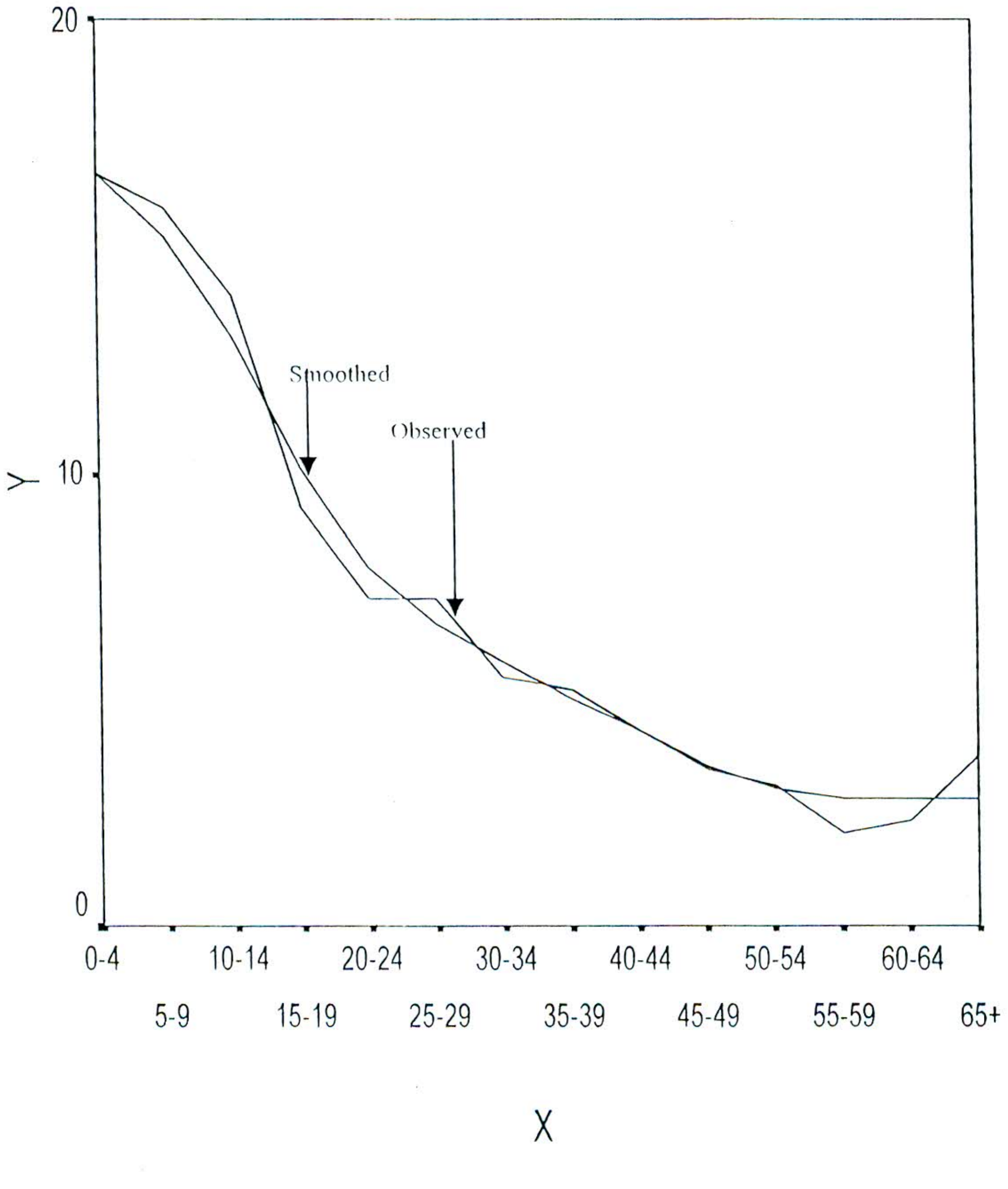


Figure 2.3(a) Observed and Smoothed Percentage Distribution of Population by Age for Male of Bangladesh in 1981. X: Age Group in Years and Y: Percent Population.

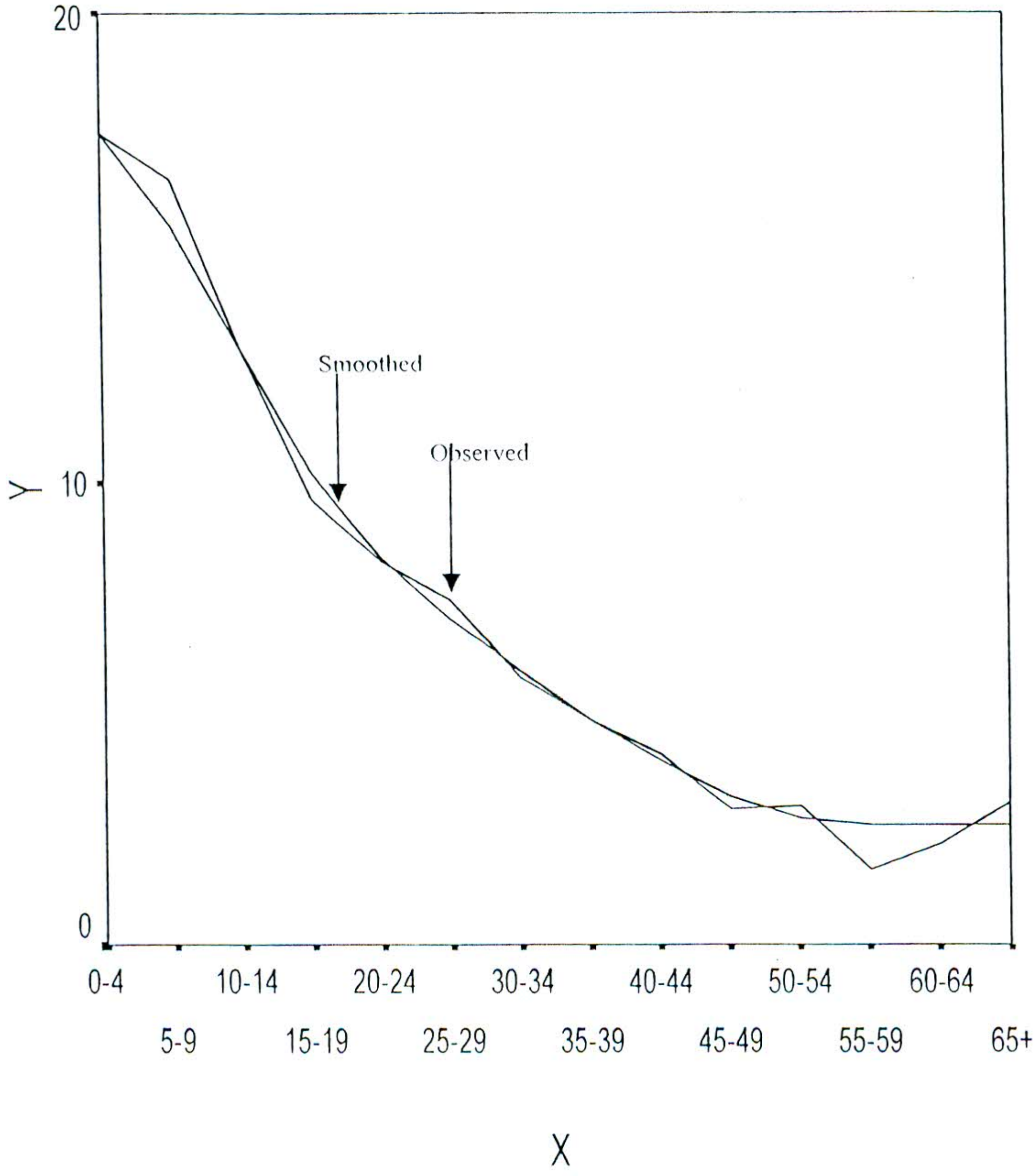


Figure 2.3(b) Observed and Smoothed Percentage Distribution of Population by Age for Female of Bangladesh in 1981. X: Age Group in Years and Y: Percent Population.

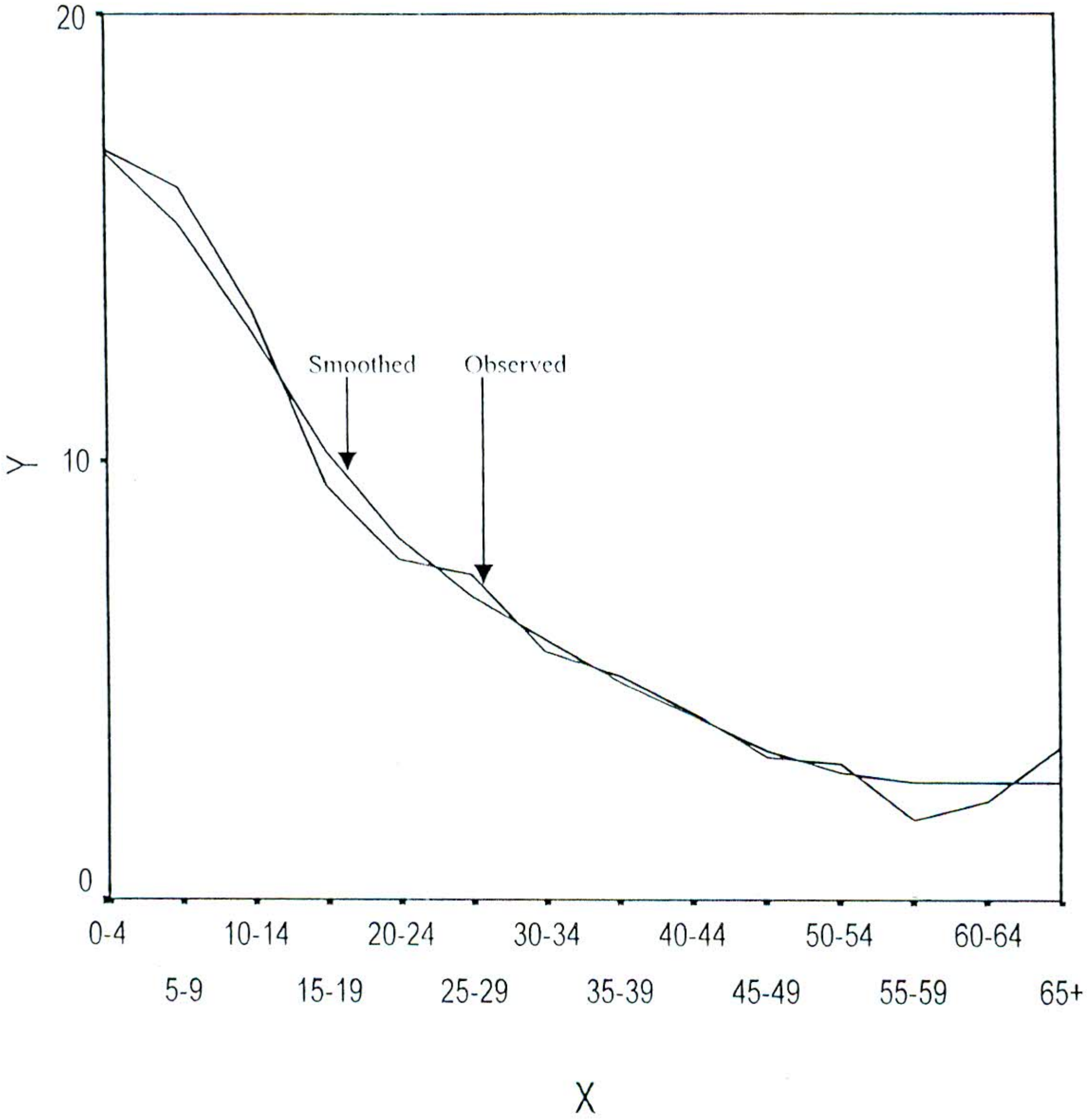


Figure 2.3(c) Observed and Smoothed Percentage Distribution of Population by Age for Both Sexes of Bangladesh in 1981. X: Age Group in Years and Y: Percent Population.

Table 2.5(a) Observed and Smoothed Population by Age and Their Percentage Distribution of Census Population of Bangladesh for Male and Female in 1991

Age Group in Years	Observed Population (000)	Smoothed Population (000)	Observed Percent	Smoothed Percent
Male				
0-4	9482	9482	16.54	16.78
5-9	9505	8602	16.58	15.22
10-14	7175	7083	12.52	12.54
15-19	4819	5496	8.41	9.73
20-24	4356	4543	7.60	8.04
25-29	4537	4048	7.92	7.16
30-34	3495	3605	6.10	6.38
35-39	3367	3098	5.87	5.48
40-44	2519	2537	4.39	4.49
45-49	1958	2011	3.42	3.56
50-54	1687	1601	2.94	2.83
55-59	1117	1279	1.95	2.26
60-64	1251	996	2.18	1.76
65-69	653	753	1.14	1.33
70-74	692	560	1.21	0.99
75-79	273	432	0.48	0.76
80+	430	379	0.75	0.67
Female				
0-4	9213	9213	17.02	17.15
5-9	8886	8081	16.41	15.04
10-14	6267	6616	11.58	12.32
15-19	4681	5403	8.65	10.06
20-24	5009	4777	9.25	8.89
25-29	4934	4266	9.11	7.94
30-34	3302	3531	6.1	6.57
35-39	2782	2770	5.14	5.16
40-44	2215	2182	4.09	4.06
45-49	1669	1745	3.08	3.25
50-54	1537	1414	2.84	2.63
55-59	898	1126	1.66	2.1
60-64	1128	845	2.08	1.57
65-69	515	613	0.95	1.14
70-74	550	452	1.02	0.84
75-79	190	356	0.35	0.66
80+	365	325	0.67	0.61

Source: Population Census 1991

Table 2.5(b) Observed and Smoothed Population by Age and Their Percentage Distribution of Census Population of Bangladesh for Both Sexes in 1991

Age Group in Years	Observed Population (000)	Smoothed Population (000)	Observed Percent	Smoothed Percent
0-4	18695	18695	16.77	16.95
5-9	18391	16660	16.5	15.10
10-14	13442	13618	12.06	12.34
15-19	9500	10818	8.52	9.81
20-24	9365	9333	8.40	8.46
25-29	9471	8393	8.50	7.61
30-34	6797	7199	6.10	6.53
35-39	6149	5904	5.52	5.35
40-44	4734	4749	4.25	4.30
45-49	3627	3786	3.25	3.43
50-54	3224	3037	2.89	2.75
55-59	2015	2414	1.81	2.19
60-64	2379	1844	2.13	1.67
65-69	1168	1370	1.05	1.24
70-74	1242	1013	1.11	0.92
75-79	463	787	0.42	0.71
80+	795	707	0.71	0.64

Source: Population Census 1991

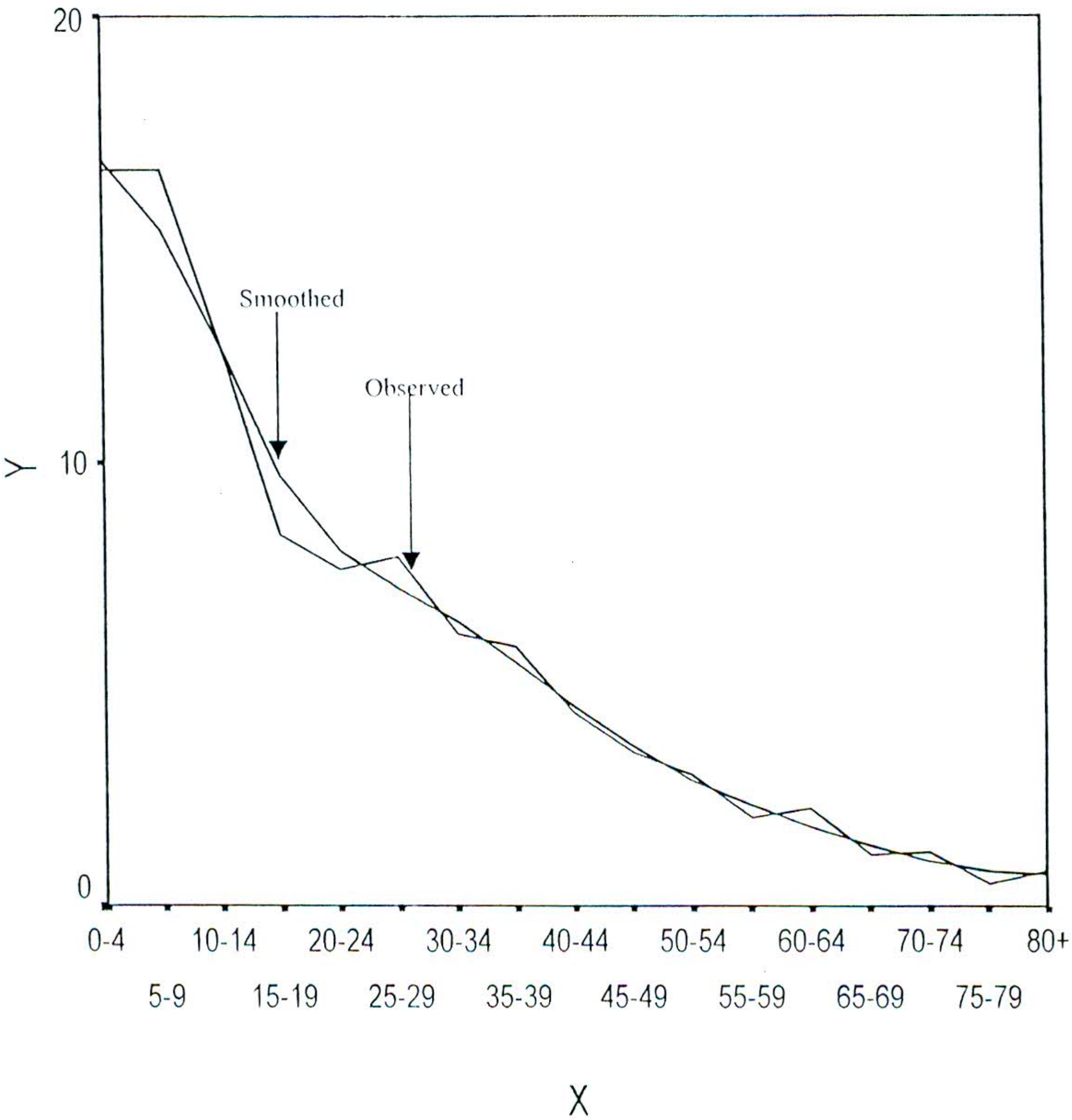


Figure 2.4(a) Observed and Smoothed Percentage Distribution of Population by Age for Male of Bangladesh in 1991. X: Age Group in Years and Y: Percent Population.

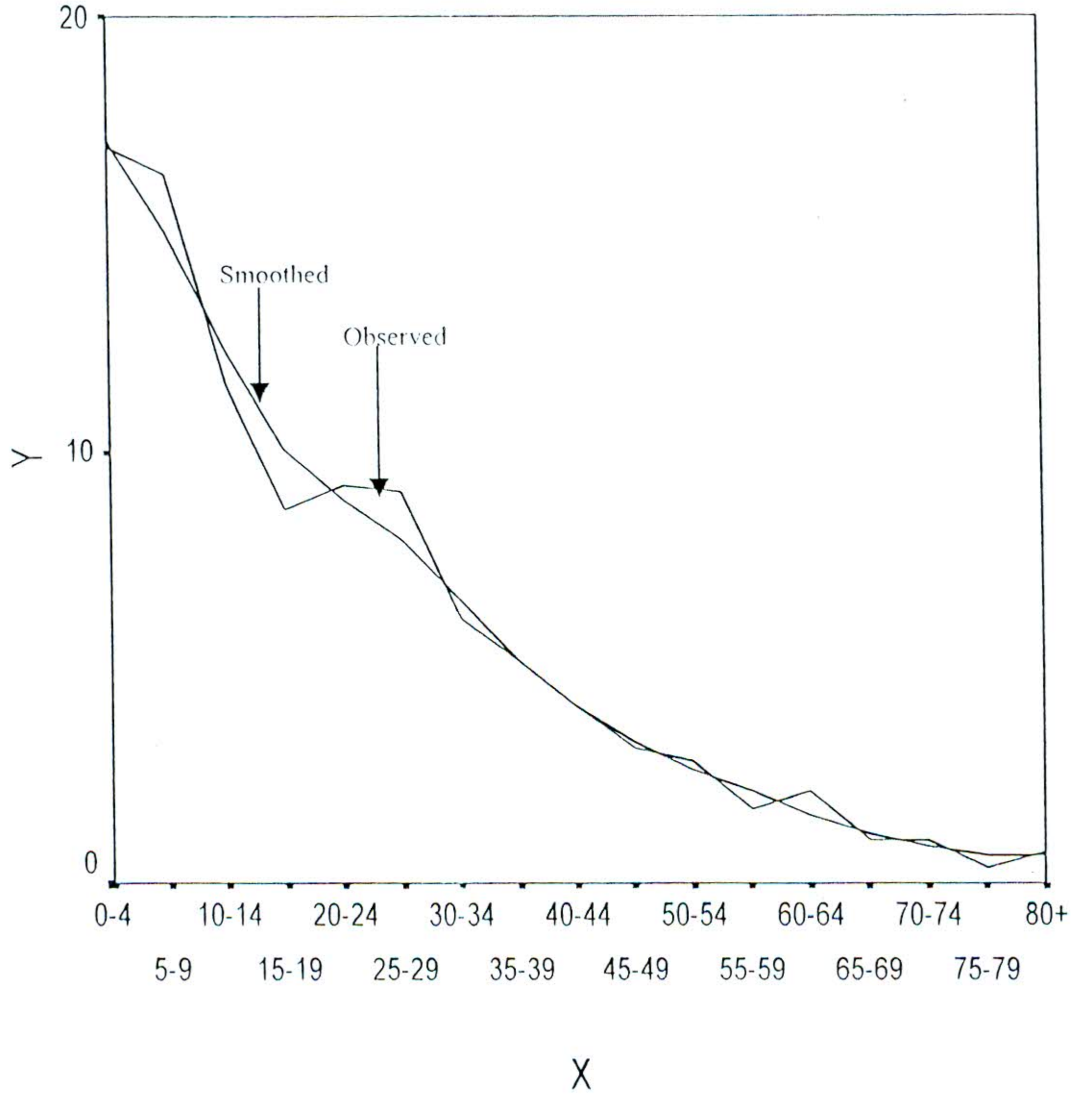


Figure 2.4(b) Observed and Smoothed Percentage Distribution of Population by Age for Female of Bangladesh in 1991. X: Age Group in Years and Y: Percent Population.

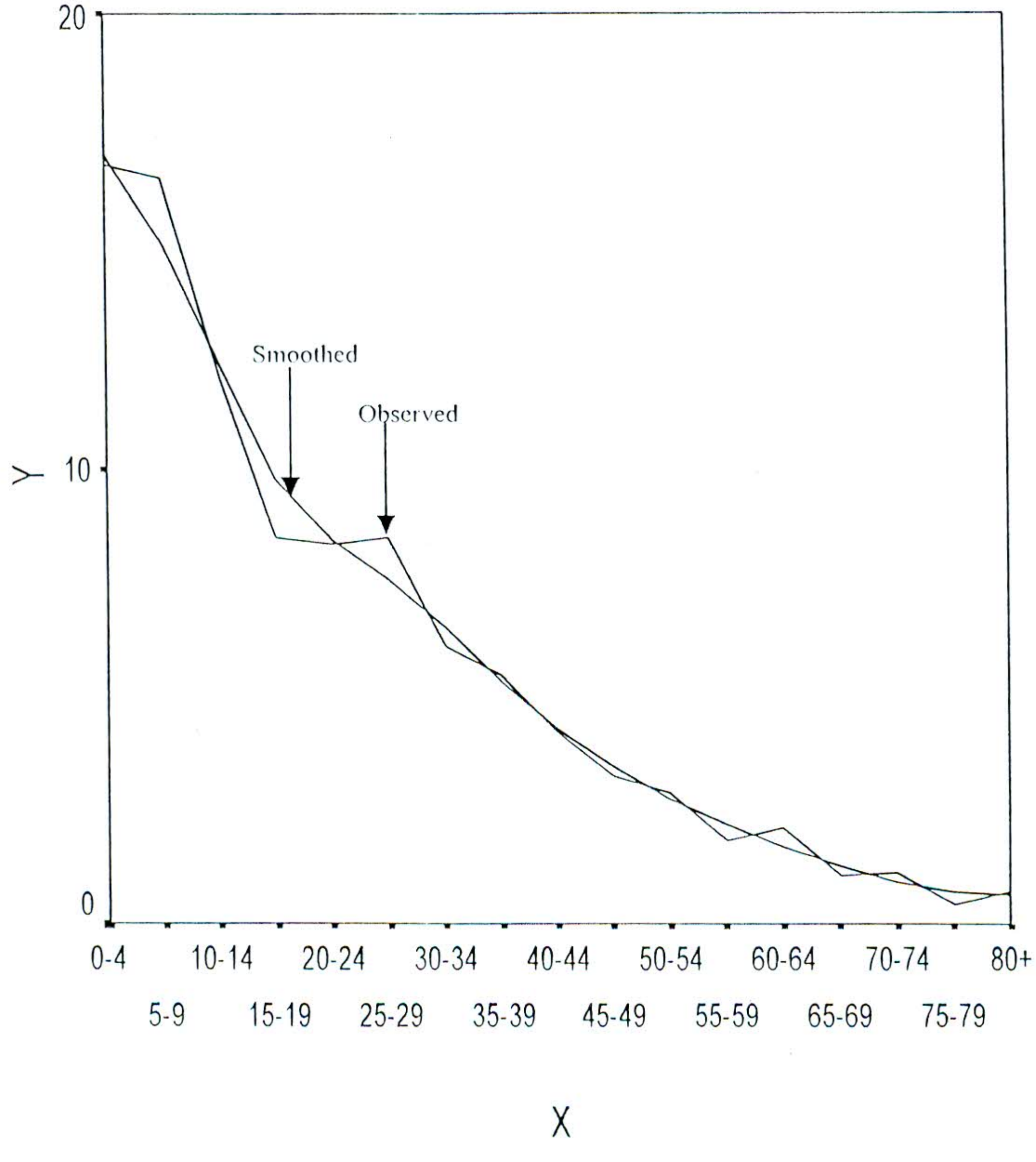


Figure 2.4(c) Observed and Smoothed Percentage Distribution of Population by Age for Both Sexes of Bangladesh in 1991. X: Age Group in Years and Y: Percent Population.

Table 2.6 Observed and Smoothed Proportion Not Widowed by Age and Sex in 1961

Age Group	Male		Female	
	Observed	Smoothed	Observed	Smoothed
10-14	0.9813	0.9819	0.9925	0.9925
15-19	0.9839	0.9814	0.9913	0.99
20-24	0.9788	0.9802	0.9792	0.981
25-29	0.9785	0.9789	0.9606	0.9591
30-34	0.9776	0.9773	0.9187	0.915
35-39	0.9758	0.9734	0.8554	0.8404
40-44	0.9644	0.966	0.7238	0.7367
45-49	0.9547	0.9553	0.6205	0.615
50-54	0.9395	0.9388	0.4592	0.4866
55-59	0.9271	0.9065	0.3811	0.3463
60+	0.8513	0.8559	0.1785	0.1852

Source: Population Census 1961

Table 2.7 Observed and Smoothed Proportion Not Widowed by Age and Sex in 1974

Age Group	Male		Female	
	Observed	Smoothed	Observed	Smoothed
10-14	0.9779	0.9779	0.9872	0.9872
15-19	0.9791	0.9805	0.9873	0.9868
20-24	0.984	0.984	0.9818	0.9831
25-29	0.9871	0.987	0.9729	0.9717
30-34	0.9884	0.988	0.9467	0.9464
35-39	0.9888	0.9875	0.9087	0.9013
40-44	0.9841	0.9849	0.8229	0.8333
45-49	0.9797	0.9797	0.7584	0.7434
50-54	0.9696	0.9719	0.6084	0.6353
55-59	0.9621	0.9581	0.5368	0.5093
60-64	0.9401	0.9321	0.3608	0.3655
65+	0.8913	0.8942	0.2115	0.2115

Source: Population Census 1974

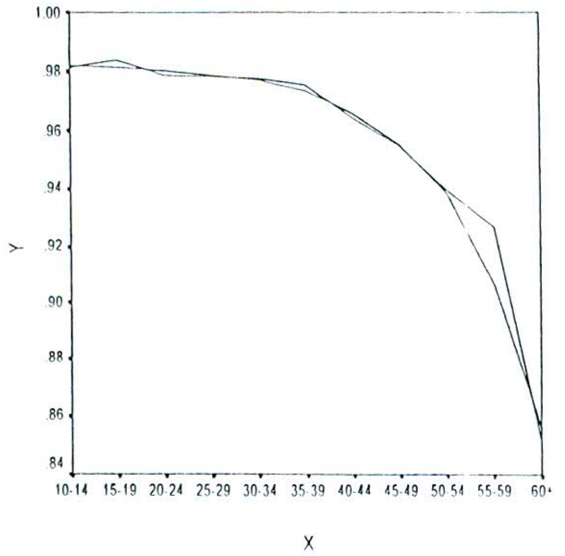


Figure 2.5(a) Observed and Smoothed Proportion Not Widowed of Male of Bangladesh by Age in 1961. X: Age Group in Years and Y: Proportion Not Widowed.

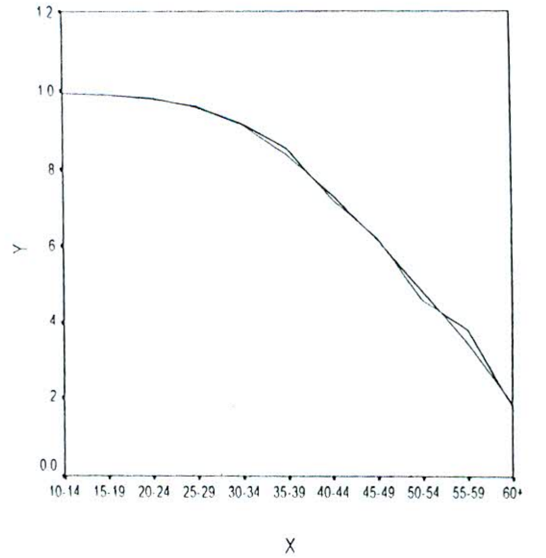


Figure 2.5(b) Observed and Smoothed Proportion Not Widowed of Female of Bangladesh by Age in 1961. X: Age Group in Years and Y: Proportion Not Widowed.

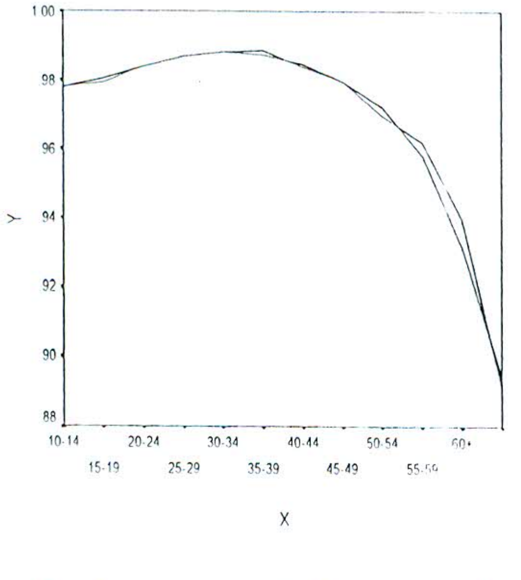


Figure 2.6(a) Observed and Smoothed Proportion Not Widowed for Male of Bangladesh by Age in 1974. X: Age Group in Years and Y: Proportion Not Widowed.

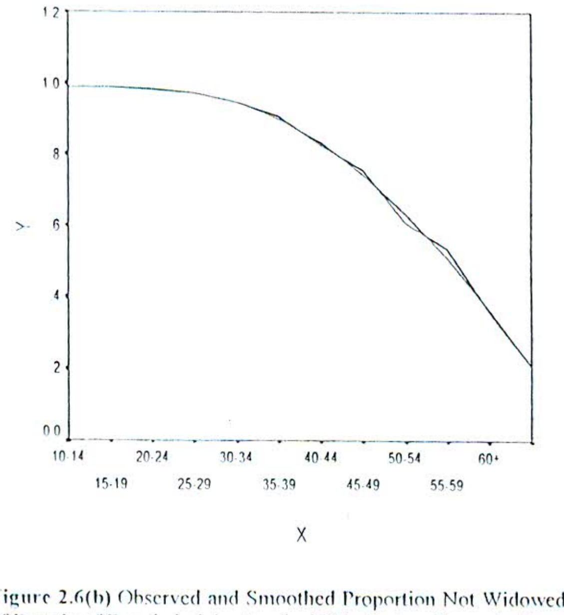


Figure 2.6(b) Observed and Smoothed Proportion Not Widowed of Female of Bangladesh by Age in 1974. X: Age Group in Years and Y: Proportion Not Widowed.

Table 2.8 Observed and Smoothed Proportion Not Widowed by Age and Sex in 1981

Age Group	Male		Female	
	Observed	Smoothed	Observed	Smoothed
10-14	1	0.995	1	1
15-19	0.9895	0.9948	0.9882	0.9912
20-24	0.9933	0.9947	0.9832	0.9836
25-29	0.995	0.9947	0.9724	0.9718
30-34	0.9948	0.9946	0.9485	0.9467
35-39	0.9939	0.9937	0.9088	0.9012
40-44	0.9907	0.9913	0.8296	0.8333
45-49	0.9876	0.9877	0.7509	0.747
50-54	0.9804	0.9822	0.6351	0.6475
55-59	0.9746	0.9688	0.5504	0.5356
60-64	0.95	0.94	0.4103	0.4106
65+	0.8935	0.8966	0.2776	0.2776

Source: Population Census 1981

Table 2.9 Observed and Smoothed Proportion Not Widowed by Age and Sex in 1991

Age Group	Male		Female	
	Observed	Smoothed	Observed	Smoothed
10-14	0.985	0.985	0.962	0.9687
15-19	0.9901	0.9903	0.9803	0.9739
20-24	0.9956	0.9946	0.9791	0.977
25-29	0.9971	0.9969	0.9734	0.9742
30-34	0.9973	0.9974	0.9579	0.9594
35-39	0.9972	0.9972	0.9336	0.9297
40-44	0.9957	0.9959	0.8799	0.8828
45-49	0.9938	0.9933	0.8244	0.8163
50-54	0.9886	0.9891	0.7127	0.7283
55-59	0.9849	0.9833	0.6493	0.6063
60-64	0.9751	0.9764	0.4352	0.4457
65+	0.9695	0.9695	0.4300	0.4413

Source: Population Census 1991

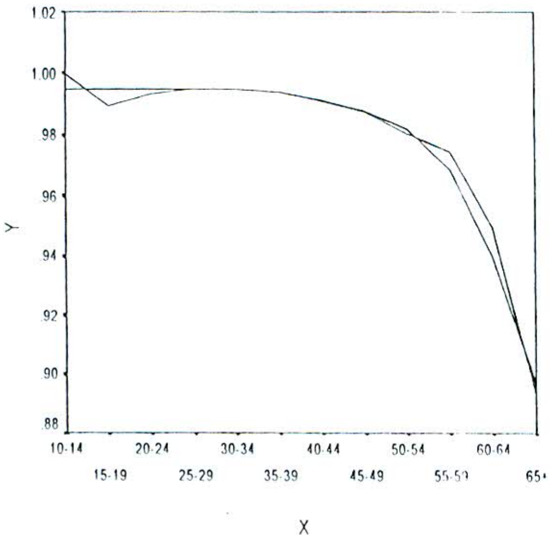


Figure 2.7(a) Observed and Smoothed Proportion Not Widowed for Male of Bangladesh by Age in 1981. X: Age Group in Years and Y: Proportion Not Widowed.

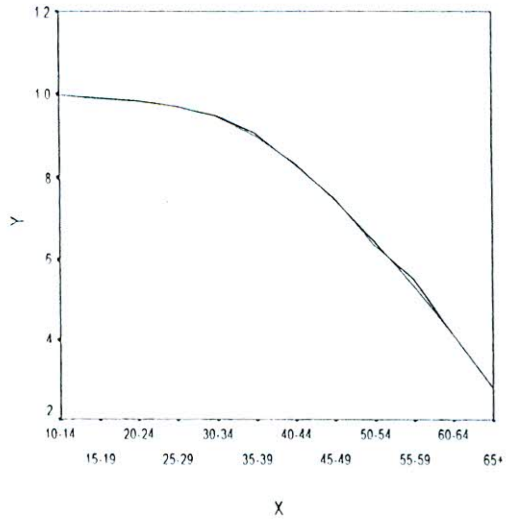


Figure 2.7(b) Observed and Smoothed Proportion Not Widowed of Female of Bangladesh by Age in 1981. X: Age Group in Years and Y: Proportion Not Widowed.

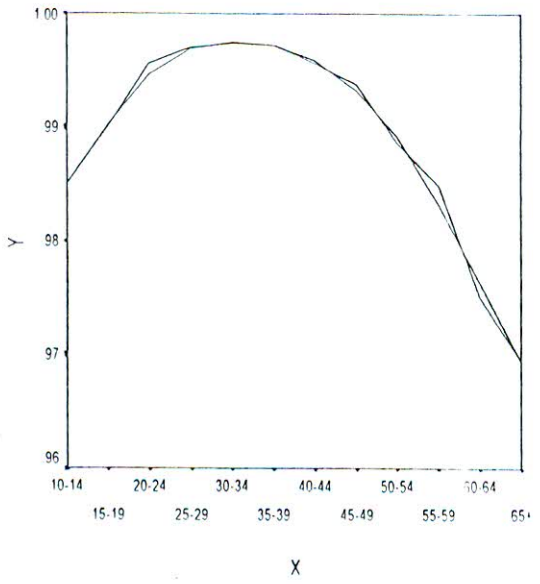


Figure 2.8(a) Observed and Smoothed Proportion Not Widowed for Male of Bangladesh by Age in 1991. X: Age Group in Years and Y: Proportion Not Widowed.

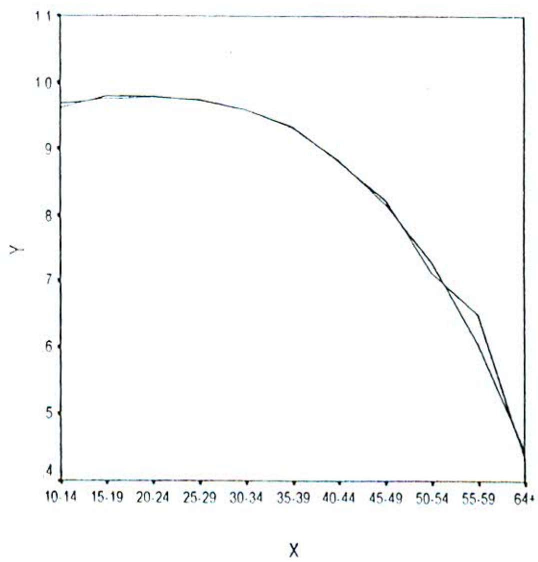


Figure 2.8(b) Observed and Smoothed Proportion Not Widowed of Female of Bangladesh by Age in 1991. X: Age Group in Years and Y: Proportion Not Widowed.

Table 2.10 Observed and Smoothed Proportion of Married Female by Age in 1961, 1974, 1981 and 1991 Censuses

Age Group in Years	61		74		81		91	
	Observed	Smoothed	Observed	Smoothed	Observed	Smoothed	Observed	Smoothed
10-14	0.3175	0.5189	0.0884	0.1633	0.0703	0.1443	0.0307	0.0721
15-19	0.8948	0.7616	0.7176	0.6231	0.6541	0.5899	0.4958	0.4830
20-24	0.9560	0.9003	0.9298	0.8886	0.9085	0.8633	0.8659	0.7834
25-29	0.9475	0.9343	0.9520	0.9521	0.9436	0.9435	0.9398	0.9214
30-34	0.9077	0.9127	0.9335	0.9370	0.9285	0.9340	0.9384	0.9442
35-39	0.8466	0.8425	0.8985	0.9030	0.8978	0.8987	0.9212	0.9243
40-44	0.7155	0.7332	0.8138	0.8353	0.8183	0.8317	0.8695	0.8807
45-49	0.6131	0.6094	0.7512	0.7399	0.7445	0.7403	0.8173	0.8132
50-54	0.4523	0.4805	0.6026	0.6302	0.6232	0.6395	0.7056	0.7244
55-59	0.3749	0.3406	0.5306	0.5042	0.5446	0.5283	0.6435	0.6021
60+	0.1739	0.1807	0.3565	0.3611	0.4036	0.4054	0.4291	0.4418

Source: Population Censuses 1961, 1974, 1981 and 1991

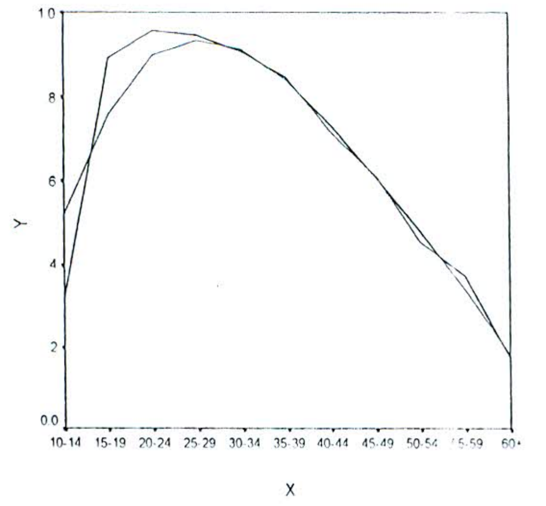


Figure 2.9(a) Observed and Smoothed Proportion of Married Female by Age in 1961. X: Age Group in Years and Y: Proportion of Married Female.

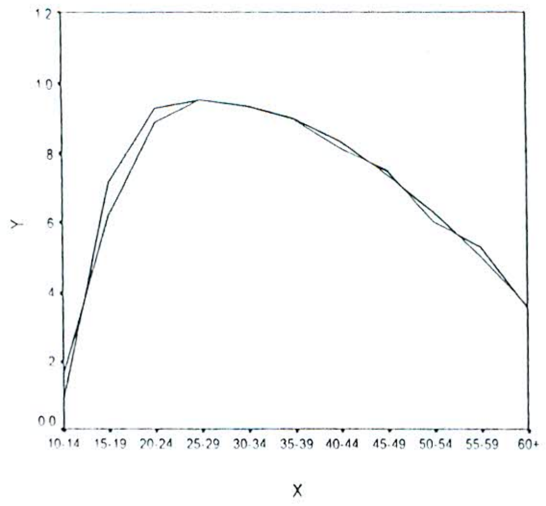


Figure 2.9(b) Observed and Smoothed Proportion of Married Female by Age in 1974. X: Age Group in Years and Y: Proportion of Married Female.

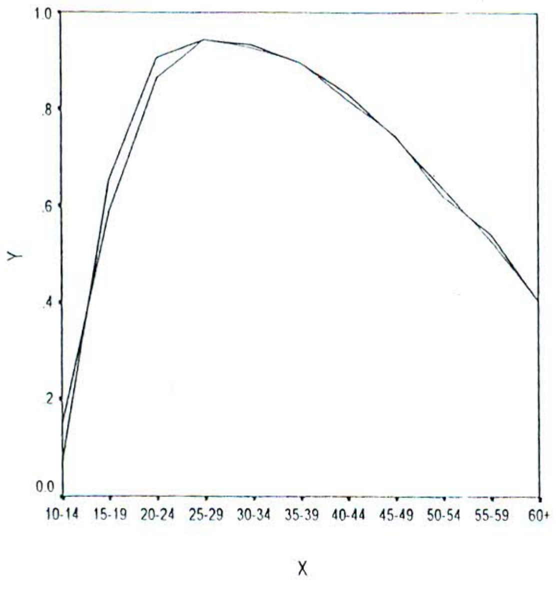


Figure 2.9(c) Observed and Smoothed Proportion of Married Female by Age in 1981. X: Age Group in Years and Y: Proportion of Married Female.

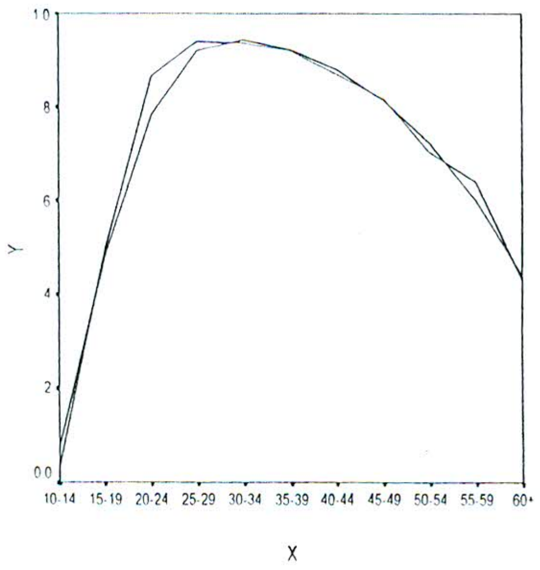


Figure 2.9(d) Observed and Smoothed Proportion of Married Female by Age in 1991. X: Age Group in Years and Y: Proportion of Married Female.

Table 2.11 Infant Mortality Rate (IMR), Crude Death Rate (CDR), Life Expectancy at Birth (e_0(m)) for Male, Life Expectancy at Birth (e_0(f)) for Female, Crude Birth Rate (CBR), Total Fertility Rate (TFR), Gross Reproduction Rate (GRR) and Net Reproduction Rate (NRR) of Bangladesh During 1980-1998

Years	IMR	CDR	Smoothed CDR	e_0(m)	e_0(f)	CBR	TFR	GRR	NRR
1980						33.4	4.99	2.46	1.83
1981				55.3	54.4	34.6	5.04	2.49	1.84
1982				54.5	54.8	34.8	5.21	2.56	1.87
1983				54.2	53.6	35	5.07	2.48	1.82
1984	121.8			54.9	54.7	34.8	4.83	2.34	1.81
1985	112			55.7	54.6	34.6	4.71	2.2	1.79
1986	116	11.9	11.84	55.2	55.3	34.4	4.7	2.29	1.8
1987	113	11.5	11.59	56.9	56	33.3	4.42	2.14	1.69
1988	116	11.3	11.43	55.9	54.4	33.2	4.39	2.21	1.74
1989	98	11.4	11.35	56	55.1	33	4.35	2.1	1.72
1990	94	11.3	11.31	56.4	55.4	32.8	4.33	2.1	1.71
1991	92	11.2	11.17	56.5	55.7	31.6	4.24	2.06	1.7
1992	88	11	10.75	56.8	55.9	30.8	4.18	2.03	1.62
1993	84	10	10.01	57.8	56.6	28.8	3.84	2.01	1.57
1994	77	9	9.18	58.2	57.9	27.8	3.58	1.81	1.48
1995	71	8.4	8.37	58.4	58.1	26.5	3.45	1.68	1.46
1996	67	8.1	7.34	59.1	58.6	25.6	3.41	1.66	1.46
1997	60	5.5	6.04	60.5	59.9	21	3.1	1.52	1.37
1998	57	4.8	4.77	60.7	60.5	19.9	2.98	1.45	1.31

Source: Statistical Year Book-1985, 1990, 1998

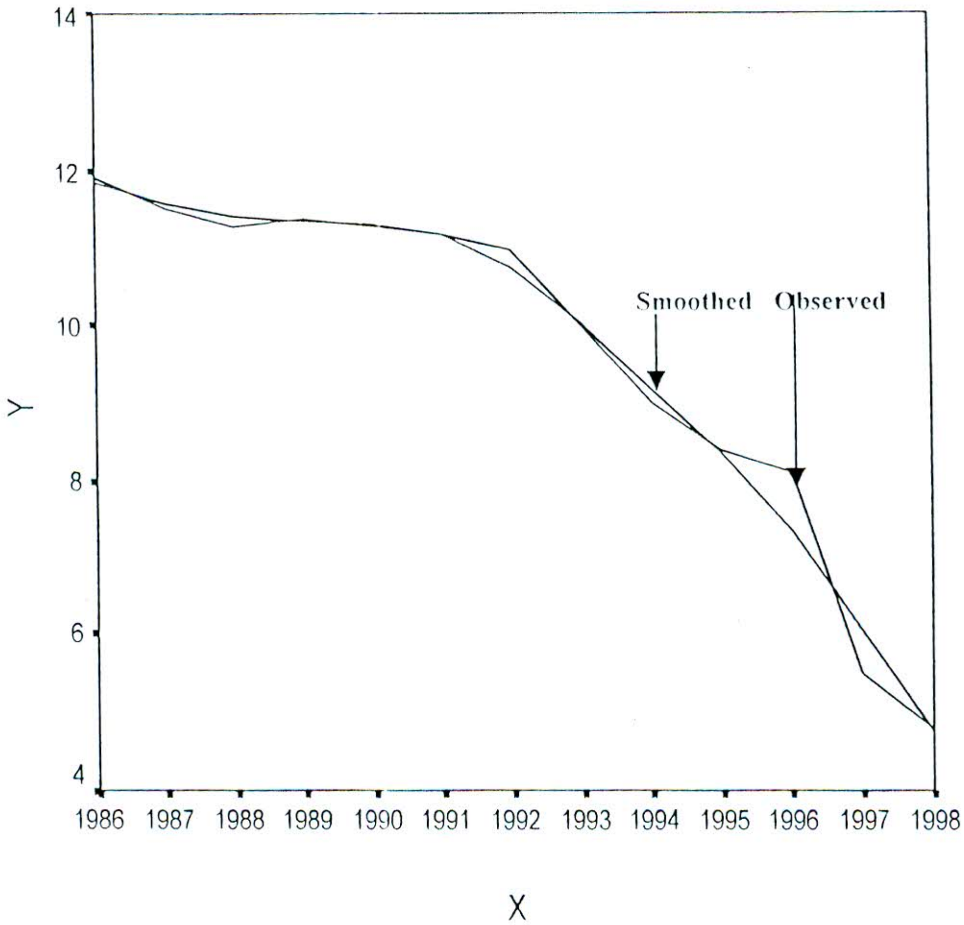


Figure 2.10 Observed and Smoothed Crude Death Rate of Bangladesh During 1986-1998. X: Calendar Years and Y: Crude Death Rate.

2.4 Methodology

The analyses have been used the following demographic, statistical and mathematical techniques and methods as per requirement of the present study.

The Widowhood method (demographic-method) has been used to estimate adult mortality which is briefly discussed in Chapter 3 at section 3.2. Combining these mortality rates with the child mortality, some abridged life tables have also been constructed in Chapter 4.

Abridged life tables have been analysed using various demographic, statistical and mathematical techniques from the estimated surviving functions. Moreover, trends, patterns, differentials and levels of different continuous functions of life tables have also been discussed.

Method of estimating fertility as well as mortality measures such as infant mortality rate, age specific death rates, crude death rate, crude birth rate, general fertility rate, child woman ratio, age specific fertility rates, total fertility rate, gross reproduction rate, replacement index and net reproduction rate have been discussed in the relevant chapter with the help of life tables. Trends and patterns of demographic parameters have also been analysed in Chapter 5.

In brief, method of constructing a mathematical model has been explained in Chapter 6. Model validation technique, cross validity prediction power (CVPP), statistical and mathematical justifications of the modeling have also been discussed in the same chapter. Techniques and methods of forecasting of the time trend models of the demographic parameters have also been discussed in Chapter 6 at section 6.7.

Statistical Package for Social Sciences (SPSS), Minitab Release 12.1, Maple, STATISTICA, C-language, Econometric Views and Microsoft Excel have been used in the data analysis to fulfill the required objectives of the present study. Moreover, Microsoft Word is also used only to type this dissertation.

Chapter 3

Estimation of Adult Mortality by Widowhood Method

Chapter 3

Estimation of Adult Mortality by Widowhood Method

3.1 Concept of Indirect Techniques

Demographic estimation has traditionally been a hazardous undertaking in most of the developing countries like Bangladesh. Vital registration system, where it exists, is not usually considered as reliable and accurate. Moreover, in our country, vital registration system has not been started yet on national basis. So, indirect techniques have been increasing demand to estimate population parameters from very few limited and defective data. The data that are collected from the national population censuses in the developing countries like Bangladesh are very defective and inaccurate. Henceforth, for the most under developed countries (like Bangladesh) of the world today, undertake special attention for mortality and fertility analyses because the basic data are highly inadequate. Over the past few decades most of the focus of demographic methodology has been turned to the development of methods and techniques for the estimation of the demographic events, viz, birth, death, marriage, widowed, divorced, separated, migration etc. from very few limited and defective data. Even if, the demographic data available in developing countries are almost less satisfactory. The processes of data collecting and analysis have been improving rapidly in the recent times. Vital statistics data have also been improved greatly, specially since the wide-spread introduction of sample surveys. But, the achievement of complete and accurate vital registration system is not easy and perhaps will not be possible in many of the developing countries as a whole at least in the near future. In such a situation the indirect methods will play a vital role to estimate demographic parameters specially fertility as well as mortality parameters in the developing countries like Bangladesh.

After fifty of past century, different types of indirect techniques have been developed to apply the use of existing inadequate and defective data to estimate mortality as well as fertility parameters. An well known indirect method of estimation of adult mortality is the Orphanhood developed by Brass and Hill (Brass and Hill, 1973). Various shortcomings of the method have been identified particularly, the multiple reckoning of parents according to the surviving children, the adoption effect for mothers, the widespread of ages at births of children for fathers, the biases due to rapid changes in mortality and fertility. In the seventy of the past century, Hill devised another sophisticated and well reputed method to estimate adult mortality from widowhood information (Hill, 1975, 1977) which is known as Widowhood Method.

3.2 Widowhood Method

One of the advantages of the methodology of estimating adult mortality by the Widowhood Method is that it uses the data on the survival of the spouse. Information needed for the application of this method can be acquired easily by including a special question; Is your first wife / husband still alive? into the census or survey questionnaire. Even if, this question has not been entered into the census or survey, data on the current marital status composition by age and sex that are usually available can be used. Female adult mortality is estimated from the information of male widowhood. Similarly, from the data on widowhood of females, male adult mortality has been estimated. But, adult mortality has been affected for the sake of extent of remarriage rates.

The estimates of adult mortality thus obtained have been linked with the estimates of infant and childhood mortality provides a complete picture of mortality in a population. The assumptions of Widowhood Method have been mentioned here: i) fertility and mortality have been approximately constant during the recent past decades and ii) death of the spouse does not affect the mortality of surviving spouse.

The present chapter is devoted to enumerate the adult mortality over a period of approximately 30 years from 1961 to 1991. Thus, the purpose of this chapter have been concentrated in the following:

i) to estimate adult mortality for male and female by Widowhood Method in the census years 1961, 1974, 1981 and 1991 and

ii) to estimate survival function l_x combining adult mortality and child mortality for male and female in the indicated time.

3.3 Estimation of Adult Mortality

The procedures of estimation of adult mortality have been briefly described in the following:

Widowhood Method has been used to estimate male and female adult mortality using the data of female and male marital status composition respectively for the census years 1961, 1974, 1981 and 1991. For this purpose, proportion not widowed $\hat{\pi}_x$ has been corrected from the proportion not widowed π_x data as

$$\hat{\pi}_x = \frac{\pi_x}{1+p},$$

where p is the proportion of remarriage.

According to (BFS, 1978) widow (female) remarriage in Bangladesh was around 3.8 percent which is negligible. So, according to Brass (1978) male adult mortality has been estimated without adjusting widow (female) remarriage. But in Bangladesh widower (male) remarriage was about 11.2 percent and hence according to Brass (1978) female adult mortality has been estimated after adjusting widower (male) remarriage. Therefore, proportion not widowed π_x has been converted to the probability of survival ratio using some weighting factors that are given by Hill (1977) corresponding to singulate mean age at marriage (SMAM) for one sex and weighted singulate mean age at marriage (WSMAM) for the other sex. Ultimately these probabilities of survival ratios have been converted to the l_x values for the adult ages corresponding to the representative model life tables in the adult ages. These estimated survivorship probabilities $l^m_{x+5} / l^m_{22.5}$ $(l^f_{x-5} / l^f_{17.5})$ for male (female) in 1961, 1974, 1981 and 1991 have been presented in Table 3.1(a) to Table 3.4(b).

The indices of infant and child mortality l_2 values for male and female have been taken from BBS data. Linking these l_2 values with the adult mortality, the l_x values for male and female have been estimated. For this purpose a linear function $Y_x = A + B\ Y^s_x$ is fitted by trial and error method according to Sivamurthy and Sitharam (1980), where Y_x is the logit of l_x and Y^s_x is the logit survival function of l^s_x of standard life table (UN, 1982). Then Y_x values have been converted to l_x values, the number of persons surviving at an exact age x and presented in Table 3.5 to Table 3.8.

It should be mentioned here that these estimated l_x values will be used to construct life tables in the next Chapter 4.

Table 3.1(a) Male Adult Mortality of Bangladesh by Widowhood Method Using 1961 Census Data of Female Marital Status

Age Group	Smoothed Proportion not Widowed Female $\pi^f(x)$	Weights W(x)	Corrected Weights W(x)	1-W(x)	$l^m_{x+5}/l^m_{22.5}$
20	0.990019	0.549282	1.045282	-0.045282	0.990429074
25	0.980963	0.382252	0.878252	0.121748	0.978299032
30	0.959082	0.416888	0.912888	0.087112	0.955243671
35	0.91502	0.447954	0.943954	0.056046	0.91083639
40	0.840374	0.473682	0.969682	0.030318	0.837229932
45	0.736671	0.485786	0.981786	0.018214	0.734454975
50	0.615005	0.47377	0.96977	0.03023	0.611122047
55	0.486558	0.45992	0.95592	0.04408	0.480376441
60	0.346323	0.395308	0.891308	0.108692	0.328812827
65	0.185224	0.328372	0.824372	0.175628	0.152693479

Note: WSMAM (Male)=22.06 and SMAM (Female)=15.22

Table 3.1(b) Female Adult Mortality of Bangladesh by Widowhood Method Using 1961 Census Data of Male Marital Status

Age Group	Smoothed Proportion not Widowed Male	Corrected Smoothed Proportion not Widowed Male $\pi^m(x)$	Weights W(x)	Corrected Weights W(x)	1-W(x)	$l^f_{x-5}/l^f_{17.5}$
20	0.98186	0.884559				
25	0.981373	0.88412	0.42691	0.32291	0.67709	0.884261757
30	0.980156	0.883023	0.171373	0.067373	0.932627	0.883096908
35	0.978942	0.881929	0.16341	0.05941	0.94059	0.881993995
40	0.977332	0.88048	0.182321	0.078321	0.921679	0.880593487
45	0.97341	0.876946	0.194447	0.090447	0.909553	0.87726564
50	0.965964	0.870238	0.201244	0.097244	0.902756	0.870890313
55	0.955296	0.860627	0.196987	0.092987	0.907013	0.861520698
60	0.938756	0.845726	0.171743	0.067743	0.932257	0.846735438
65	0.906521	0.816686	0.149768	0.045768	0.954232	0.818015103

Note: WSMAM (Female)=15.73 and SMAM (Male)=23.02

Table 3.2(a) Male Adult Mortality of Bangladesh by Widowhood Method Using 1974 Census Data of Female Marital Status

Age Group	Smoothed Proportion not Widowed Female $\pi^f(x)$	Weights W(x)	Corrected Weights W(x)	1-W(x)	$l^m_{x+5}/l^m_{22.5}$
20	0.986821	0.568408	0.886408	0.113592	0.986393099
25	0.983054	0.410688	0.728688	0.271312	0.979960501
30	0.971652	0.460664	0.778664	0.221336	0.96607057
35	0.946435	0.512428	0.830428	0.169572	0.93878442
40	0.901318	0.561468	0.879468	0.120532	0.893124596
45	0.833341	0.596476	0.914476	0.085524	0.825652906
50	0.743447	0.605404	0.923404	0.076596	0.735160002
55	0.635256	0.608704	0.926704	0.073296	0.626022756
60	0.509284	0.558632	0.876632	0.123368	0.491542695
65	0.365476	0.501024	0.819024	0.180976	0.299333615

Note: WSMAM (Male)=23.04 and SMAM (Female)=16.41

Table 3.2(b) Female Adult Mortality of Bangladesh by Widowhood Method Using 1974 Census Data of Male Marital Status

Age Group	Smoothed Proportion not Widowed Male	Corrected Smoothed Proportion not Widowed Male $\pi^m(x)$	Weights W(x)	Corrected Weights W(x)	1-W(x)	$l^f_{x-5}/l^f_{17.5}$
20	0.980535	0.8820				
25	0.984034	0.8865	0.446468	0.364468	0.635532	0.884859894
30	0.986964	0.8892	0.19864	0.116640	0.883360	0.888885072
35	0.988022	0.8904	0.19330	0.111300	0.888700	0.89026644
40	0.987495	0.8908	0.216036	0.134036	0.865964	0.890746386
45	0.984856	0.8866	0.23384	0.151840	0.848160	0.887237728
50	0.979681	0.8826	0.247236	0.165236	0.834764	0.883260944
55	0.971909	0.8735	0.249608	0.167608	0.832392	0.875025233
60	0.958061	0.8667	0.230328	0.148328	0.851672	0.86770863
65	0.932064	0.8469	0.21276	0.130760	0.869240	0.849489048

Note: WSMAM (Female)=16.08 and SMAM (Male)=24.00

Table 3.3(a) Male Adult Mortality of Bangladesh by Widowhood Method Using 1981 Census Data of Female Marital Status

Age Group	Smoothed Proportion not Widowed Female $\pi^f(x)$	Weights W(x)	Corrected Weights W(x)	1-W(x)	$l^m_{x+5}/l^m_{22.5}$
20	0.99116	0.583	0.623	0.377	0.98831365
25	0.98361	0.4344	0.4744	0.5256	0.977413176
30	0.97182	0.5006	0.5406	0.4594	0.960293654
35	0.94673	0.5731	0.6131	0.3869	0.929095098
40	0.90115	0.6447	0.6847	0.3153	0.87974113
45	0.83325	0.7015	0.7415	0.2585	0.810949205
50	0.74698	0.7303	0.7703	0.2297	0.724136335
55	0.64753	0.7504	0.7904	0.2096	0.624069472
60	0.5356	0.7154	0.7554	0.2446	0.505029892
65	0.41062	0.6696	0.7096	0.2904	0.291375952

Note: WSMAM (Male)=23.07 and SMAM (Female)=17.80

Table 3.3(b) Female Adult Mortality of Bangladesh by Widowhood Method Using 1981 Census Data of Male Marital Status

Age Group	Smoothed Proportion not Widowed Male	Corrected Smoothed Proportion not Widowed Male $\pi^m(x)$	Weights W(x)	Corrected Weights W(x)	1-W(x)	$l^f_{x-5}/l^f_{17.5}$
20	0.995013	0.8898				
25	0.994824	0.8897	0.583	0.823	0.177	0.8897823
30	0.994677	0.8897	0.4344	0.6744	0.3256	0.8897
35	0.994668	0.8897	0.5006	0.7406	0.2594	0.8897
40	0.994642	0.8889	0.5731	0.8131	0.1869	0.88955048
45	0.993749	0.8867	0.6447	0.8847	0.1153	0.88864634
50	0.991296	0.8835	0.7015	0.9415	0.0585	0.8865128
55	0.987725	0.8785	0.7303	0.9703	0.0297	0.8833515
60	0.982164	0.8666	0.7504	0.9904	0.0096	0.87838576
65	0.968845	0.8408	0.7154	0.9554	0.0446	0.86544932

Note: WSMAM (Female)=16.54 and SMAM (Male)=25.80

Table 3.4(a) Male Adult Mortality of Bangladesh by Widowhood Method Using 1991 Census Data of Female Marital Status

Age Group	Smoothed Proportion not Widowed Female $\pi^f(x)$	Weights W(x)	Corrected Weights W(x)	1-W(x)	$l^m_{x+5}/l^m_{22.5}$
20	0.9803	0.5875	0.5695	0.4305	0.9798
25	0.979005	0.4429	0.4249	0.5751	0.9758
30	0.972973	0.5161	0.4981	0.5019	0.9656
35	0.957813	0.5972	0.5792	0.4208	0.9477
40	0.929189	0.6777	0.6597	0.3403	0.9153
45	0.882765	0.743	0.725	0.275	0.8646
50	0.816303	0.7796	0.7616	0.2384	0.7978
55	0.733264	0.8066	0.7886	0.2114	0.6993
60	0.63572	0.7783	0.7603	0.2397	0.6168
65	0.524177	0.7385	0.7205	0.2795	

Note: WSMAM (Male)=24.39 and SMAM (Female)=17.90

Table 3.4(b) Female Adult Mortality of Bangladesh by Widowhood Method Using 1991 Census Data of Male Marital Status

Age Group	Smoothed Proportion not Widowed Male	Corrected Smoothed Proportion not Widowed Male $\pi^m(x)$	Weights W(x)	Corrected Weights W(x)	1-W(x)	$l^f_{x-5}/l^f_{17.5}$
20	0.984961	0.9085				
25	0.990277	0.9125	0.49713	0.19313	0.80687	0.91172748
30	0.994575	0.9146	0.267142	-0.036858	1.036858	0.914677402
35	0.996867	0.9151	0.271202	-0.032798	1.032798	0.915116399
40	0.997432	0.9148	0.305458	0.001458	0.998542	0.914800437
45	0.997163	0.9137	0.339266	0.035266	0.964734	0.913738793
50	0.995931	0.9113	0.370982	0.066982	0.933018	0.911460757
55	0.993304	0.9075	0.391368	0.087368	0.912632	0.907831998
60	0.989136	0.9021	0.388252	0.084252	0.915748	0.902554961
65	0.983276	0.8957	0.38335	0.07935	0.92065	0.89620784

Note: WSMAM (Female)=17.73 and SMAM (Male)=24.9

Table 3.5 Estimation of Male and Female Mortality (Smoothing of Child Mortality l_2 to Adult Mortality) of Bangladesh in 1961

	Male				Female			
Age x	$\frac{l^m_{x+5}}{l^m_{22.5}}$	Estimate lx	Model Life Table lx	Final lx	$\frac{l^f_{x-5}}{l^f_{17.5}}$	Estimate lx	Model Life Table lx	Final lx
0			1	1			1	1
1			0.82733	0.83418993			0.84359	0.848420108
2		0.77813	0.77281	0.778128975		0.78989	0.78796	0.789889348
3			0.74546	0.749719912			0.75925	0.759315211
4			0.72925	0.732812026			0.74205	0.740912651
5			0.71874	0.721825432			0.73083	0.72888012
10			0.69731	0.699372522			0.70802	0.704362988
15			0.6887	0.690334523			0.69858	0.694198886
20	0.9904		0.67884	0.679973929	0.884262		0.68512	0.679692722
25	0.9783	0.673294	0.66741	0.667951119	0.883097	0.613134	0.66907	0.662379303
30	0.9552	0.665068	0.65385	0.653672727	0.881994	0.612368	0.65236	0.644342202
35	0.9108	0.649364	0.6374	0.636333078	0.880593	0.611396	0.63346	0.623935544
40	0.8372	0.61918	0.61624	0.614006851	0.877266	0.609086	0.61267	0.601492548
45	0.7345	0.569145	0.58797	0.584156447	0.87089	0.604659	0.58894	0.57589677
50	0.6111	0.499328	0.55023	0.544303073	0.861521	0.598154	0.5599	0.544628706
55	0.4804	0.415438	0.499	0.490282957	0.846735	0.587888	0.51893	0.500675051
60	0.3288	0.326586	0.43488	0.422972761	0.818015	0.567948	0.46182	0.439860338
65	0.1527	0.223525	0.35395	0.338848049	0.77104		0.3853	0.359550284
70			0.2627	0.245712638			0.29489	0.26704926
75			0.17056	0.154390711			0.197	0.170859865
80			0.09463	0.082162514			0.10771	0.088082661
85+			0.04392	0.036291227			0.04701	0.035798074

Note: l_2=.77813, $l_{22.5}$=.6798, A=-0.017, B=1.05 for Male
and l_2=.78989, $l_{17.5}$=.6943, A=-0.039, B=1.068 for Female

Table 3.6 Estimation of Male and Female Mortality (Smoothing of Child Mortality l_2 to Adult Mortality) of Bangladesh in 1974

	Male				Female			
Age x	$\frac{l^m_{x+5}}{l^m_{22.5}}$	Estimate lx	Model Life Table lx	Final lx	$\frac{l^f_{x-5}}{l^f_{17.5}}$	Estimate lx	Model Life Table lx	Final lx
0			1	1			1	1
1			0.83298	0.821259337			0.85235	0.830306191
2		0.78756	0.78092	0.770667251		0.7943	0.80141	0.778147972
3			0.75483	0.74566748			0.77529	0.751980182
4			0.73936	0.730932604			0.75966	0.736477625
5			0.72931	0.721391312			0.74946	0.72641811
10			0.70873	0.701921186			0.7286	0.705975269
15			0.70042	0.694082798		0.623562	0.71993	0.697526539
20	0.9864		0.69086	0.68508038	0.8849	0.626381	0.70759	0.68554657
25	0.98	0.684266	0.67977	0.674655763	0.8889	0.627368	0.69288	0.671330837
30	0.9661	0.679826	0.66659	0.662289996	0.8903	0.62772	0.6774	0.656442003
35	0.9388	0.670184	0.65057	0.64728986	0.8908	0.625183	0.65973	0.639528677
40	0.8931	0.651246	0.62987	0.627948842	0.8872	0.622365	0.64001	0.620746913
45	0.8257	0.619543	0.60206	0.602020652	0.8832	0.616586	0.61711	0.599047492
50	0.7352	0.572788	0.56467	0.567223436	0.875	0.611513	0.58869	0.572261919
55	0.626	0.510008	0.51352	0.51964548	0.8678	0.598617	0.5481	0.534233951
60	0.4915	0.434256	0.44899	0.459472523	0.8495	0.567894	0.49084	0.480916262
65	0.2993	0.340954	0.36694	0.382319499	0.8059		0.41305	0.408780683
70			0.27379	0.293111459			0.3196	0.321939182
75			0.17896	0.199301835			0.21659	0.224834802
80			0.10008	0.11738527			0.12073	0.131418944
85+			0.04682	0.05849559			0.05377	0.062457182

Note: l_2=.78756, $l_{22.5}$=.6937, A=.0141, B=.9314 for Male and l_2=.7943, $l_{17.5}$=.7047, A=0.021, B=0.9298 for Female

Table 3.7 Estimation of Male and Female Mortality (Smoothing of Child Mortality l_2 to Adult Mortality) of Bangladesh in 1981

	Male				Female			
Age x	$l^m_{x+5} / l^m_{22.5}$	Estimate lx	Model Life Table lx	Final lx	$l^f_{x-5} / l^f_{17.5}$	Estimate lx	Model Life Table lx	Final lx
0			1	1			1	1
1			0.85497	0.861334916			0.86526	0.870058525
2		0.8175	0.8123	0.81749961		0.82571	0.82107	0.825710019
3			0.79103	0.795418022			0.79863	0.803035195
4			0.77839	0.782236474			0.78523	0.789453827
5			0.77011	0.773580361			0.77649	0.780580542
10			0.75284	0.755476914			0.75844	0.762221047
15			0.74571	0.747985244			0.75089	0.754528936
20	0.9883		0.73739	0.739231438	0.8898		0.7402	0.743625986
25	0.9774	0.73066	0.7277	0.729021425	0.8897	0.6584	0.72742	0.73057462
30	0.9602	0.72260	0.71607	0.716748252	0.8897	0.6583	0.7138	0.716647035
35	0.9290	0.70994	0.70189	0.701759224	0.8896	0.6583	0.69801	0.700479111
40	0.8797	0.68688	0.68321	0.681978369	0.8886	0.65828	0.67999	0.682003177
45	0.8109	0.65039	0.6575	0.654703142	0.8865	0.65754	0.65852	0.659960792
50	0.7241	0.59953	0.62194	0.616919483	0.8834	0.65599	0.63127	0.631947758
55	0.6240	0.53535	0.57172	0.563536793	0.8784	0.65369	0.59165	0.591167908
60	0.5050	0.46137	0.50633	0.494195606	0.8654	0.64999	0.53474	0.532548324
65	0.2913	0.37336	0.42065	0.404063291	0.8373	0.64037	0.45576	0.451269672
70			0.3206	0.300625852			0.35843	0.351555922
75			0.21536	0.195059553			0.24823	0.239838858
80			0.12432	0.107676023			0.14248	0.134641382
85+			0.06011	0.049353772			0.06547	0.060112817

Note: l_2=0.8175, $l_{22.5}$=.7393, A=-0.025, B=1.057 for Male

and l_2=0.82571, $l_{17.5}$=0.7399, A=-0.0065, B=1.029 for Female

Table 3.8 Estimation of Male and Female Mortality (Smoothing of Child Mortality l_2 to Adult Mortality) of Bangladesh in 1991

	Male				Female			
Age x	$\frac{l^m_{x+5}}{l^m_{22.5}}$	Estimate lx	Model Life Table lx	Final lx	$\frac{l^f_{x-5}}{l^f_{17.5}}$	Estimate lx	Model Life Table lx	Final lx
0			1	1			1	1
1			0.89426	0.90903			0.89041	0.89507371
2		0.87999	0.86728	0.87999		0.86469	0.85859	0.863394292
3			0.854	0.86523			0.84279	0.847563635
4			0.84605	0.85626			0.8334	0.838128101
5			0.84078	0.85026			0.82726	0.83194811
10			0.82936	0.83714			0.81438	0.818959643
15			0.8244	0.83139			0.80891	0.813434056
20	0.9798		0.81847	0.82448	0.9117		0.80128	0.805717581
25	0.9758	0.81492	0.81148	0.81629	0.9147	0.741121	0.79213	0.796450815
30	0.9656	0.81162	0.80297	0.80624	0.9151	0.74356	0.78209	0.786267254
35	0.9477	0.80314	0.79252	0.79381	0.9148	0.743885	0.7701	0.774086071
40	0.9153	0.78821	0.77822	0.77664	0.9137	0.743641	0.75576	0.759491417
45	0.8646	0.7613	0.75756	0.75157	0.9115	0.742747	0.73767	0.74104367
50	0.7978	0.71914	0.72727	0.71439	0.9078	0.740958	0.71363	0.716473084
55	0.6993	0.66353	0.68163	0.65776	0.9026	0.737951	0.67729	0.679232827
60	0.6168	0.58163	0.61824	0.57891	0.8962	0.733724	0.62312	0.623563209
65		0.51301	0.53001	0.47104	0.8896	0.728521	0.54458	0.542684139
70			0.42102	0.34468			0.44254	0.437688257
75			0.29853	0.21664			0.32036	0.312783597
80			0.27888	0.19778			0.19542	0.186989822
85+							0.09591	0.089284671

Note: l_2=0.87999, $l_{22.5}$=.8317, A=-0.1301, B=1.201 for Male and l_2=0.86469, $l_{17.5}$=0.8129, A=-0.025, B=1.001 for Female

Chapter 4

Construction of Abridged Life Tables

Chapter 4

Construction of Abridged Life Tables

4.1 Concept of Life Tables

There are various definitions of life tables. According to Nathan Keyfitz life table is a scheme for expressing the form of mortality in terms of probability. It is clearly designed essentially to measure mortality, but it is used by a variety of specialists in different branches. It is used by actuarial sciences, demographers, economists, sociologists, public health workers, geographers and many others in studies of longevity, fertility, migration, population renewal and population growth as well as in making estimates and projections of population size, distribution and characteristics. It is also used in the studies of widowhood, orphanhood, length of married life, length of working life and length of disability free life. Life table is a basic model of population dynamics and is an important tool in the hands of demographers.

For the lack of vital statistics, the life tables have been constructed using indirect method. These life tables have been used here for calculating age specific death rate (ASDR), crude death rate (CDR) and infant mortality rate (IMR) for the census years 1961, 1974, 1981 and 1991.These life tables have also been used to estimate replacement index and net reproduction rate (NRR). As the demographic parameters are inter-related, so, the above life tables indirectly help us to estimate other parameters of fertility and reproductivity such as crude birth rate (CBR), age specific fertility rates (ASFR), total fertility rate (TFR) and gross reproduction rate (GRR).

To fulfil the purpose of this study an attempt has been made to the following points:

i) Abridged life tables have been constructed for male and female in the census years 1961, 1974, 1981 and 1991,

ii) The trends and patterns of the different continuous functions of the newly constructed abridged life tables have been studied,

iii) The life expectancy at birth for both sexes have been estimated, and

iv) The trends of life expectancy at birth for male, female and both sexes have been studied.

In this chapter abridged life tables for male and female in the census years 1961, 1974, 1981, and 1991 have been constructed using the estimated continuous functions l_x from the fifth and last columns of Table 3.5 to Table 3.8 in the Chapter 3. These life tables have been presented in Table 4.1 to Table 4.8. It is to be noted that life table described in this chapter which are all called single decrement life tables as well as period life tables too.

Moreover, the expectation of life at birth for both sexes at the indicated census years has been estimated using the formula

$e_0^0 = \frac{e_0^{f\prime} + s\,e_0^m}{1+s}$; where, e_0^m is the expectation of life at birth for male, e_0^f is the expectation of life at birth for female and s is the sex ratio at birth at the same year.

The estimated life expectancy at birth for male, female and both sexes in the censuses 1961, 1974, 1981 and 1991 have been estimated and presented in Table 4.9 and depicted in Figure 4.5.

4.2 Trends and Patterns of Survival Functions (l_x) at Exact Age x for Male and Female of Bangladesh in the Censuses 1961, 1974, 1981 and 1991

The surviving function for male and female of Bangladesh at different censuses have been computed and presented in the second column of Table 4.1 to Table 4.8. To see the trends and patterns of the surviving functions for male and female in the censuses, l_x values have been plotted in the graph paper and depicted in Figure 4.1 and Figure 4.2 respectively. From these figures, it has been seen that surviving function at the different censuses for male and female exhibit traditionally decreasing pattern as well as approximately similar pattern. It can be observed that with passing of time the peaks of the curves for male and female are showing increasing trend, that is, they show upward pattern with respect to time as expected. It is also seen that the surviving functions for male and female in 1961 and 1974 are more or less identical in the age interval [0, 10] years, that is, the difference is negligible. On the contrary, sharp difference is existent in the interval [10, 80+] years. But, it has been observed that the surviving function in 1991 higher than that of 1981 census and the surviving function in 1981 census also higher than that of 1974 census at each age in the whole domain [0, 80+] of the given function.

4.3 Trends and Patterns of Life Expectancy (e_x) at exact Age x for Male and Female of Bangladesh in the Census Years 1961, 1974, 1981 and 1991

The expectations of life for male and female of Bangladesh at the different censuses have already been estimated and presented in the last column of Table 4.1 to Table 4.8. To observe the trends and patterns for male and female in the censuses e_x

values have been plotted in the graph paper and depicted in Figures 4.3 and 4.4 respectively. From these figures, it is seen that the expectations of life for both male and female exhibit almost similar but traditional pattern. It is observed that with passing of time the peaks of the curves of the expectation of life for male and female are showing increasing trend, that is, they show upward pattern with respect to time. The expectation of life at birth for male are 43.43, 45.15, 48.34 and 55.13 and that of females are 43.75, 45.80, 49.82 and 55.84 in 1961, 1974, 1981 and 1991 respectively. Both the series exhibit increasing trend with passing of time. It is also observed that for male and female the life expectancy are gradually increasing in the age interval [0, 4] and smoothly decreasing in the ages (5, 80+] for the census years 1961, 1974 and 1981. It is also observed that the expectation of life for male and female in 1991 is strictly increasing curve in [0, 3] and afterwards strictly decreasing in the sub domain (3, 80+] of the whole domain. It is also found that the maximum life expectancy for male and female are 54.91 and 54.69 in 1961, 57.45 and 57.86 in 1974, 57.57 and 58.84 in1981 census respectively in the same age 4 years where as 60.6 and 62.74 for 1991 census in the age 3 years.

It should be mentioned here that the number of deaths (d_x), the probability of dying (q_x) and the probability of surviving (p_x) have been estimated and presented in the 3rd, 4th and 5th columns of Table 4.1 to Table 4.8. Discussions about d_x , q_x and p_x are self explanatory and need no explanation.

4.4 Time Trends of Expectation of Life at Birth (e_0^0) of Bangladesh During 1961 – 1991

To observe the time trends of expectation of life at birth for male, female and both sexes those rates have been estimated and presented in Table 4.9 and depicted in Figure 4.5. From the figure, it is seen that the curves of expectation of life for male, female and both sexes are showing strictly increasing trend with the increase of time. The rate of increment of e_0^0 for male during 1961 – 1974 was 3.96% which was relatively slower and increased at a greater speed at 7.07% and 14.05% during 1974 – 1981 and 1981 - 1991 respectively. Also, the rate of increment of life expectancy at birth for female was 4.69% during 1961 – 1974 which was slower than that of the faster speed at 8.87% and 11.99% during 1974 – 1981 and 1981 - 1991 respectively. Again, the rate of increment of e_0^0 for both sexes during 1961 – 1974 was 4.31% which was slower and it rapidly increased at a speed at 7.94% and 13.04% during 1974 – 1981 and 1981 to 1991 respectively.

Table 4.1 Abridged Life Table for Male of Bangladesh in 1961

Age Group x	l_x	${}_nd_x$	${}_nq_x$	${}_np_x$	${}_nL_x$	T_x	e_x
0	100000	16581	0.16581	0.83419	88393	4342518	43.42518
1	83419	5606	0.067203	0.932797	80055	4254125	50.99708
2	77813	2841	0.036511	0.963489	76393	4174070	53.64232
3	74972	1691	0.022555	0.977445	74127	4097677	54.6561
4	73281	1099	0.014997	0.985003	72732	4023550	54.90577
5	72182	2245	0.031102	0.968898	355298	3950818	54.73412
10	69937	904	0.012926	0.987074	347425	3595520	51.41084
15	69033	1036	0.015007	0.984993	342575	3248095	47.05134
20	67997	1202	0.017677	0.982323	336980	2905520	42.73012
25	66795	1428	0.021379	0.978621	330405	2568540	38.45408
30	65367	1734	0.026527	0.973473	322500	2238135	34.23952
35	63633	2233	0.035092	0.964908	312583	1915635	30.10443
40	61400	2984	0.048599	0.951401	299540	1603052	26.10834
45	58416	3986	0.068235	0.931765	282115	1303512	22.3143
50	54430	5402	0.099247	0.900753	258645	1021397	18.76533
55	49028	6731	0.137289	0.862711	228313	762752	15.55748
60	42297	8413	0.198903	0.801097	190453	534439	12.63539
65	33884	9313	0.274849	0.725151	146138	343986	10.15187
70	24571	9132	0.371658	0.628342	100025	197848	8.052094
75	15439	3594	0.232787	0.767213	68210	97823	6.336097
80+	11845	11845	1	0	29613	29613	2.500042

Table 4.2 Abridged Life Table for Female of Bangladesh in 1961

Age Group x	l_x	$_nd_x$	$_nq_x$	$_np_x$	$_nL_x$	T_x	e_x
0	100000	15158	0.15158	0.84842	89389	4375311	43.75311
1	84842	5853	0.068987	0.931013	81330	4285922	50.51651
2	78989	3057	0.038702	0.961298	77461	4204592	53.2301
3	75932	1841	0.024245	0.975755	75012	4127131	54.35299
4	74091	1203	0.016237	0.983763	73490	4052119	54.69111
5	72888	2452	0.033641	0.966359	358310	3978629	54.58551
10	70436	1016	0.014424	0.985576	349640	3620319	51.3987
15	69420	1451	0.020902	0.979098	343473	3270679	47.11436
20	67969	1731	0.025467	0.974533	335518	2927206	43.06678
25	66238	1804	0.027235	0.972765	326680	2591688	39.12691
30	64434	2040	0.03166	0.96834	317070	2265008	35.15237
35	62394	2245	0.035981	0.964019	306358	1947938	31.21996
40	60149	2559	0.042544	0.957456	294348	1641580	27.29189
45	57590	3127	0.054298	0.945702	280133	1347232	23.39351
50	54463	4395	0.080697	0.919303	261328	1067099	19.5931
55	50068	6082	0.121475	0.878525	235135	805771	16.09353
60	43986	8031	0.182581	0.817419	199853	570636	12.97313
65	35955	9250	0.257266	0.742734	156650	370783	10.31242
70	26705	9619	0.360195	0.639805	109478	214133	8.018461
75	17086	4698	0.274962	0.725038	73685	104655	6.12519
80+	12388	12388	1	0	30970	30970	2.5

Table 4.3 Abridged Life Table for Male of Bangladesh in 1974

Age Group x	l_x	$_nd_x$	$_nq_x$	$_np_x$	$_nL_x$	T_x	e_x
0	100000	17874	0.17874	0.82126	87488	4515313	45.15313
1	82126	5059	0.0616	0.9384	79091	4427825	53.91502
2	77067	2500	0.032439	0.967561	75817	4348734	56.42797
3	74567	1474	0.019767	0.980233	73830	4272917	57.30306
4	73093	954	0.013052	0.986948	72616	4199087	57.44855
5	72139	1947	0.02699	0.97301	355828	4126471	57.20167
10	70192	784	0.011169	0.988831	349000	3770643	53.71899
15	69408	900	0.012967	0.987033	344790	3421643	49.29753
20	68508	1042	0.01521	0.98479	339935	3076853	44.91232
25	67466	1237	0.018335	0.981665	334238	2736918	40.56737
30	66229	1500	0.022649	0.977351	327395	2402680	36.27837
35	64729	1934	0.029878	0.970122	318810	2075285	32.06113
40	62795	2593	0.041293	0.958707	307493	1756475	27.97157
45	60202	3480	0.057805	0.942195	292310	1448982	24.06867
50	56722	4757	0.083865	0.916135	271718	1156672	20.39195
55	51965	6018	0.115809	0.884191	244780	884954	17.02981
60	45947	7715	0.167911	0.832089	210448	640174	13.93288
65	38232	8921	0.233339	0.766661	168858	429726	11.23996
70	29311	9381	0.32005	0.67995	123103	260868	8.900003
75	19930	2342	0.117511	0.882489	93795	137765	6.912444
80+	17588	17588	1	0	43970	43970	2.5

Table 4.4 Abridged Life Table for Female of Bangladesh in 1974

Age Group x	l_x	${}_nd_x$	${}_nq_x$	${}_np_x$	${}_nL_x$	T_x	e_x
0	100000	16969	0.16969	0.83031	88122	4580202	45.80202
1	83031	5216	0.06282	0.93718	79901	4492080	54.10124
2	77815	2617	0.033631	0.966369	76507	4412179	56.70088
3	75198	1550	0.020612	0.979388	74423	4335672	57.65675
4	73648	1006	0.01366	0.98634	73145	4261249	57.85967
5	72642	2044	0.028138	0.971862	358100	4188104	57.65403
10	70598	845	0.011969	0.988031	350878	3830004	54.25089
15	69753	1198	0.017175	0.982825	345770	3479126	49.8778
20	68555	1422	0.020742	0.979258	339220	3133356	45.70573
25	67133	1489	0.02218	0.97782	331943	2794136	41.6209
30	65644	1691	0.02576	0.97424	323993	2462193	37.50827
35	63953	1878	0.029365	0.970635	315070	2138200	33.43393
40	62075	2170	0.034958	0.965042	304950	1823130	29.36979
45	59905	2679	0.044721	0.955279	292828	1518180	25.34313
50	57226	3803	0.066456	0.933544	276623	1225352	21.4125
55	53423	5331	0.099788	0.900212	253788	948729	17.75881
60	48092	7214	0.150004	0.849996	222425	694941	14.45024
65	40878	8684	0.212437	0.787563	182680	472516	11.55918
70	32194	9711	0.30164	0.69836	136693	289836	9.002796
75	22483	3096	0.137704	0.862296	104675	153143	6.811502
80+	19387	19387	1	0	48468	48468	2.500026

Table 4.5 Abridged Life Table for Male of Bangladesh in 1981

Age Group x	l_x	$_nd_x$	$_nq_x$	$_np_x$	$_nL_x$	T_x	e_x
0	100000	13866	0.13866	0.86134	90294	4833692	48.33692
1	86134	4384	0.050897	0.949103	83504	4743398	55.06998
2	81750	2208	0.027009	0.972991	80646	4659894	57.00176
3	79542	1318	0.01657	0.98343	78883	4579248	57.57019
4	78224	866	0.011071	0.988929	77791	4500365	57.53177
5	77358	1810	0.023398	0.976602	382265	4422574	57.17022
10	75548	749	0.009914	0.990086	375868	4040309	53.48003
15	74799	876	0.011711	0.988289	371805	3664441	48.99051
20	73923	1021	0.013812	0.986188	367063	3292636	44.54143
25	72902	1227	0.016831	0.983169	361443	2925573	40.13022
30	71675	1499	0.020914	0.979086	354628	2564130	35.7744
35	70176	1978	0.028186	0.971814	345935	2209502	31.48515
40	68198	2728	0.040001	0.959999	334170	1863567	27.32583
45	65470	3778	0.057706	0.942294	317905	1529397	23.36027
50	61692	5338	0.086527	0.913473	295115	1211492	19.63775
55	56354	6934	0.123044	0.876956	264435	916377	16.26108
60	49420	9014	0.182396	0.817604	224565	651942	13.19187
65	40406	10343	0.255977	0.744023	176173	427377	10.57707
70	30063	10557	0.351163	0.648837	123923	251204	8.355919
75	19506	3803	0.194966	0.805034	88023	127281	6.525223
80+	15703	15703	1	0	39258	39258	2.500032

Table 4.6 Abridged Life Table for Female of Bangladesh in 1981

Age Group x	l_x	$_nd_x$	$_nq_x$	$_np_x$	$_nL_x$	T_x	e_x
0	100000	12994	0.12994	0.87006	90904	4981527	49.81527
1	87006	4435	0.050973	0.949027	84345	4890623	56.21018
2	82571	2267	0.027455	0.972545	81438	4806278	58.20782
3	80304	1359	0.016923	0.983077	79625	4724840	58.83692
4	78945	887	0.011236	0.988764	78502	4645215	58.84116
5	78058	1836	0.023521	0.976479	385700	4566713	58.5041
10	76222	769	0.010089	0.989911	379188	4181013	54.8531
15	75453	1090	0.014446	0.985554	374540	3801825	50.38666
20	74363	1306	0.017562	0.982438	368550	3427285	46.08858
25	73057	1392	0.019054	0.980946	361805	3058735	41.86779
30	71665	1617	0.022563	0.977437	354283	2696930	37.63246
35	70048	1848	0.026382	0.973618	345620	2342647	33.44345
40	68200	2204	0.032317	0.967683	335490	1997027	29.28192
45	65996	2801	0.042442	0.957558	322978	1661537	25.17633
50	63195	4078	0.06453	0.93547	305780	1338559	21.18141
55	59117	5862	0.099159	0.900841	280930	1032779	17.47008
60	53255	8128	0.152624	0.847376	245955	751849	14.1179
65	45127	9971	0.220954	0.779046	200708	505894	11.21045
70	35156	11172	0.317784	0.682216	147850	305186	8.680908
75	23984	4509	0.188	0.812	108648	157336	6.56004
80+	19475	19475	1	0	48688	48688	2.500026

Table 4.7 Abridged Life Table for Male of Bangladesh in 1991

Age Group x	l_x	$_nd_x$	$_nq_x$	$_np_x$	$_nL_x$	T_x	e_x
0	100000	9097	0.091	0.909	93632	5513200	55.13
1	90903	2904	0.0319	0.9681	89161	5419568	59.62
2	87999	1476	0.0168	0.9832	87261	5330407	60.57
3	86523	897	0.0104	0.9896	86075	5243146	60.6
4	85626	600	0.007	0.993	85326	5107271	59.65
5	85026	1312	0.0154	0.9846	421850	5021945	59.06
10	83714	575	0.0069	0.9931	417133	4600095	54.95
15	83139	691	0.0083	0.9917	413968	4182963	50.31
20	82448	819	0.0099	0.9901	410193	3768995	45.71
25	81629	1005	0.0123	0.9877	405633	3358803	41.15
30	80624	1243	0.0154	0.9846	400013	2953170	36.63
35	79381	1717	0.0216	0.9784	392613	2553158	32.16
40	77664	2507	0.0323	0.9677	382053	2160545	27.82
45	75157	3718	0.0495	0.9505	366490	1778493	23.66
50	71439	5663	0.0793	0.9207	343038	1412003	19.77
55	65776	7885	0.1199	0.8801	309168	1068965	16.25
60	57891	10787	0.1863	0.8137	262488	759798	13.12
65	47104	12636	0.2683	0.7317	203930	497310	10.56
70	34468	12804	0.3715	0.6285	140330	293380	8.51
75	21664	1886	0.0871	0.9129	103605	153050	7.06
80+	19778	19778	1	0	49445	49445	2.5

Table 4.8 Abridged Life Table for Female of Bangladesh in 1991

Age Group x	l_x	${}_nd_x$	${}_nq_x$	${}_np_x$	${}_nL_x$	T_x	e_x
0	100000	10493	0.10493	0.89507	92655	5583656	55.83656
1	89507	3168	0.035394	0.964606	87606	5491001	61.34717
2	86339	1583	0.018335	0.981665	85548	5403395	62.58348
3	84756	943	0.011126	0.988874	84285	5317847	62.74302
4	83813	618	0.007374	0.992626	83504	5233562	62.44332
5	83195	1299	0.015614	0.984386	412728	5150058	61.90346
10	81896	553	0.006752	0.993248	408098	4737330	57.84568
15	81343	771	0.009478	0.990522	404788	4329232	53.22194
20	80572	927	0.011505	0.988495	400543	3924444	48.70729
25	79645	1018	0.012782	0.987218	395680	3523901	44.2451
30	78627	1218	0.015491	0.984509	390090	3128221	39.78558
35	77409	1460	0.018861	0.981139	383395	2738131	35.37226
40	75949	1845	0.024293	0.975707	375133	2354736	31.00417
45	74104	2457	0.033156	0.966844	364378	1979603	26.71385
50	71647	3724	0.051977	0.948023	348925	1615225	22.54421
55	67923	5567	0.08196	0.91804	325698	1266300	18.64317
60	62356	8088	0.129707	0.870293	291560	940602	15.08439
65	54268	10499	0.193466	0.806534	245093	649042	11.95994
70	43769	12491	0.285385	0.714615	187618	403949	9.229112
75	31278	3651	0.116727	0.883273	147263	216331	6.916395
80+	27627	27627	1	0	69068	69068	2.500018

Table 4.9 Estimated Life Expectancy at Birth (e_0^0) for Male, Female and Both Sexes in the Censuses 1961, 1974, 1981 and 1991

Census Years	Life Expectancy at Birth (e_0^0)		
	Male	Female	Both Sexes
1961	43.43	43.75	43.59
1974	45.15	45.80	45.47
1981	48.34	49.86	49.08
1991	55.13	55.84	55.48

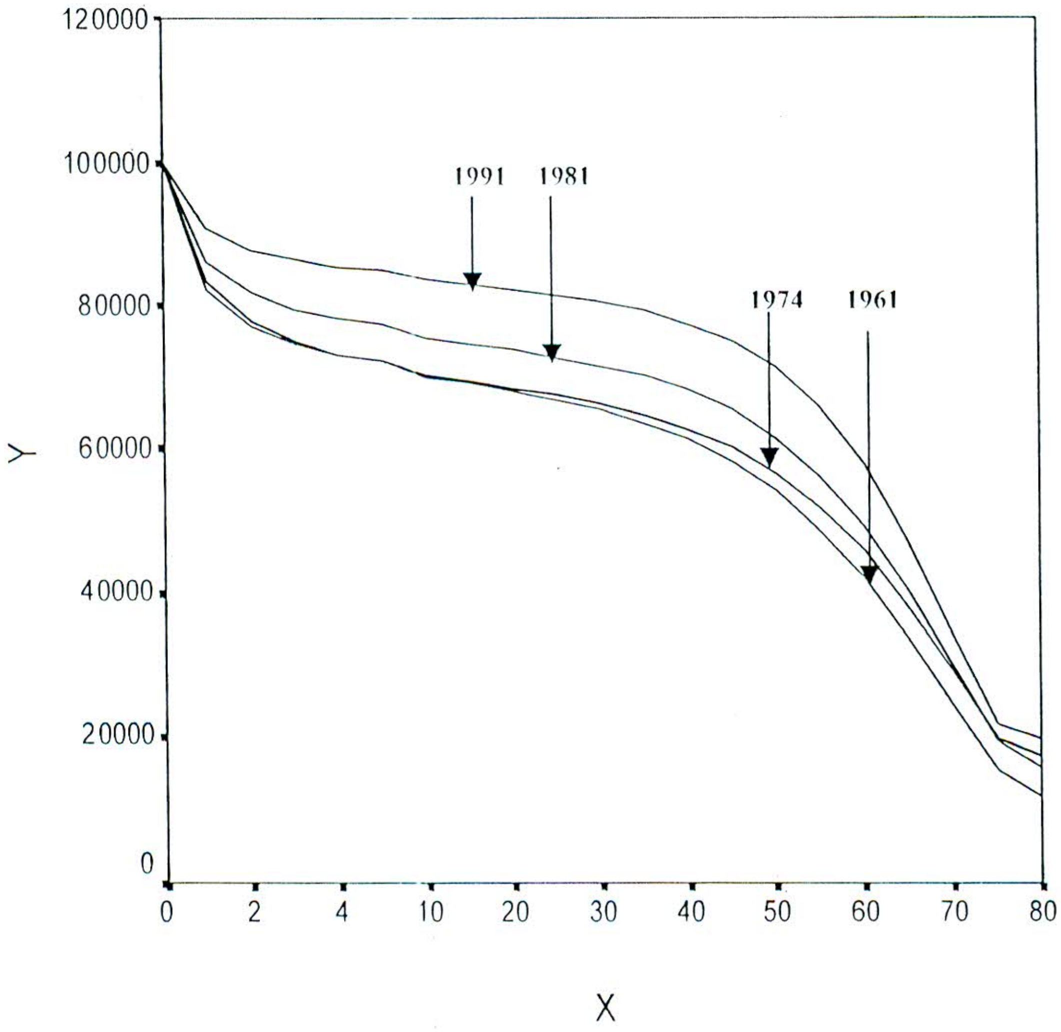


Figure 4.1 Life Table Survival Function (l_x) at an Exact Age x for Male of Bangladesh During 1961-1991. X: Age Group in Years and Y: Survival Function.

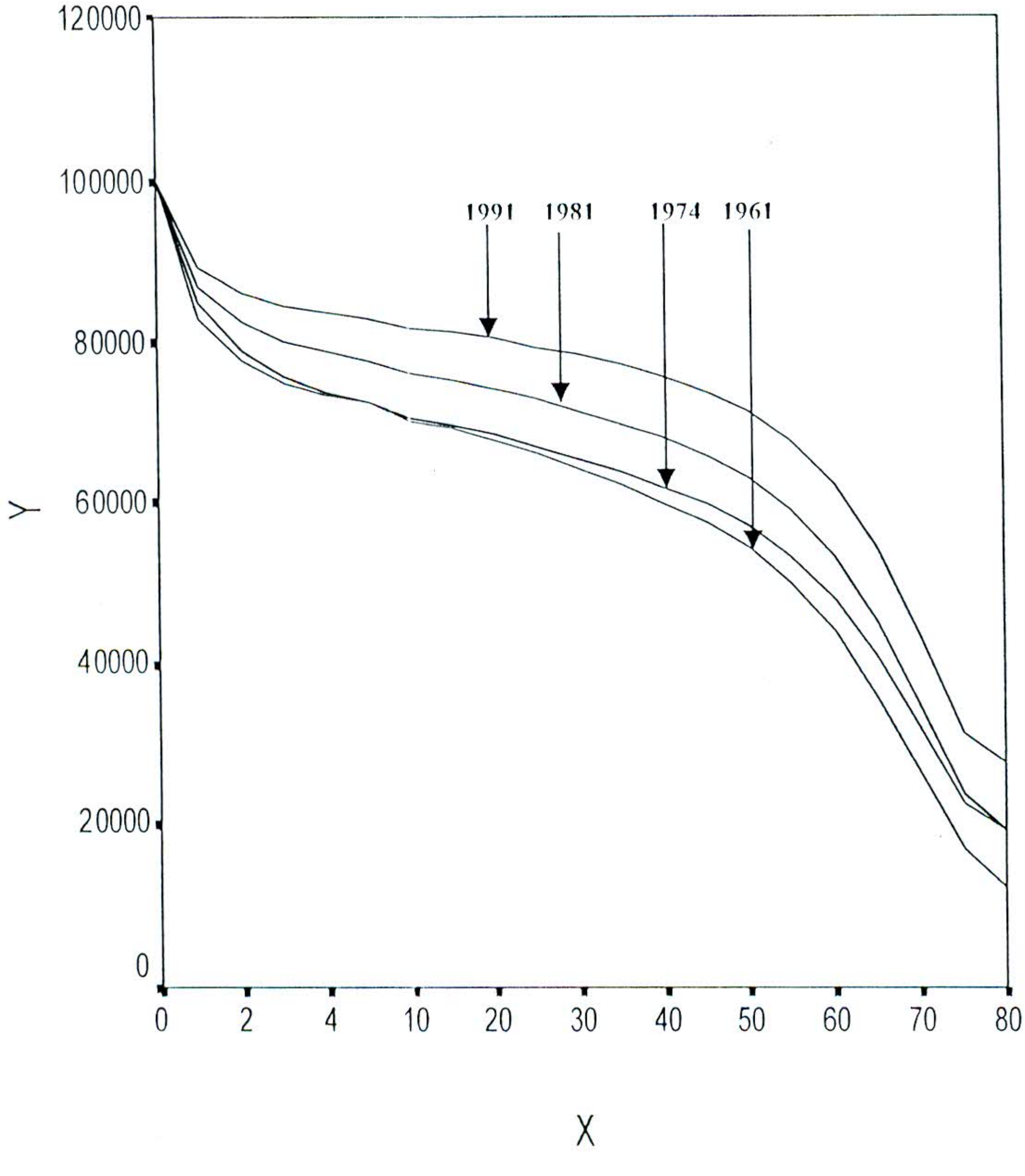


Figure 4.2 Life Table Survival Function (l_x) at an Exact Age x for Female of Bangladesh During 1961-1974. X: Age Group in Years and Y: Survival Function.

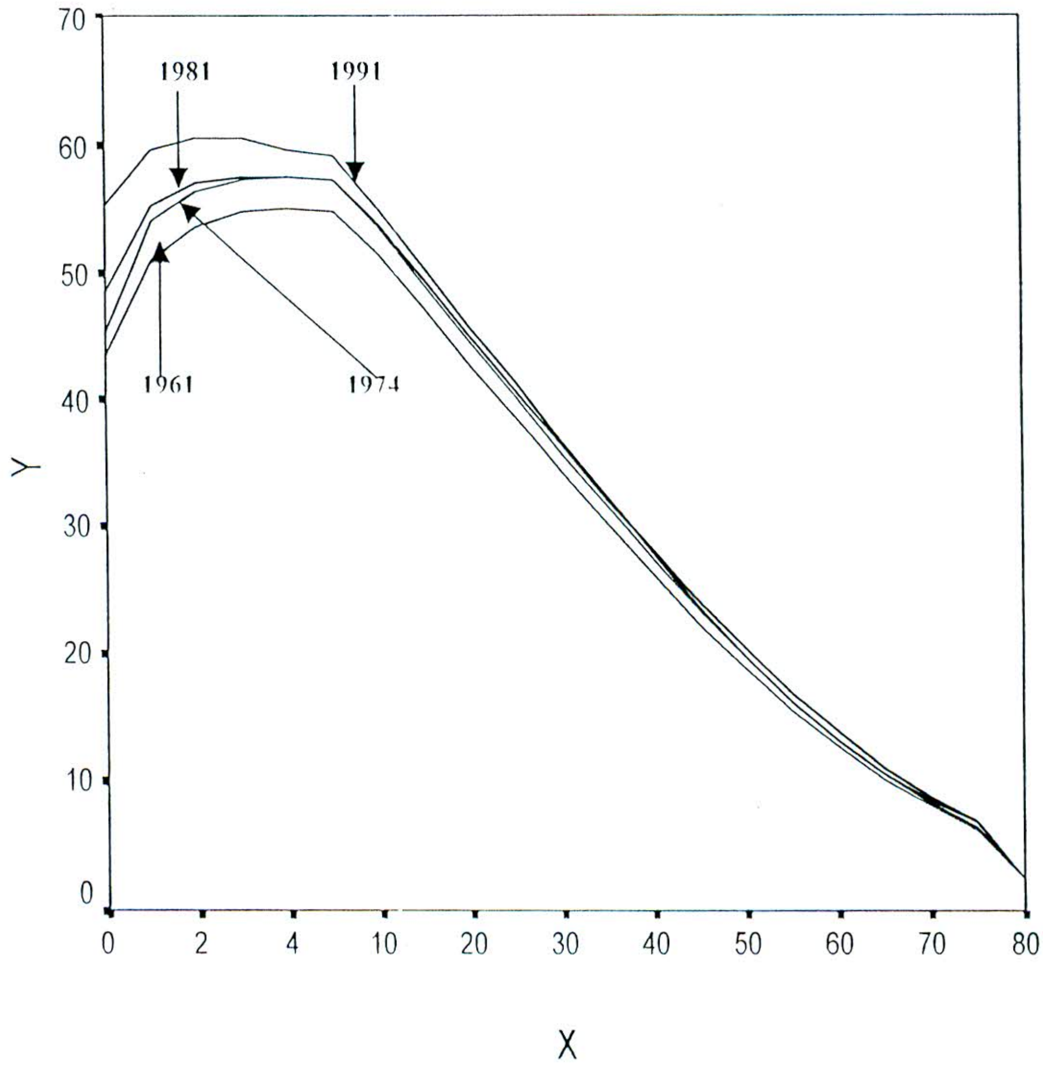


Figure 4.3 Life Expectancy for Male of Bangladesh During 1961-1991. X: Age Group in Years and Y: Life Expectancy.

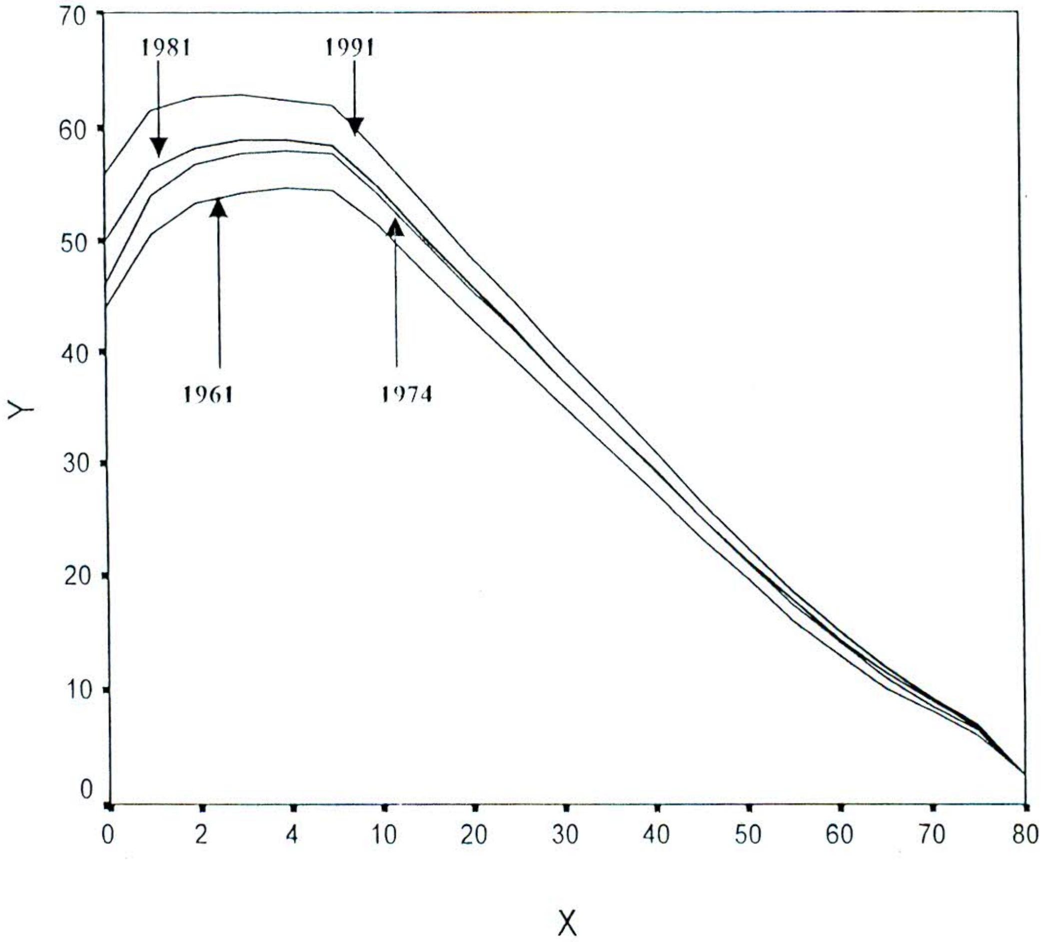


Figure 4.4 Life Expectancy for Female of Bangladesh During 1961-1991. X: Age Group in Years and Y: Life Expectancy.

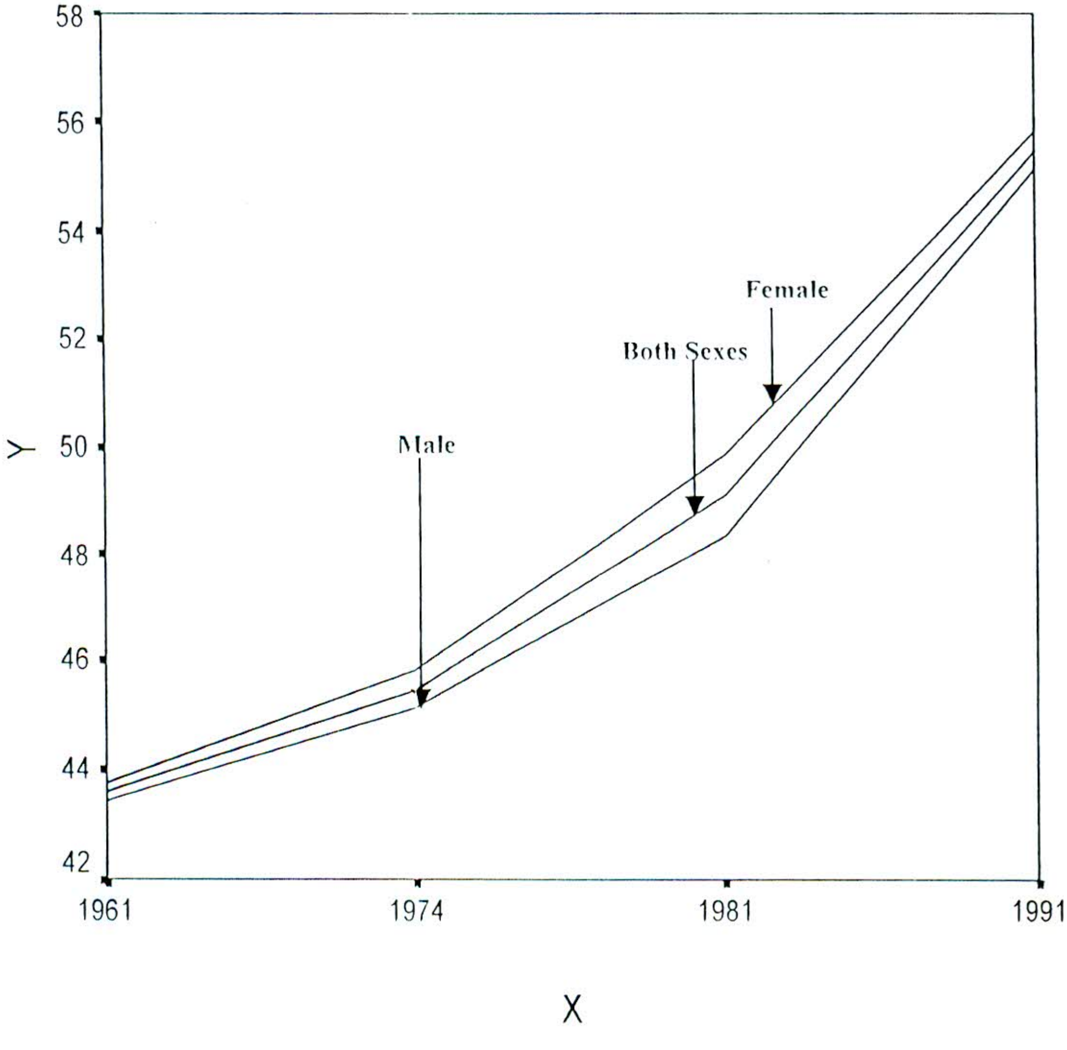


Figure 4.5 Life Expectancy at Birth for Male, Female and Both Sexes of Bangladesh During 1961 to 1991. X: Age Group in Years and Y: Life Expectancy at Birth.

Chapter 5

Estimation of Demographic Parameters

Chapter 5

Estimation of Demographic Parameters

By the word demographic parameters we mean the demographic characteristics of a population. As for example, birth rate, death rate, growth rate, expectation of life etc. To estimate above parameters some specific raw data is needed, e.g. to estimate the CDR, we need number of deaths from registration data and mid-year population from census data. But unfortunately the registration data is not available in the developing countries like Bangladesh. The same problems arise at the time of estimating all most all the parameters. In such a case, for the limited and defective data, parameters should be estimated using the related alternative data sources e.g. estimation of mortality using marital status data etc. Such an estimating procedure is called an indirect estimation of parameters.

Based on the above concepts the different demographic parameters of Bangladesh have been estimated using various related data.

5.1 Estimation of Infant Mortality Rate (IMR) and Infant Death Rate (IDR) Using Life Table

Infant mortality rate is a probability based measure. It is approximately the probability of death among infants in a calendar year. It is defined as the number of infant deaths per 1000 live births during the year. That is

$$IMR = \frac{D_0}{B} \times 1000$$; where, D_0 represents death of infants during a calendar year and B represents live births during the same year.

Infant death rate is the death among infants per 1000 mid-year population under age 1 year. It is defined as

$$IDR = \frac{D_0}{P_0} \times 1000$$; where, D_0 represents death of infants during a calendar year and P_0 is the mid-year population under age 1 year during the same year.

For the lack of sufficient data, IMR and IDR have been estimated using indirect method. Here, IMR and IDR for male, female and both sexes have been estimated using the information from estimated abridged life tables shown in Chapter 4. The estimated IMR and IDR are presented in Table 5.1. These rates have been plotted in graph papers exhibited in Figure 5.1 and Figure 5.2 respectively. From these figures, it is observed that they exhibited an increasing trend during 1961 to 1974 and then they declined sharply during the intercensal periods 1974-1981 and 1981-1991.

Table 5.1 Infant Mortality Rate (IMR) and Infant Death Rate (IDR) for Male, Female and Both Sexes of Bangladesh During 1961-1991

Census Year	Infant Mortality Rate (per thousand)			Infant Death Rate (per thousand)		
	Male	Female	Both Sexes	Male	Female	Both Sexes
1961	166	152	159	182	164	173
1974	179	170	174	193	185	189
1981	139	130	134	148	139	144
1991	91	104	98	93	107	100

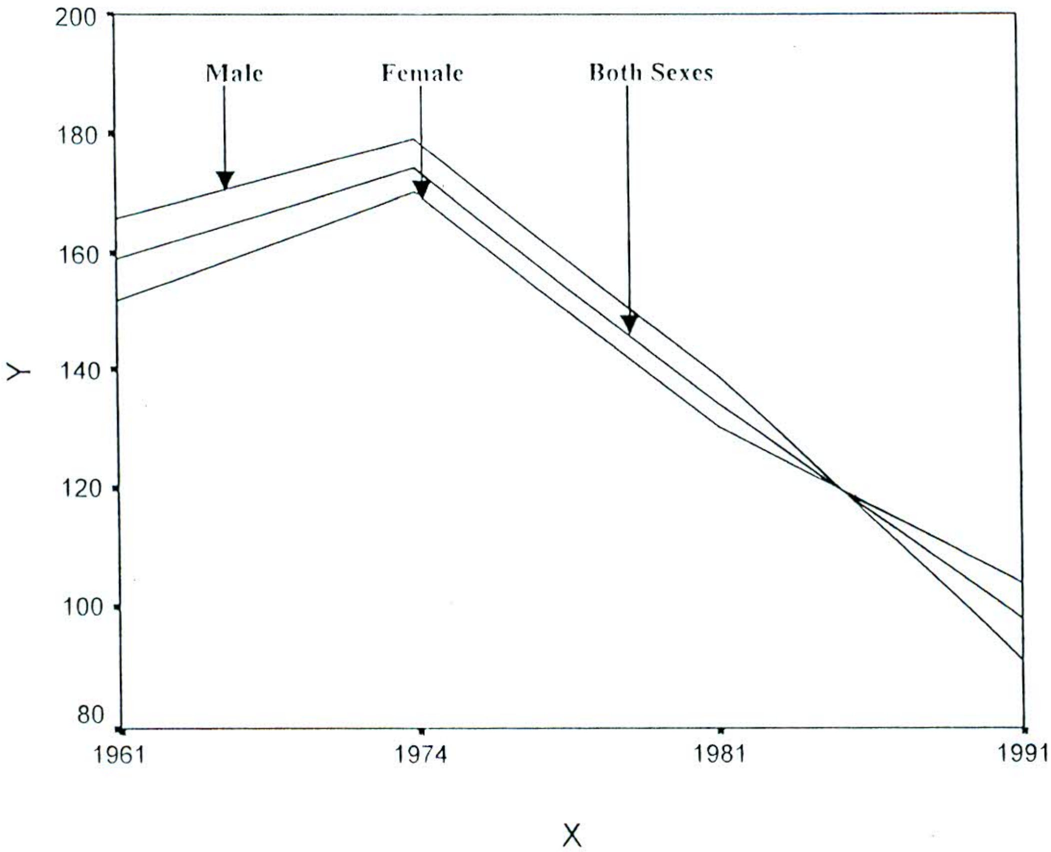


Figure 5.1 Infant Mortality Rate (IMR) for Male, Female and Both Sexes of Bangladesh During 1961-1991. X: Census Years and Y: Infant Mortality Rate.

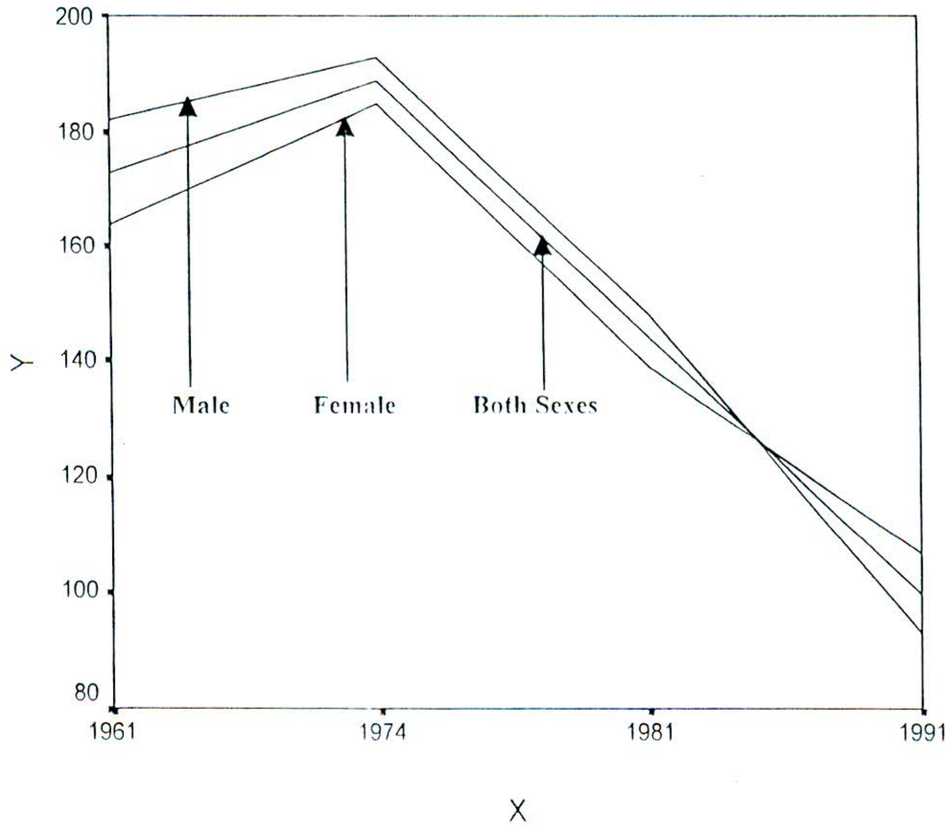


Figure 5.2 Infant Death Rate (IMR) for Male, Female and Both Sexes of Bangladesh During 1961-1991. X: Census Years and Y: Infant Death Rate.

5.2 Estimation of Age Specific Death Rates (ASDR)

Age specific death rates are more effective and sophisticated measure than CDR, as they comprehend mortality rate at different ages. It is the number of deaths of persons of a given age during a year (D_a) per 1,000 mid-year population in that age (P_a), that is, $\frac{D_a}{P_a} \times 1000$. For the deficiency of sufficient information, different ASDR have been estimated using the information from different constructed life tables [Table 4.1 to Table 4.8]. Thus, we have used the following formula for ASDR:

$$ASDR = {}_n d_x \,/\, {}_n L_x$$

where, ${}_n d_x$ is the number of deaths in the age interval x to x+n and ${}_n L_x$ is the stationary population in a life table corresponding to the census years 1961, 1974, 1981 and 1991. Estimated ASDR have been presented in Table 5.2 to Table 5.5.

The estimated and smoothed age specific death rates for male, female and both sexes of Bangladesh at different censuses have been presented in Table 5.2 to Table 5.5. Now, to observe the trends and patterns of ASDR for male, female and both sexes have been plotted in the graph paper and depicted in Figures 5.3, 5.4 and 5.5. From these figures, it is observed that ASDR for male, female and both sexes exhibited traditional U-shape pattern. It is also found that with passing of time the curves of ASDR for male, female and both sexes are going down, that is, they indicate downward pattern with respect to time. The minimum values of ASDR for male are .00285 in 1961, .00246 in 1974, .00223 in 1981 and .00159 in 1991 and the minimum values of ASDR for female are .003999, .00326, .00275 and .0081 in 1961, 1974, 1981 and 1991 respectively. Where as .00344, .00287, .002495 and .0017 are corresponding minimum values of ASDR for both sexes. During the study period. the minimum values of ASDR for male, female and both sexes have been showing decreasing trend. The lowest values of ASDR ranges from .00285 to .00159 and .003999 to .0081 and .00344 to .0017 for male, female and both sexes during 1961-1991.

Table 5.2 Estimated and Smoothed Age Specific Death Rates (ASDR) for Male, Female and Both Sexes of Bangladesh in 1961

Age Group x	Male		Female		Both Sexes	
	Estimated ASDR	Smoothed ASDR	Estimated ASDR	Smoothed ASDR	Estimated ASDR	Smoothed ASDR
0	0.18758	0.18207	0.16957	0.16434	0.17853	0.17316
1	0.07003	0.09744	0.07197	0.09225	0.071	0.09483
2	0.03719	0.04431	0.03947	0.04532	0.03834	0.04482
3	0.02281	0.02162	0.02454	0.02341	0.02368	0.02252
4	0.01511	0.01231	0.01637	0.01331	0.01574	0.01281
5-9	0.00632	0.00654	0.00684	0.00719	0.00658	0.00686
10-14	0.0026	0.00334	0.00291	0.00419	0.00275	0.00377
15-19	0.00302	0.00285	0.00422	0.00399	0.00363	0.00344
20-24	0.00357	0.00337	0.00516	0.0047	0.00436	0.00405
25-29	0.00432	0.00404	0.00552	0.00538	0.00492	0.00472
30-34	0.00538	0.0052	0.00643	0.00616	0.0059	0.00568
35-39	0.00714	0.00707	0.00733	0.00714	0.00723	0.00711
40-44	0.00996	0.00992	0.00869	0.00854	0.00933	0.00922
45-49	0.01413	0.01437	0.01116	0.01144	0.01265	0.01287
50-54	0.02089	0.02114	0.01682	0.01727	0.01884	0.01915
55-59	0.02948	0.03167	0.02587	0.02715	0.02765	0.02937
60-64	0.04417	0.04489	0.04018	0.04046	0.04213	0.04269
65-69	0.06373	0.05765	0.05905	0.0547	0.06131	0.05612
70-74	0.0913	0.08806	0.08786	0.08649	0.0895	0.08698
75-79	0.05269	0.17548	0.06376	0.17519	0.05844	0.17494
80+	0.39999	0.32259	0.4	0.32435	0.4	0.32311

Table 5.3 Estimated and Smoothed Age Specific Death Rates (ASDR) for Male, Female and Both Sexes of Bangladesh in 1974

Age Group x	Male		Female		Both Sexes	
	Estimated ASDR	Smoothed ASDR	Estimated ASDR	Smoothed ASDR	Estimated ASDR	Smoothed ASDR
0	0.2043	0.19329	0.19256	0.18519	0.19841	0.18923
1	0.06396	0.09908	0.06528	0.09647	0.06463	0.09777
2	0.03297	0.04126	0.03421	0.04171	0.03359	0.04149
3	0.01996	0.01851	0.02083	0.01943	0.0204	0.01897
4	0.01314	0.01051	0.01375	0.01095	0.01345	0.01073
5-9	0.00547	0.00566	0.00571	0.00599	0.00559	0.00582
10-14	0.00225	0.00288	0.00241	0.00344	0.00233	0.00316
15-19	0.00261	0.00246	0.00346	0.00326	0.00304	0.00287
20-24	0.00307	0.0029	0.00419	0.00384	0.00363	0.00338
25-29	0.0037	0.00347	0.00449	0.00437	0.00409	0.00393
30-34	0.00458	0.00443	0.00522	0.005	0.0049	0.00472
35-39	0.00607	0.00601	0.00596	0.00581	0.00601	0.00592
40-44	0.00843	0.00843	0.00712	0.00703	0.00778	0.00773
45-49	0.01191	0.01218	0.00915	0.00948	0.01053	0.01083
50-54	0.01751	0.01781	0.01375	0.01422	0.01561	0.016
55-59	0.02459	0.02644	0.02101	0.02232	0.02276	0.02434
60-64	0.03666	0.03677	0.03243	0.03214	0.03449	0.03439
65-69	0.05283	0.04631	0.04754	0.04121	0.05008	0.04367
70-74	0.0762	0.07449	0.07104	0.06997	0.07349	0.07214
75-79	0.02497	0.16109	0.02958	0.16	0.0274	0.16051
80+	0.4	0.30786	0.4	0.31295	0.4	0.3105

Table 5.4 Estimated and Smoothed Age Specific Death Rates (ASDR) for Male, Female and Both Sexes of Bangladesh in 1981

Age Group x	Male		Female		Both Sexes	
	Estimated ASDR	Smoothed ASDR	Estimated ASDR	Smoothed ASDR	Estimated ASDR	Smoothed ASDR
0	0.15357	0.14807	0.14294	0.13871	0.14824	0.14392
1	0.0525	0.0772	0.05258	0.07383	0.05254	0.075647
2	0.02738	0.03345	0.02784	0.03326	0.02761	0.033358
3	0.01671	0.01568	0.01707	0.01607	0.01689	0.015873
4	0.01113	0.00896	0.0113	0.00909	0.01122	0.00903
5-9	0.00473	0.00487	0.00476	0.00498	0.00475	0.004927
10-14	0.00199	0.00258	0.00203	0.0029	0.00201	0.00274
15-19	0.00236	0.00223	0.00291	0.00275	0.00263	0.002495
20-24	0.00278	0.00261	0.00354	0.00324	0.00316	0.002939
25-29	0.00339	0.00315	0.00385	0.00375	0.00362	0.003463
30-34	0.00423	0.00409	0.00456	0.00438	0.0044	0.004252
35-39	0.00572	0.00565	0.00535	0.00522	0.00553	0.005442
40-44	0.00816	0.00813	0.00657	0.00643	0.00736	0.007276
45-49	0.01188	0.01215	0.00867	0.00888	0.01027	0.010484
50-54	0.01809	0.01843	0.01334	0.01376	0.01567	0.016022
55-59	0.02622	0.02838	0.02087	0.02223	0.02346	0.025177
60-64	0.04014	0.04065	0.03305	0.03316	0.03643	0.036699
65-69	0.05871	0.0522	0.04968	0.04401	0.0539	0.047827
70-74	0.08519	0.08219	0.07556	0.07401	0.07995	0.077779
75-79	0.0432	0.17073	0.0415	0.16448	0.04226	0.167331
80+	0.39999	0.32004	0.4	0.31775	0.4	0.31875

Table 5.5 Estimated and Smoothed Age Specific Death Rates (ASDR) for Male, Female and Both Sexes of Bangladesh in 1991

Age Group x	Male		Female		Both Sexes	
	Estimated ASDR	Smoothed ASDR	Estimated ASDR	Smoothed ASDR	Estimated ASDR	Smoothed ASDR
0	0.09716	0.09323	0.11325	0.10766	0.10516	0.10040
1	0.03257	0.04843	0.03616	0.05528	0.03435	0.05183
2	0.01691	0.02083	0.01850	0.02309	0.01770	0.02195
3	0.01042	0.00974	0.01119	0.01036	0.0108	0.01004
4	0.00703	0.00566	0.0074	0.00586	0.00721	0.00575
5-9	0.00311	0.00318	0.00315	0.00327	0.00313	0.00323
10-14	0.00138	0.0018	0.00136	0.00191	0.00137	0.00186
15-19	0.00167	0.00159	0.0019	0.0018	0.00179	0.0017
20-24	0.002	0.00184	0.00231	0.00212	0.00215	0.002
25-29	0.00248	0.00228	0.00257	0.0025	0.00252	0.00241
30-34	0.00311	0.00305	0.00312	0.00302	0.00311	0.00305
35-39	0.00437	0.00431	0.00381	0.00373	0.00409	0.00403
40-44	0.00656	0.00652	0.00492	0.0048	0.00575	0.00567
45-49	0.01014	0.01044	0.00674	0.00691	0.00845	0.0087
50-54	0.01651	0.01703	0.01067	0.01108	0.01357	0.01403
55-59	0.0255	0.02821	0.01709	0.01845	0.02119	0.02314
60-64	0.0411	0.0422	0.02774	0.02764	0.03407	0.03446
65-69	0.06196	0.05519	0.04284	0.03642	0.05152	0.04503
70-74	0.09124	0.08483	0.06658	0.06542	0.07713	0.07417
75-79	0.0182	0.16745	0.02479	0.1566	0.02207	0.1614
80+	0.4	0.30532	0.4	0.31157	0.4	0.30861

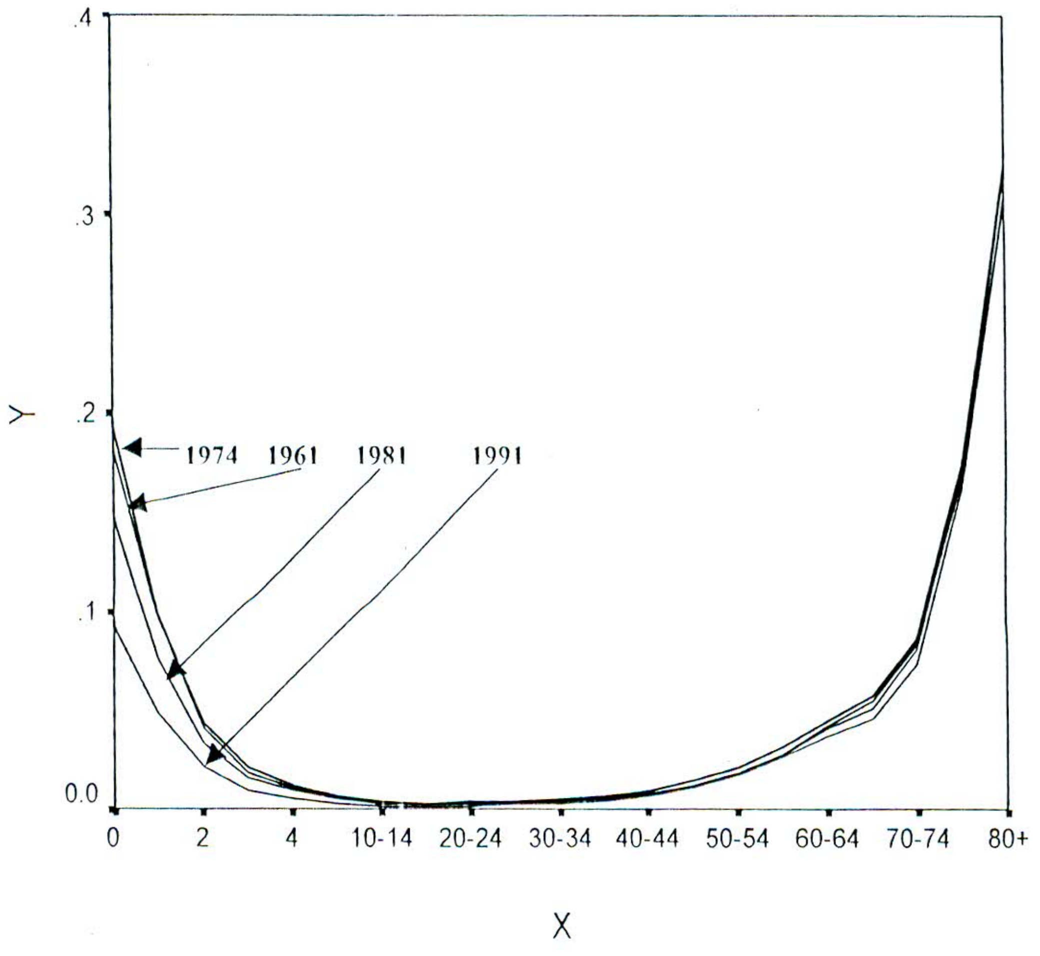


Figure 5.3 Smoothed Age Specific Death Rates (ASDR) for Male of Bangladesh During 1961-1991. X: Age Group in Years and Y: ASDR.

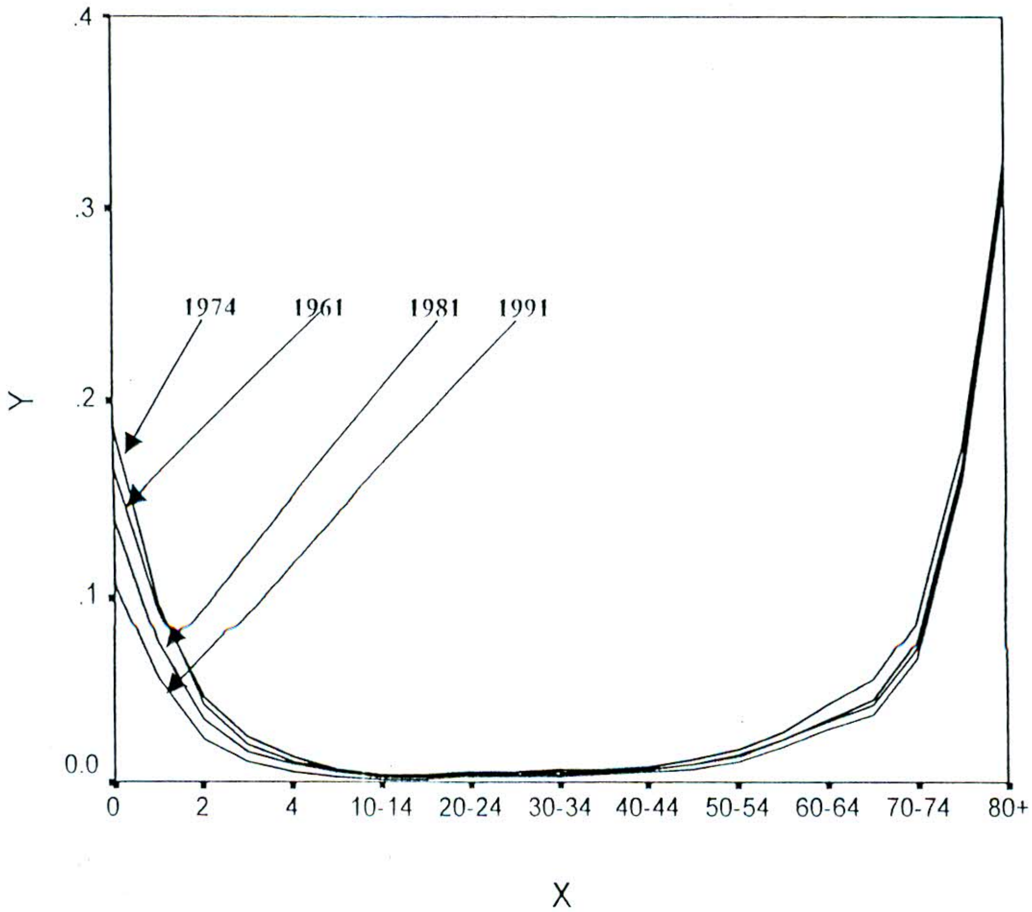


Figure 5.4 Smoothed Age Specific Death Rates (ASDR) for Female of Bangladesh During 1961-1991. X: Age Group in Years and Y: ASDR.

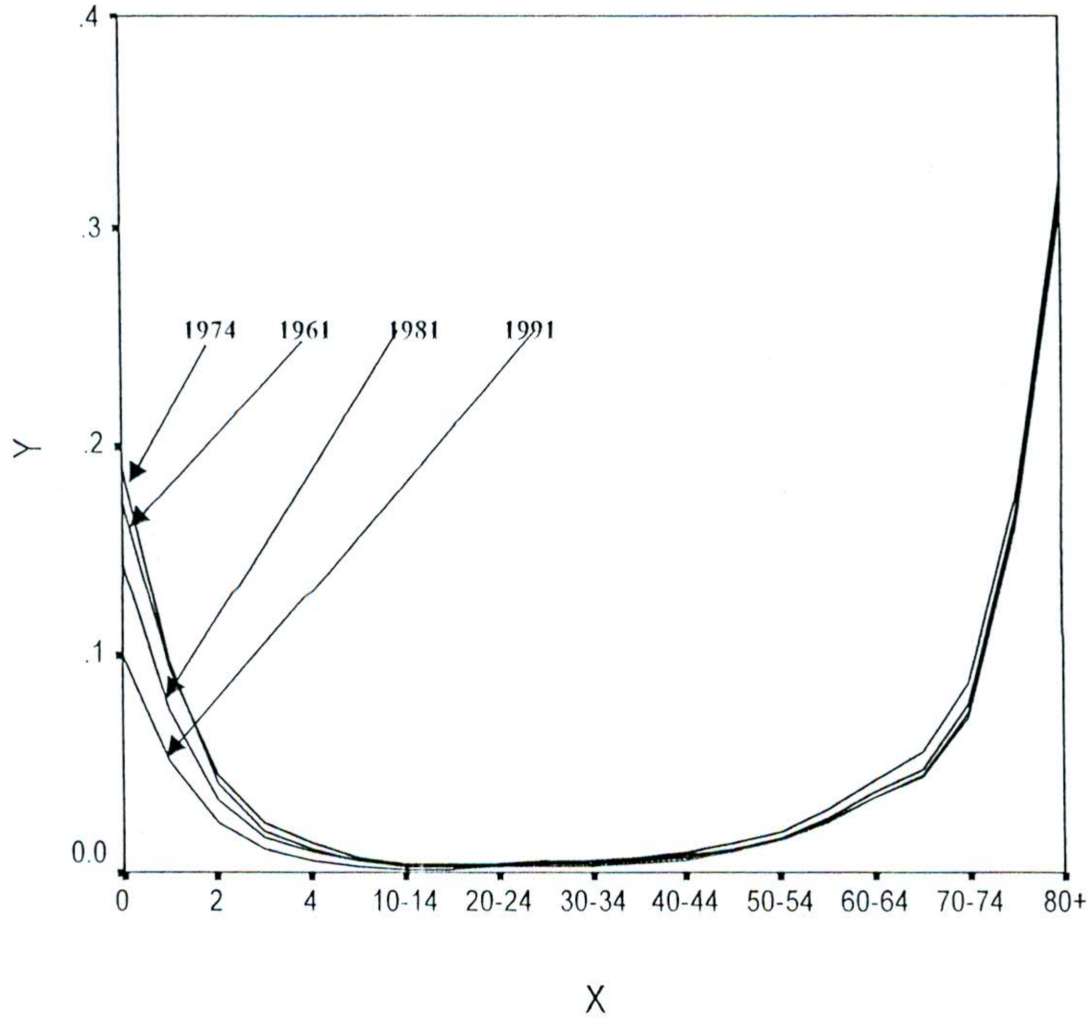


Figure 5.5 Smoothed of Age Specific Death Rates (ASDR) for Both Sexes of Bangladesh During 1961-1991. X: Age Group in Years and Y: ASDR.

5.3 Estimation of Crude Death Rate (CDR)

It is the simplest measure of mortality. It is defined as the number of deaths in a year per 1,000 mid-year population and its traditional formula is $\frac{D}{P} \times 1000$. This formula is applicable of those countries in which vital registration system exists. But in this study we have estimated different CDR using the estimated smoothed ASDR from the constructed life tables [Table 4.1 to Table 4.8] and census smoothed age structure as follows:

$$CDR = \sum_x ASDR_x \ C_x$$

where, $ASDR_x$ is the age specific death rates in the age group x and C_x is the smoothed age structure for male, female and both sexes at the times of census years. Estimated CDR have been presented in Table 5.6 and depicted in Figure 5.6. These table and figure reveals that CDR for male, female and both sexes were showing strictly decreasing trend with passing of time. The rate of decrement of CDR for male during 1961-1974 was 4.82% which was slower and it decreased at a greater speed at 11.28% and 20.04% during 1974-1981 and 1981-1991 respectively. Also, the rate of decrement of CDR for female was 5.5% during 1961 to 1974 which was slower and it decreased at a faster speed at 11.74% and 23.03% during 1974-1981 and 1981-1991 respectively. Again, the rate of decrement of CDR for both sexes during 1961-1974 was 5.32% which was slower and it rapidly decreased at a larger speed at 11.58% and 21.27% during 1974-1981 and 1981-1991 respectively.

5.4 Estimation of Intercensal Growth Rate (r)

Growth of population is an important aspect of population that is most often talked about not only in the demographic circle but also by people concerned with economic growth, national planning and social welfare. It refers to the change of population size between two dates. Growth rate of a population during an intercensal period is a percentage of population or per person of population at the beginning census of the intercensal period. There are various estimating methods of population growth rate. The exponential growth rate method is suitable for our country. So, an attempt has been made to estimate the growth rate applying the exponential growth rate method. The formula is as follows:

$$p_{t_2} = p_{t_1} \exp(r(t_2 - t_1)$$

$$\text{i. e. } r = \frac{\ln\left(\frac{p_{t_2}}{p_{t_1}}\right)}{t_2 - t_1}$$

where, p_{t_1} is the initial population at time t_1, p_{t_2} is the terminal population at time t_2 and $t_2 - t_1$ is the exact number of years between the intercensal period t_1 to t_2.

Using the above formula, growth rates have been estimated for male, female and both sexes during the period 1951 – 1961, 1961 – 1974, 1974 – 1981 and 1981 – 1991 and presented in Table 5.6. These rates have been plotted in the graph paper and displayed in Figure 5.7. From the figure, it has been observed that they were all increasing trends up to 1981 and then, they were decreasing up to 1991. From the table and the figure, it is observed that growth rate curve for male, female and both sexes were increasing trend during 1961-1981 and then, they started to reduce up to 1991. The rate of increment of growth rate for male during 1961-1974 was 29.21% which was faster and the rate of increment during 1974-1981 was 4.98% which was slower but the rate of decrement of it during 1981 to 1991 was 11.31%. Also, the rate of increment of it for female during 1961-1974 was 16.59% which was faster than that of it during 1974-1981 was 11.92% but it decreased at a greater speed at 14.43% during 1981-1991. Again, during 1961-1974 the rate of increment of growth rate for both sexes was 22.64% which was faster than that of it during 1974-1981 was 8.46% which was slower, it rapidly started to decrease at a speed at 12.77% in the interval 1981 to 1991.

5.5 Estimation of Crude Birth Rate (CBR)

It is ratio of the total number of births during a year to the average or mid-year population. It is conventionally estimated by

$$CBR = \frac{B}{P} \times 1000 \cdot$$

It has been estimated using the inter-relation between CBR, CDR and growth rate in the present study. Thus applying the balancing equation, CBR = CDR + r [assuming the net migration rate is zero]. Estimated CBR for male, female and both sexes at the time of census years have been shown in Table 5.6 and depicted in Figure 5.8. From the table and figure, it is observed that CBR for male and both sexes were strictly increasing during 1961 to 1974 and afterwards, started to decrease till 1991. But, CBR for female increases with passing of time up to 1981 and then, started to decrease up to 1991. The rate of increment of CBR for male during 1961-1974 was 11.55% which was faster but it decreased at a slow speed at 2.22% and at a greater speed at 14.8% during 1974-1981 and 1981 to 1991 respectively. Also, the rate of increment of CBR for female were 5.76% and

1.56% during 1961-1974 and 1974-1981 respectively but it decreased at a greater speed at 17.71% in the interval 1981 to 1991. Again, during 1961-1974 the rate of increment of CBR for both sexes was 8.53% which was faster but it decreased slowly at a speed 0.54% during 1974-1981 and decreased at a greater speed at 16.08% during 1981-1991.

Table 5.6 Crude Death Rate (CDR), Intercensal Growth Rate (r) and Crude Birth Rate (CBR) for Male, Female and Both Sexes of Bangladesh in the Census Years 1961, 1974, 1981 and 1991

Census Years	Male			Female			Both Sexes		
	CDR	Growth Rate (r)	CBR	CDR	Growth Rate (r)	CBR	CDR)	Growth Rate (r)	CBR
1961	21.80	0.0202	42.00	21.45	0.0223	43.75	21.61	0.0212	42.81
1974	20.75	0.0261	46.85	20.27	0.0260	46.27	20.46	0.0260	46.46
1981	18.41	0.0274	45.81	17.89	0.0291	46.99	18.01	0.0282	46.21
1991	14.72	0.0243	39.03	13.77	0.0249	38.67	14.18	0.0246	38.78

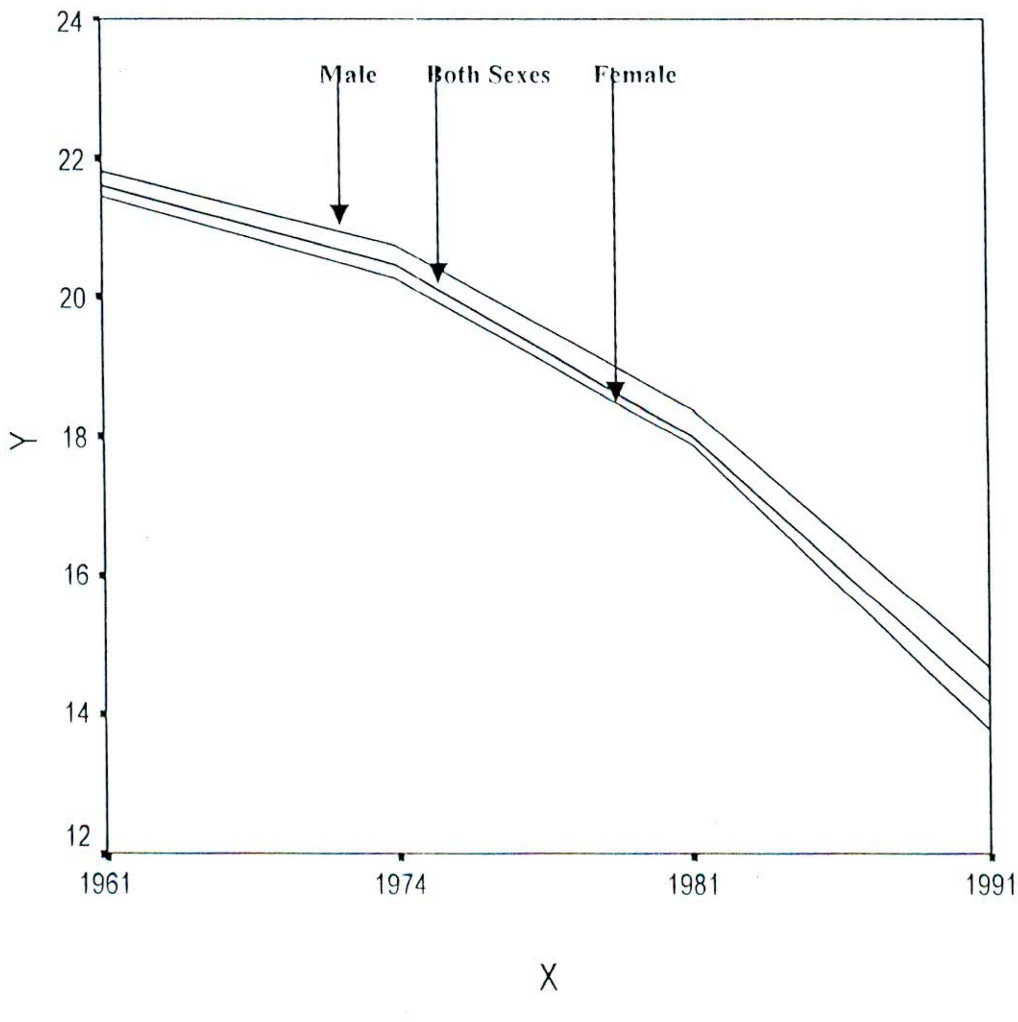


Figure 5.6 Crude Death Rate (CDR) for Male, Female and Both Sexes of Bangladesh in the Census Year 1961, 1974, 1981 and 1991. X: Census Years and Y: Crude Death Rate.

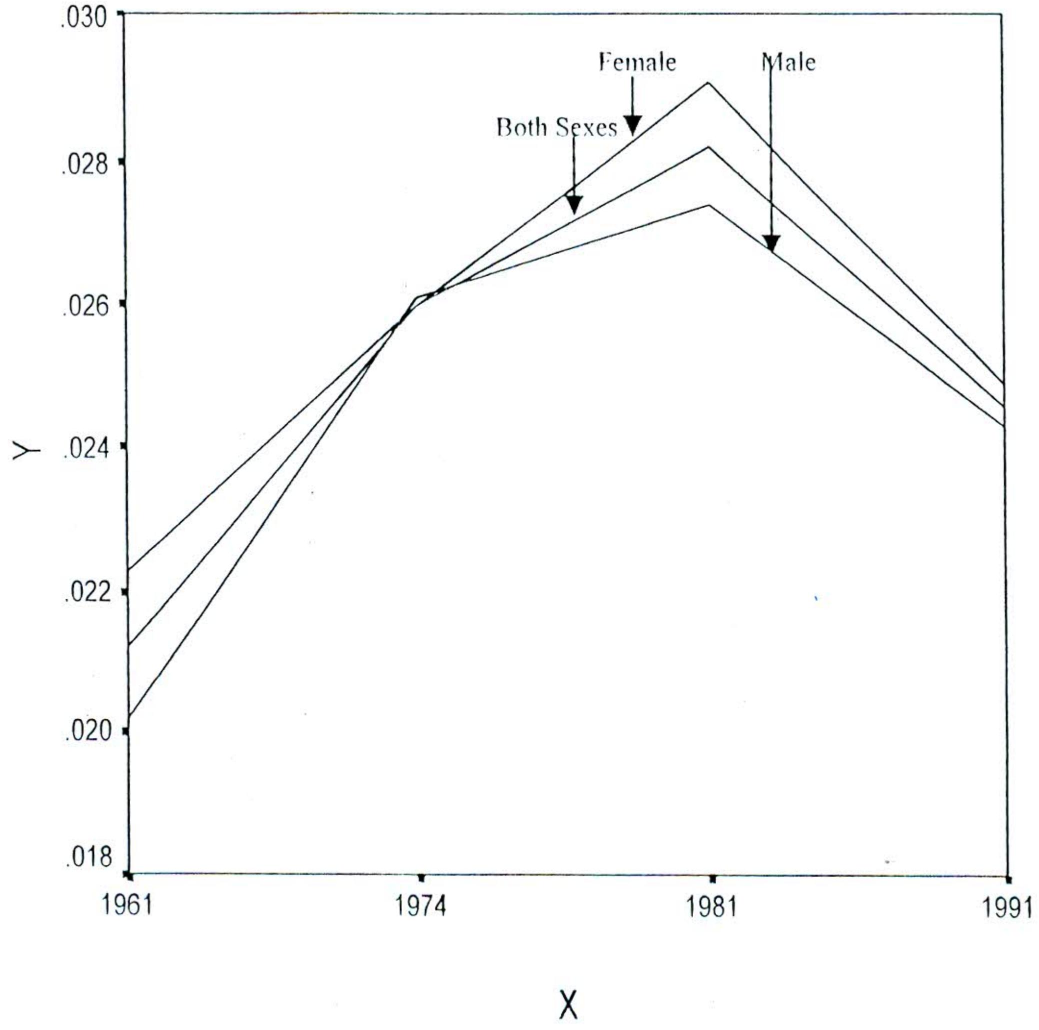


Figure 5.7 Growth Rate (r) for Male, Female and Both Sexes of Bangladesh in the Census Years 1961, 1974, 1981 and 1991. X: Census Years and Y: Growth Rate.

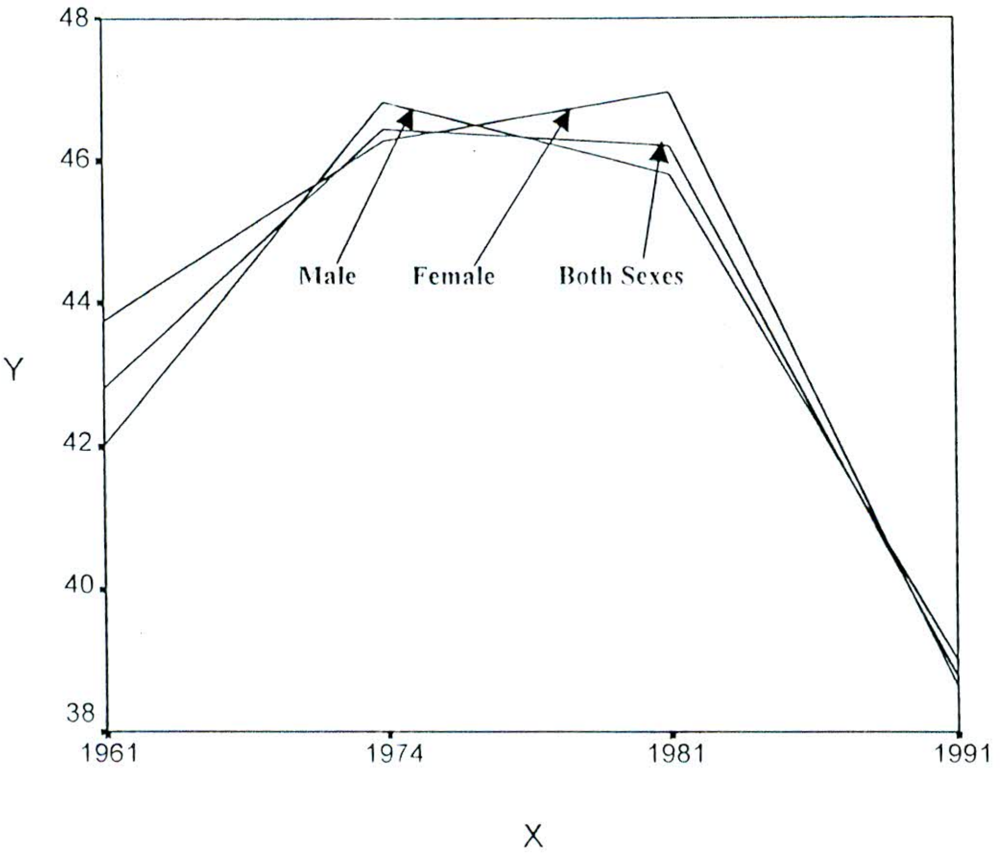


Figure 5.8 Crude Birth Rate (CBR) for Male, Female and Both Sexes of Bangladesh in the Census Years 1961, 1974, 1981 and 1991. X: Census Years and Y: Crude Birth Rate.

5.6 Estimation of Age Specific Fertility Rates (ASFR)

Age specific fertility rates are calculated separately at various ages in the reproductive life of women. The effectiveness of this rate is very strong in the analysis of fertility. An age specific fertility is defined as the number of births to women of a given age group per 1000 mid-year women in that age group. That is,

$$ASFR = \frac{B_a}{P_a^f} \times 1000$$

where B_a is the total number of births in the age group a to a+5 and P_a^f is the mid-year women in the same age group. But, in the present study ASFR have been estimated by indirect method. The complete procedure of estimating ASFR has been shortly discussed below:

To estimate the age specific fertility rates for the census years 1961, 1974, 1981 and 1991, proportion of married women have been taken from censuses and standard age pattern of female marital fertility rates have been taken from Manual-IV (UN, 1967). Mean age at fertility schedule $\overline{m}$ at the indicated census years have been estimated. Now, for $\overline{m}$ standard fertility schedule has been interpolated from Coale-Demeny Model Life Tables (1966). Multiplying this fertility schedule by the smoothed female population of the census year, the hypothetical female births assuming GRR as 1 has been estimated. Total hypothetical births is also found for the censuses applying the formula (hypothetical female birth) × 2.05 mentioning the sex ratio at birth as 1.05 (an average of 1961, 1974, 1981 and 1991 censuses). But, crude birth rate estimated before gives total births. The ASFR is obtained by multiplying the standard fertility schedule by

$\frac{(1+\text{sex ratio at birth})\times\text{total births}}{\text{hypothetical total births}}$. Thus, ASFR of Bangladesh at the indicated census year have been estimated and presented in Table 5.7 and depicted in Figure5.9

To see the trend of fertility, the data have been plotted in graph paper shown in Figure 5.9. From the figure, it is observed that all fertility schedules show the traditional (reciprocal of V-shape) pattern. It can also be observed that with passing of time the peaks of the curves are showing declining trend of fertility during 1961-1991 excepting 1961 census. That is, ASFR in 1974 is higher than that of 1961 at each age in the reproductive age group. Then, ASFR at each age strictly started to decrease during 1974-1981 slowly where as it rapidly commenced to decrease during 1981 census to 1991 census. It is observed that all these curves are unimodal. It is also found that the highest ASFRs in the age group 25-29 years were 354, 415, 389 and 221 per thousand for the years 1961, 1974, 1981 and 1991 respectively where as corresponding lowest ASFRs are 11, 15, 15, 10 in the last age group of the reproductive life respectively.

Table 5.7 Age Specific Fertility Rates (ASFR) of Bangladesh in the Census Years 1961, 1974, 1981 and 1991

Age Group in Years	Age Specific Fertility Rates (ASFR)			
	1961	1974	1981	1991
15-19	0.131385	0.132824	0.119225	0.058075
20-24	0.288695	0.308664	0.286265	0.153931
25-29	0.35409	0.414449	0.388549	0.220901
30-34	0.269555	0.327574	0.309873	0.184026
35-39	0.16588	0.209014	0.198513	0.120122
40-44	0.05423	0.076145	0.074444	0.050994
45-49	0.011165	0.015331	0.015131	0.010751

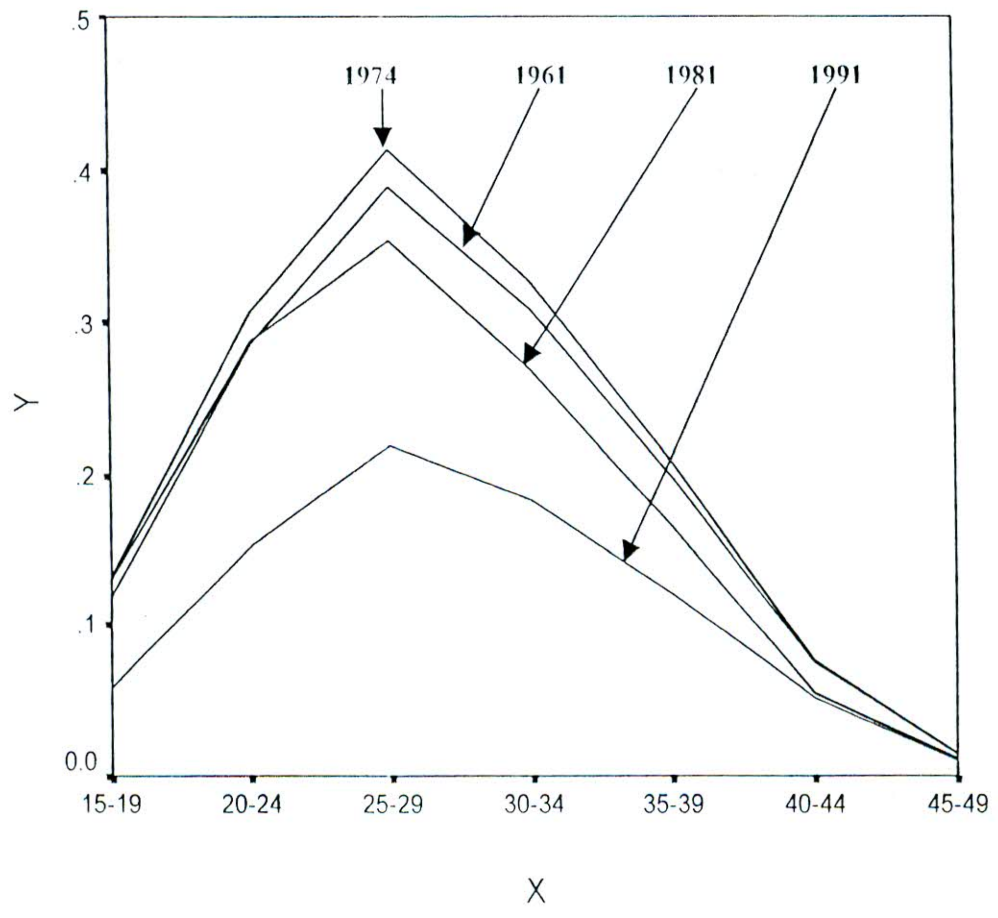


Figure 5.9 Age Specific Fertility Rates (ASFR) of Bangladesh in the Census Years 1961, 1974, 1981 and 1991. X: Age Group in Years and Y: Age Specific Fertility Rates.

5.8 Estimation of Total Fertility Rate (TFR), Gross Reproduction Rate (GRR) and Net Reproduction Rate (NRR)

Total fertility rates have been estimated using the traditional formula of

$$TFR = 5 \sum_{a=15-19}^{45-49} ASFR_a$$

under usual assumptions from the estimated ASFR which can be approximated by

$$TFR = \int_{a=15}^{49} f(a)\, da$$

Gross reproduction rate can be estimated using the formula

$$GRR = \frac{B^F}{B^T} \int_{a=15}^{49} f(a)\, da \quad \text{(Keyfitz, 1968)}$$

which can be approximated as

$$GRR = 5 \frac{B^F}{B^T} \sum_{a=15-19}^{45-49} f_a .$$

where $\frac{B^F}{B^T}$ is the proportion of all births which are female and f_a is the age specific fertility rate estimated before at the age group a to a +5 in the childbearing ages. To estimate GRR the sex ratio at birth is taken to be 1.05 (an average of 1961, 1974, 1981 and 1991).

Net reproduction rate can be estimated using the formula

$$NRR = \frac{B^F}{B^T} \int_{a=15}^{49} p(a)\, f(a)\, da \quad \text{(Keyfitz, 1968)}$$

which can be approximately estimated as

$$NRR = 5 \frac{B^F}{B^T} \sum_{a=15-19}^{45-49} p(a)\ f_a$$

where f_a is the age specific fertility rate estimated before at age a or age group a to a+5 and p(a) is the probability of surviving from birth to age a.

TFR, GRR and NRR have been estimated and presented in Table 5.8. To see the trends of them from the table, it is observed that they were increasing in the interval 1961 to 1974 years but they were decreasin1g in the interval 1974 to 1991 years. The rate of increment of TFR during the intercensal period 1961 to 1974 was 16.14% which was faster but it decreased at a speed at 6.07% during 1974-1981 and it rapidly decreased at a greater speed at 42.67% during 1981-1991 which was more faster than the previous intercensal period. The rate of increment of GRR during 1961-1974 was 16.08% which was faster but it started to decrease at a slower speed at 6.09% during 1974-1981 and at a greater speed at 42.77% during 1981-1991. The NRR increased at a larger rate of 17.82% during 1961 to 1991 but it started to decrease at a slower rate of 5.88% during 1974-1981 and at a greater rate of 32.14% during 1981 to 1991.

5.9 Estimation of Reproduction Survival Ratio (RSR)

Reproduction survival ratio can be approximately estimated using the formula

$$RSR = \frac{\int_{a=15}^{49} p(a)\ f(a)\ da}{\int_{a=15}^{49} f(a)\ da} \qquad \text{(Keyfitz,1968)}$$

where f(a) is the age specific fertility rate at age a and p(a) is the probability of surviving from birth to age a. It is the proportion of potential reproductivity that survives the effects of mortality.

RSR has been estimated and presented in Table 5.8. From the table it is observed that RSR was strictly increasing trend during 1961-1991. The rate of increment of RSR were 1.54%, 1.52% and 16.42% during the intercensal periods 1961-1974, 1974-1981 and 1981-1991 respectively in which the rate of increment during 1981-1991 was a greater speed than other intercensal periods.

5.10 Estimation of Replacement Index (J)

In the absence of birth statistics, replacement index is very effective and significant measure in the developing countries like Bangladesh measured from census statistics and life tables at the same year. The replacement index can be defined by the age distribution of actual female population in the childbearing ages and female life table population in the same ages for the same period, the replacement index has as its numerator, the ratio of children under age five years to females at the reproductive ages in the actual population and, as its denominator, the corresponding ratio in the stationary or life table population. Usually it is denoted by J and can be expressed in the more conventional demographic notation, that is

$$J = \frac{P_{0-4}}{P^{f}_{15-49}} \div \frac{L^{f}_{0-4} + \frac{B_m}{B_f} L^{m}_{0-4}}{L^{f}_{15-49}}$$ (Shryock, Siegel and Associates, 1975)

Where, P_{0-4} is the number of children under age 5 years, P^{f}_{15-49} is the female population in the reproductive ages 15 to 49 years, L^{f}_{0-4} is life table female population under age 5 years, L^{m}_{0-4} is life table male population under age 5 years, $\frac{B_m}{B_f}$ is the sex ratio at birth and L^{f}_{15-49} is the female life table population in the childbearing ages 15 to 49 years.

It is also to be mentioned here that the replacement index may be used as a substitute measure of net reproduction rate, where data are not available for the computation of the net reproduction rate.

The replacement index for the census years have been estimated and presented in Table 5.8. To see the trend, it is observed from the table that it was increasing trend in the intercensal period 1961-1974 and decreasing trend during 1974-1991. The rate of increment during 1961-1974 was 6.7% but the rate of decrement were 7.45% and 19.92% during the intercensal periods 1974-1981 and 1981-1991 respectively in which rate of decrement during 1981-1991 was more faster than previous period.

5.11 Estimation of Vital Index (VI)

The vital index is the ratio of the number of births to the number of deaths during a calendar year, times 100. It does indicate the extent to which the force of natality exceeds that of mortality at a given time.

The vital index for the census years have been estimated and presented in Table 5.8. The table reveals that VI was showing increasing trend during the study period. The rate of increment during 1961-1974, 1974-1981 and 1981-1991 were 14.65%, 13.22% and 6.62% respectively in which we see that the rate of increment were gradually decreasing with the increase of time.

Table 5.8 General Fertility Rate (GFR), Child Woman Ratio (CWR), Total Fertility Rate (TFR), Gross Reproduction Rate (GRR), Net Reproduction Rate (NRR), Reproduction Survival Ratio (RSR), Replacement Index (J) and Vital Index (VI)

Census	GFR	CWR	TFR	GRR	NRR	RSR	J	VI
1961	206	876	6.38	3.11	2.02	65	2.39	198
1974	229	833	7.41	3.61	2.38	66	2.55	227
1981	220	812	6.96	3.39	2.24	67	2.36	257
1991	126	758	3.99	1.94	1.52	78	1.89	274

In this chapter mortality measures IMR, CDR, ASDR and fertility measures CBR, ASFR, TFR, GRR, NRR, RI have been estimated using indirect techniques and GFR, CWR have been estimated using census data. Trends of the above demographic parameters have been exhibited in this chapter and those parameters will be used for construction of various models.

Chapter 6

Mathematical Modeling

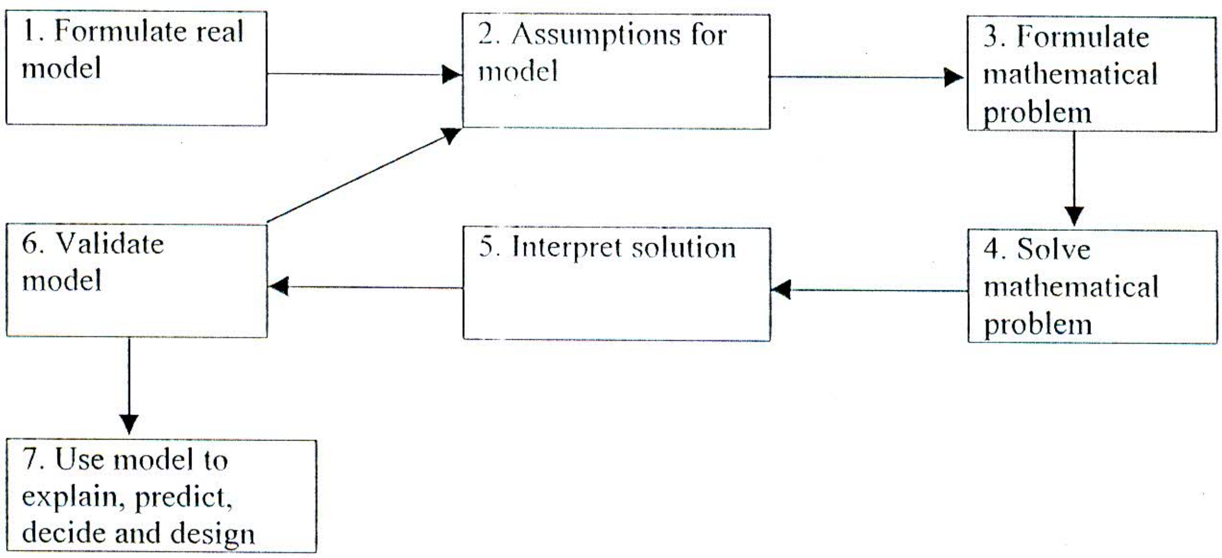
Chapter 6

Mathematical Modeling

6.1 Concept of Mathematical Modeling

Mathematical models would be developed in this chapter which lead to the differential equations and or functioning form of demographic parameters.

The principal phases in modeling problems in the real world are illustrated in the following diagram:



The problem may be for explaining observed data, or making some predictions, or taking a decision. To achieve this goal the real problem has been translated into a mathematical model under some assumptions. The variables used in mathematical modeling as important must be identified and relationship between them may be postulated. The relationships under assumptions construct the mathematical model. Then, an attempt has been made to validate the model to check that the theoretical solution is in good argument with the observations from the real situation. On the other hand, if the correlation between the theoretical and observed result do not fit adequate we must come back to assumptions made in the model and decide which need modifying or what either additions or subtractions should be made.

In the above diagram the far left-hand column represents the real world, the far right hand column the mathematical world and the linking between these two worlds is in the middle column. Most mathematics teaching aspects are solely concerned with box 4, nevertheless in applied mathematics is sometimes a token movement through the boxes 1, 2 and 3, but usually none back through the boxes 5, 6 and 7. The boxes are there just to give some notion of the underlying relationship between real world problems and the mathematical techniques applied to find out solutions to them. The important concepts to appreciate are the two translation stages, firstly form a real problem into a mathematical one through the model and secondly back from the mathematical solution to its interpretation in terms of the real problem (Andews and Mclone, 1976).

Mathematical models are of great help to demographers in understanding the process in distinguishing between important and unimportant variables, in studying the relationships between various events and population characteristics, in evaluation of data, and finally in making population projections and estimation. In demography, mathematical models mainly are of two types. These are stochastic models and deterministic models. Deterministic models are used to describe the functional relationship between variables which take definite values. These models are generally used with data at the aggregate level. Also, these models can provide the values of the unknown parameters of the model which are definite with probability one under the given circumstances. On the other hand, the variables in the stochastic models are in the form of probability distribution. In any case, stochastic models either involve the variables which follow probability distribution or an unknown error described in terms of probabilistic distribution. Life tables, population projections and estimation are examples of deterministic models. It is because, under the given schedule of vital events, definite values of life table function and of the future population size can be ascertained. Probability distribution of birth intervals that includes chance of conception in a month, models for the family size distributions under various stopping rules, birth-death and migration process are the examples of stochastic models.

In this study, deterministic models have only been introduced in this chapter. Moreover, deterministic models are also classified into two classes-stationary population models and time series population models. These two types of model have also been discussed in this section.

Thus, the purpose of this chapter has been mentioned below:

i) The mathematical models for age distribution and of ASDR for male, female and both sexes in the census years 1961, 1974, 1981 and 1991 have been constructed in this chapter,

ii) The mathematical modeling of l_x values for male and female and of ASFR in the indicated census years have also been discussed here,

iii) Time trend models of IMR, CDR, e_0, CBR, TFR, GRR, NRR and

iv) Cross validity prediction power (CVPP), ρ^2_{cv}, has been applied on these models to verify the stability of the fitted models.

It should be mentioned here that all these mathematical models studied in this chapter have been estimated using the software STATISTICA and Econometric views.

6.2 Model Validation Technique

To check, how much those models are stable over the population, the cross validity prediction power (CVPP), ρ^2_{cv}, is applied.

$$\text{Here } \rho^2_{cv} = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}(1-R^2)$$

where, n is the sample size or number of cases, k is the number of predictors in the model and the cross-validated R is the correlation between observed and predicted values of the dependent variable. Using the above statistics, it can be concluded that if the prediction equation is applied to many other samples from the same population, then (ρ^2_{cv} X 100) % of the variance on the predicted variable would be explained by the model (Stevens, 1996).

6.3 Mathematical Models for Age Distribution (Male, Female and Both Sexes)

It is evident that the population is a function of age or age groups. Using the scattered plot of ages and population from the Figures 6.1(a) to Figure 6.4(c), it is observed that population for the census years 1961, 1981 and 1991 follow modified negative exponential distribution and population for the census year 1974 follows negative exponential distribution. Therefore, for age distribution of population with respect to ages, a negative exponential model and modified negative exponential model may be considered. The models are

$$y = c + e^{-(ax+b)} + u$$

$$\text{or } y = e^{-(ax+b)} + u$$

where, x represents the age group in years; y represents population; a, b, c are parameters treated as constants and u is the stochastic error term of the model.

The fitted equations are as follows:

y = (904.034)+exp(-(0.0622303)x+(8.23487)) for male in 1961 (1)

y = (637.141)+exp(-(0.056758)x+(8.307225)) for female in 1961 (2)

y = (1551.106)+exp(-(0.059452)x+(8.96384)) for both sexes in 1961 (3)

y = exp(-(0.0353837)x+(8.740813)) for male in 1974 (4)

y = exp(-(0.0382171)x+(8.72389)) for female in 1974 (5)

y = exp(-(0.0369396)x+(9.42882)) for both sexes in 1974 (6)

y = 542.3384+exp(-(0.0408924)x+(8.93262)) for male in 1981 (7)

y = 331.367+exp(-(0.0401485)x+(8.92893)) for female in 1981 (8)

y = 816.718+exp(-(0.0398995)x+(9.627715)) for both sexes in 1981 (9)

y = -489.5943+exp(-(0.0315718)x+(9.226313)) for male in 1991 (10)

y = -755.1627+exp(-(0.0299607)x+(9.214934)) for female in 1991 (11)

y = -1269.7+exp(-(0.0305276)x+(9.912446)) for both sexes in 1991 (12)

The estimated CVPP, ρ^2_{cv}, corresponding to their R^2 is shown in Table 6.1 From this table we see that all the fitted models in equation (1) to equation (12) are highly cross-validated and their shrinkage`s are very small. These imply that the fitted models are more than 98% stable. In all the cases, the stability of R^2 is about more than 99%.

The information on model fitting has been presented in Table 6.2. From Table 6.2, it is shown that all the parameters of the fitted models are highly significant with large significant proportion of variance explained.

Table 6.1 Estimated Cross Validity Prediction Power (ρ^2_{cv}) of the Predicted Equations of Age Distribution for Male, Female and Both Sexes of Bangladesh in the Census Years 1961, 1974, 1981 and 1991

Models	n	k	R^2	ρ^2_{cv}	Shrinkage
Equation 1	13	1	.98497	.980577	0.004393
Equation 2	13	1	.98972	.986715	0.003005
Equation 3	13	1	.98860	.985268	0.003332
Equation 4	16	1	.98412	.980532	0.003588
Equation 5	16	1	.99071	.988611	0.002099
Equation 6	16	1	.98823	.985570	0.00266
Equation 7	14	1	.98982	.987110	0.00271
Equation 8	14	1	.99501	.993681	0.001329
Equation 9	14	1	.99296	.991086	0.001874
Equation 10	17	1	.99541	.994446	0.000964
Equation 11	17	1	.99771	.997229	0.000481
Equation 12	17	1	.99731	.996745	0.000565

Table 6.2 Information on Model Fitting for the Population by Age Distribution of Bangladesh

Models	Proportion of Variance Explained	Parameters	Significant Probability (p)
Model 1	0.98497		
		a	0.000003
		b	0.00000
		c	0.00000
Model 2	0.98972		
		a	0.000002
		b	0.000
		c	0.0002
Model 3	0.98860		
		a	0.000
		b	0.000
		c	0.000
Model 4	0.98412		
		a	0.000
		b	0.000

Model 5	0.99071		
		a	0.000
		b	0.000
Model 6	0.98823		
		a	0.000
		b	0.000
Model 7	0.98982		
		a	0.000
		b	0.000
		c	0.0344
Model 8	0.9950		
		a	0.000
		b	0.000
		c	0.0411
Model 9	0.99296		
		a	0.000
		b	0.000
		c	0.0432
Model 10	0.99541		
		a	0.000
		b	0.000
		c	0.027
Model 11	0.99771		
		a	0.000
		b	0.000
		c	0.000
Model 12	0.99731		
		a	0.000
		b	0.000
		c	0.000

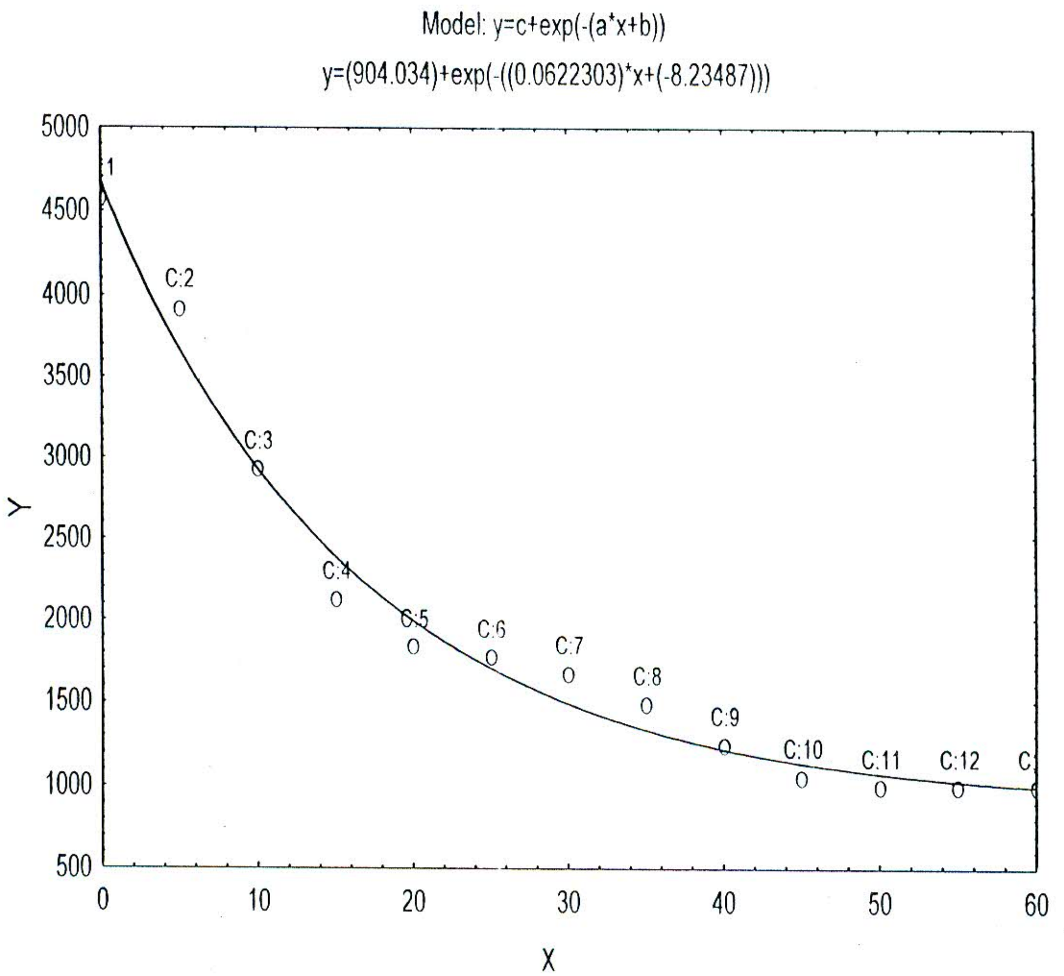


Figure 6.1(a) Observed and Fitted Model for Male Age Distribution of Bangladesh in 1961. X: Age Group in Years and Y: Male Population.

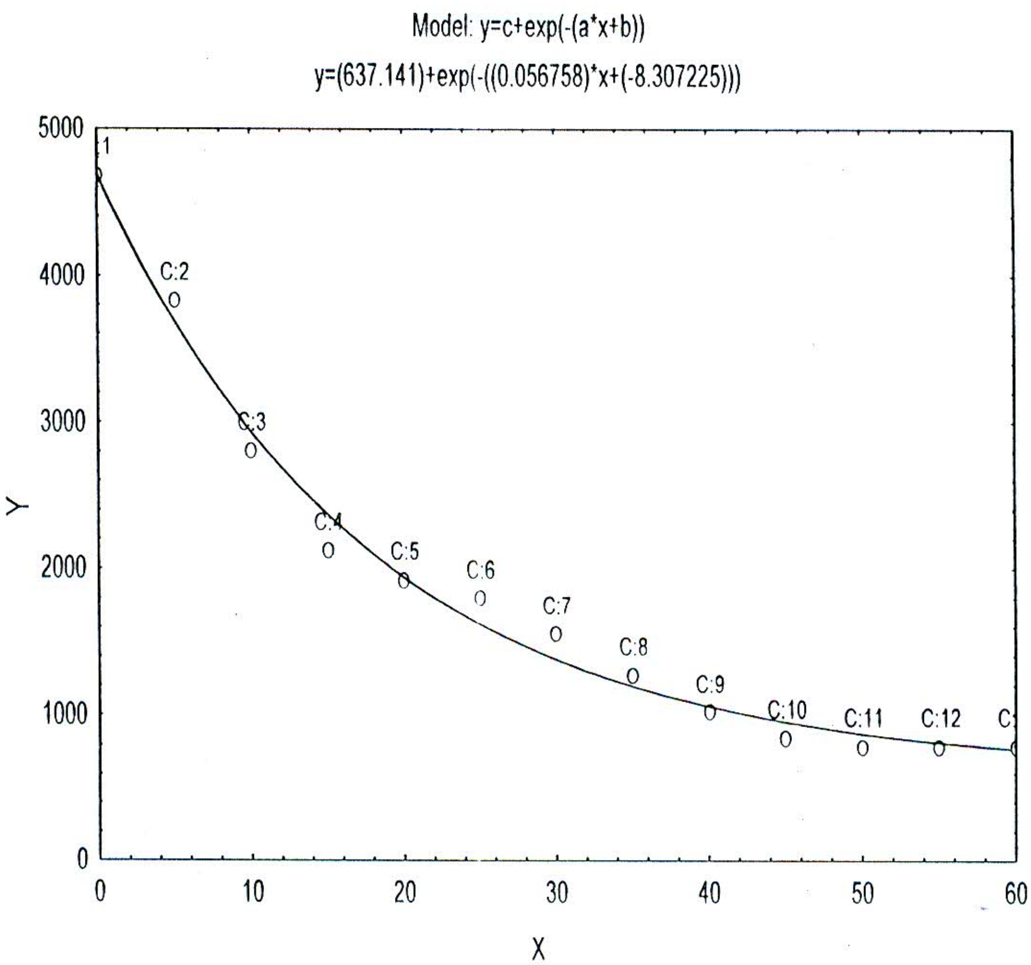


Figure 6.1(b) Observed and Fitted Model for Female Age Distribution of Bangladesh in 1961. X: Age Group in Years and Y: Female Population.

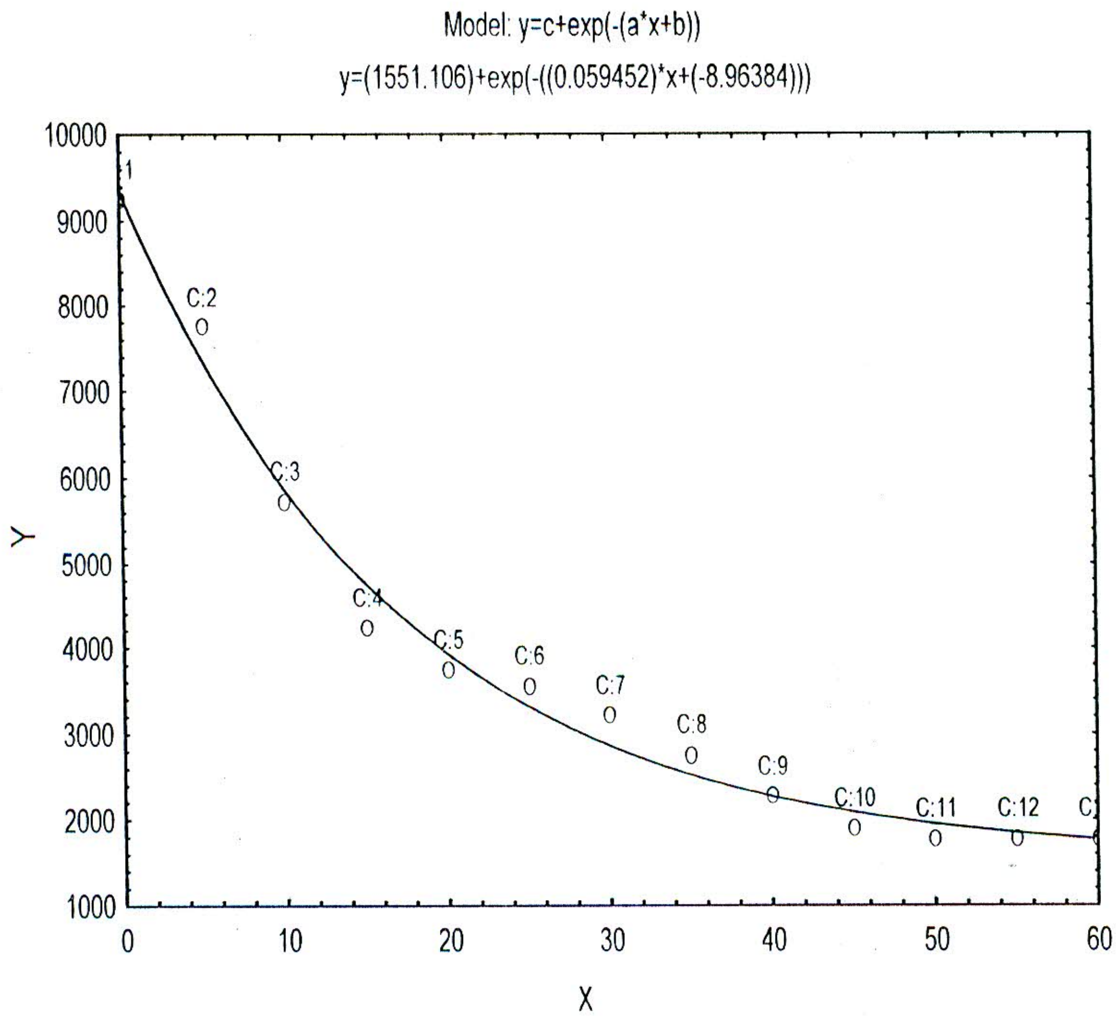


Figure 6.1(c) Observed and Fitted Model for Age Distribution of Both Sexes of Bangladesh in 1961. X: Age Group in Years and Y: Population for Both Sexes.

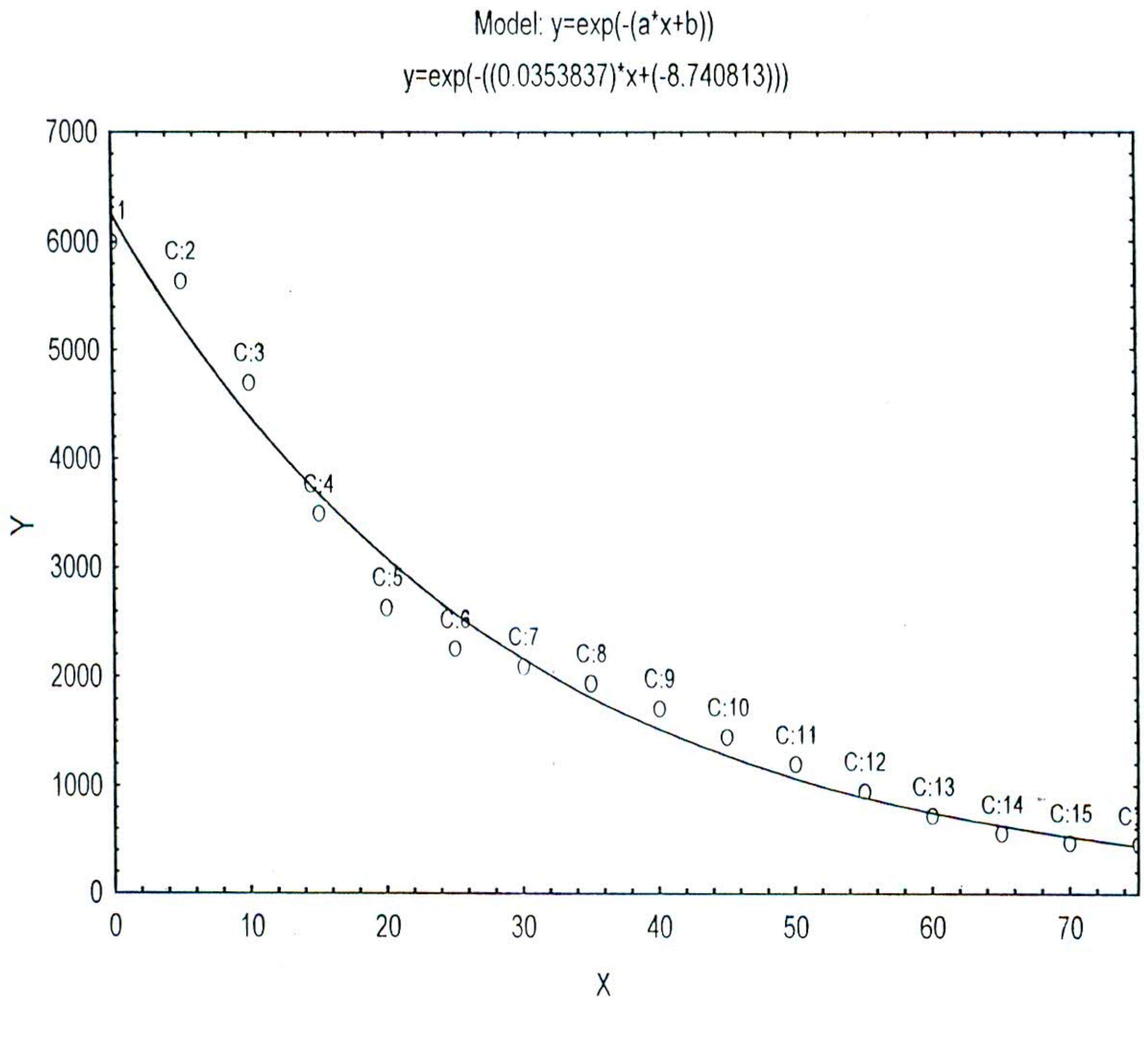


Figure 6.2(a) Observed and Fitted Model for Male Age Distribution of Bangladesh in 1974. X: Age Group in Years and Y: Male Population.

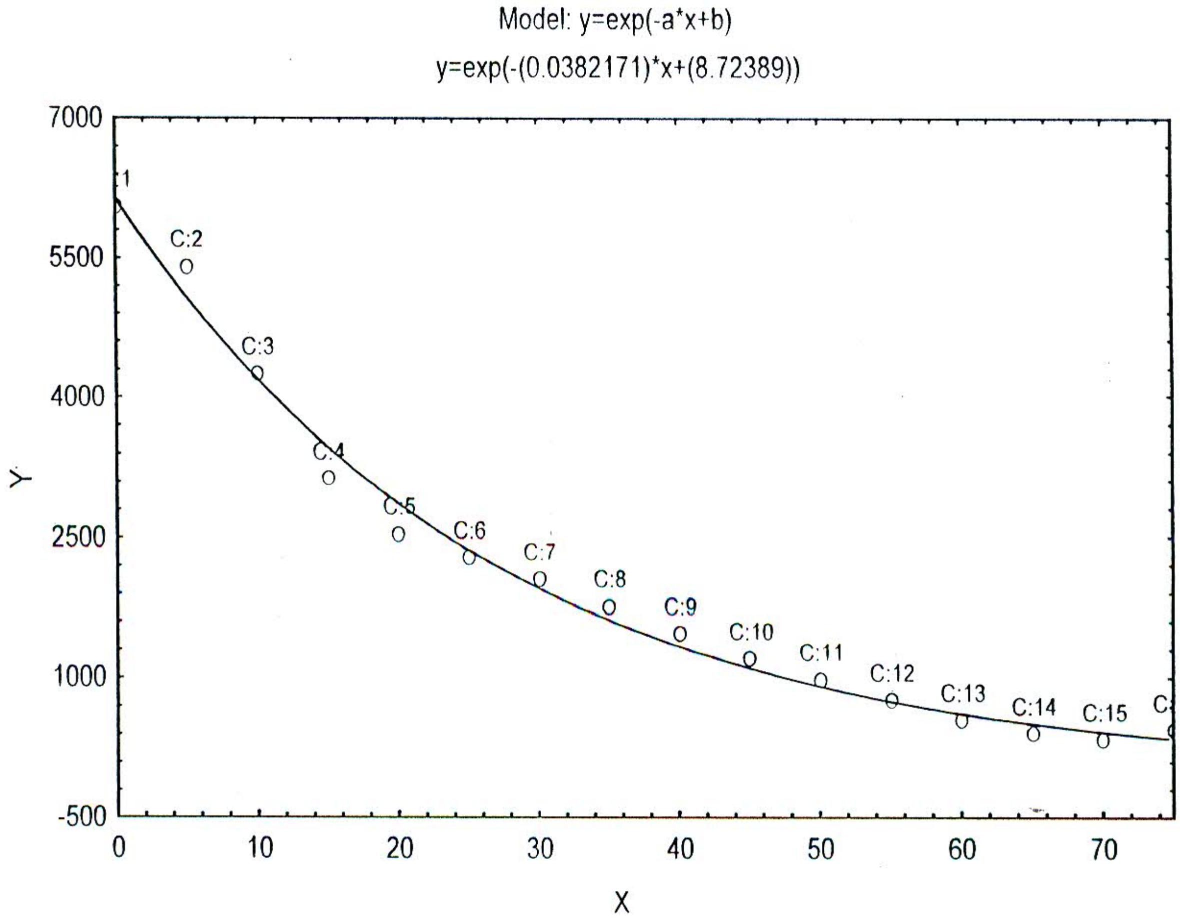


Figure 6.2(b) Observed and Fitted Model for Female Age Distribution of Bangladesh in 1974. X: Age Group in Years and Y: Female Population.

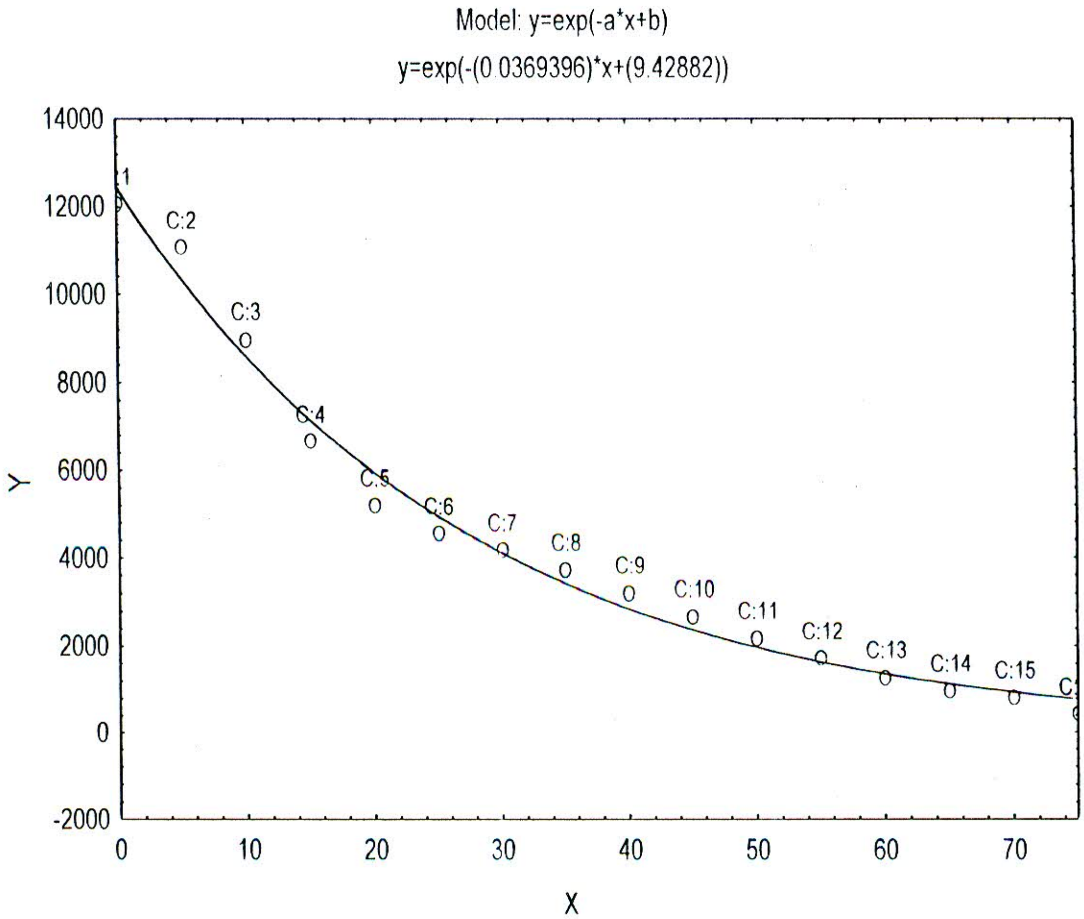


Figure 6.2(c) Observed and Fitted Model for Age Distribution of Both Sexes of Bangladesh in 1974. X: Age Group in Years and Y: Population for Both Sexes.

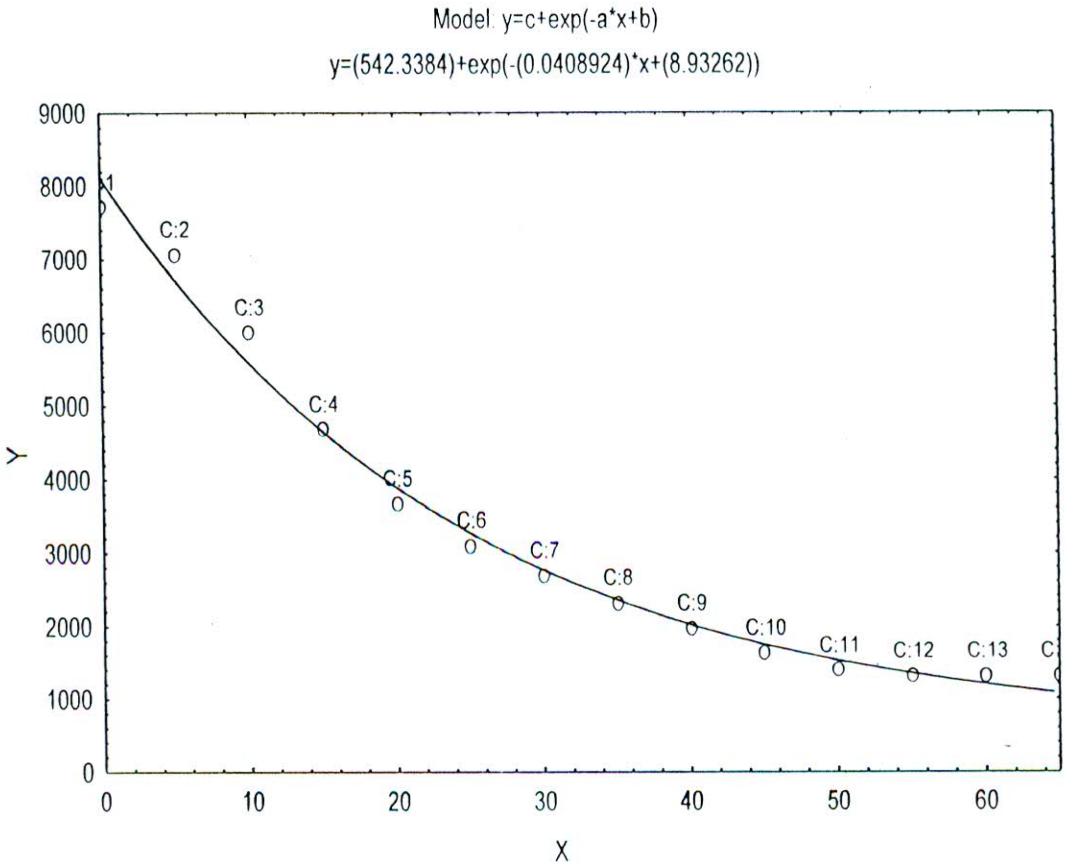


Figure 6.3(a) Observed and Fitted Model for Male Age Distribution of Bangladesh in 1981. X: Age Group in Years and Y: Male Population.

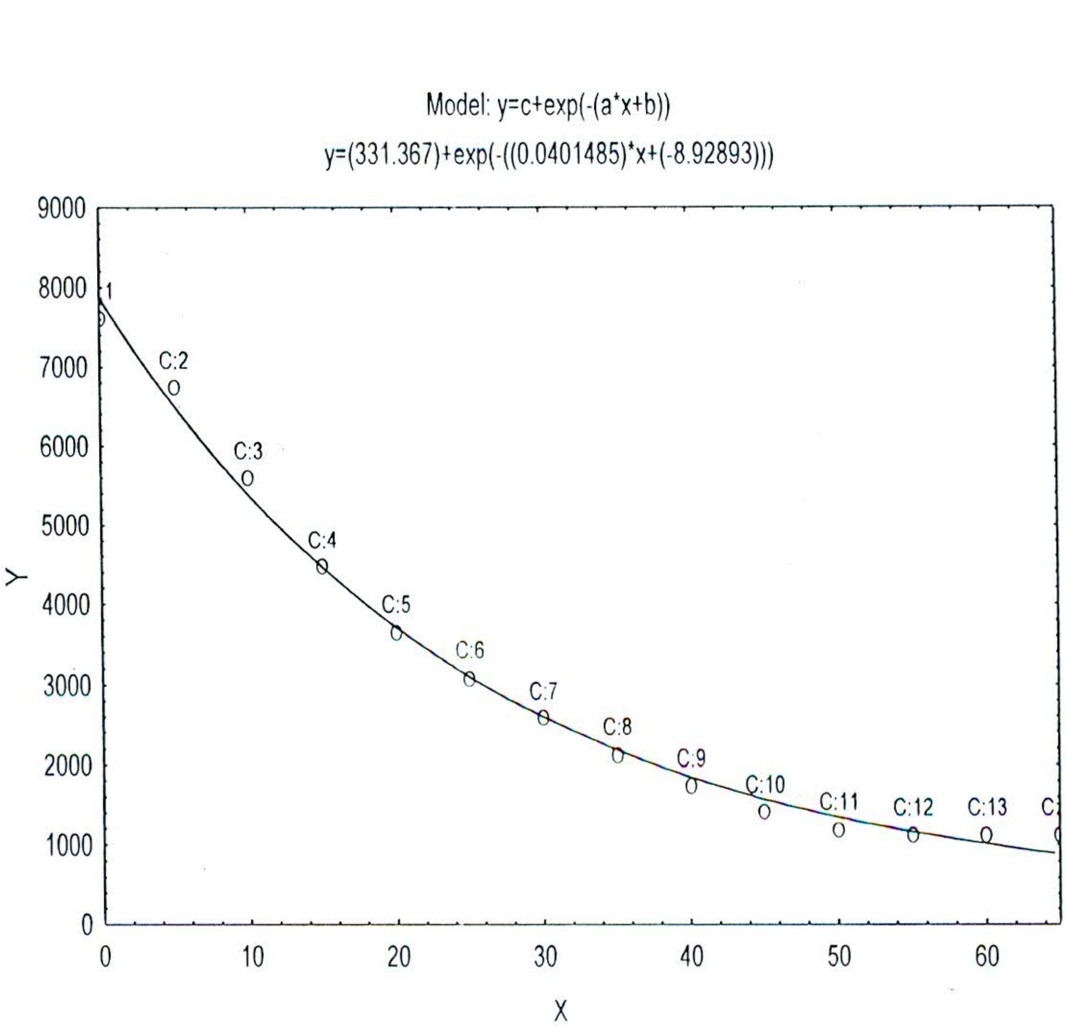


Figure 6.3(b) Observed and Fitted Model for Female Age Distribution of Bangladesh in 1981. X: Age Group in Years and Y: Female Population.

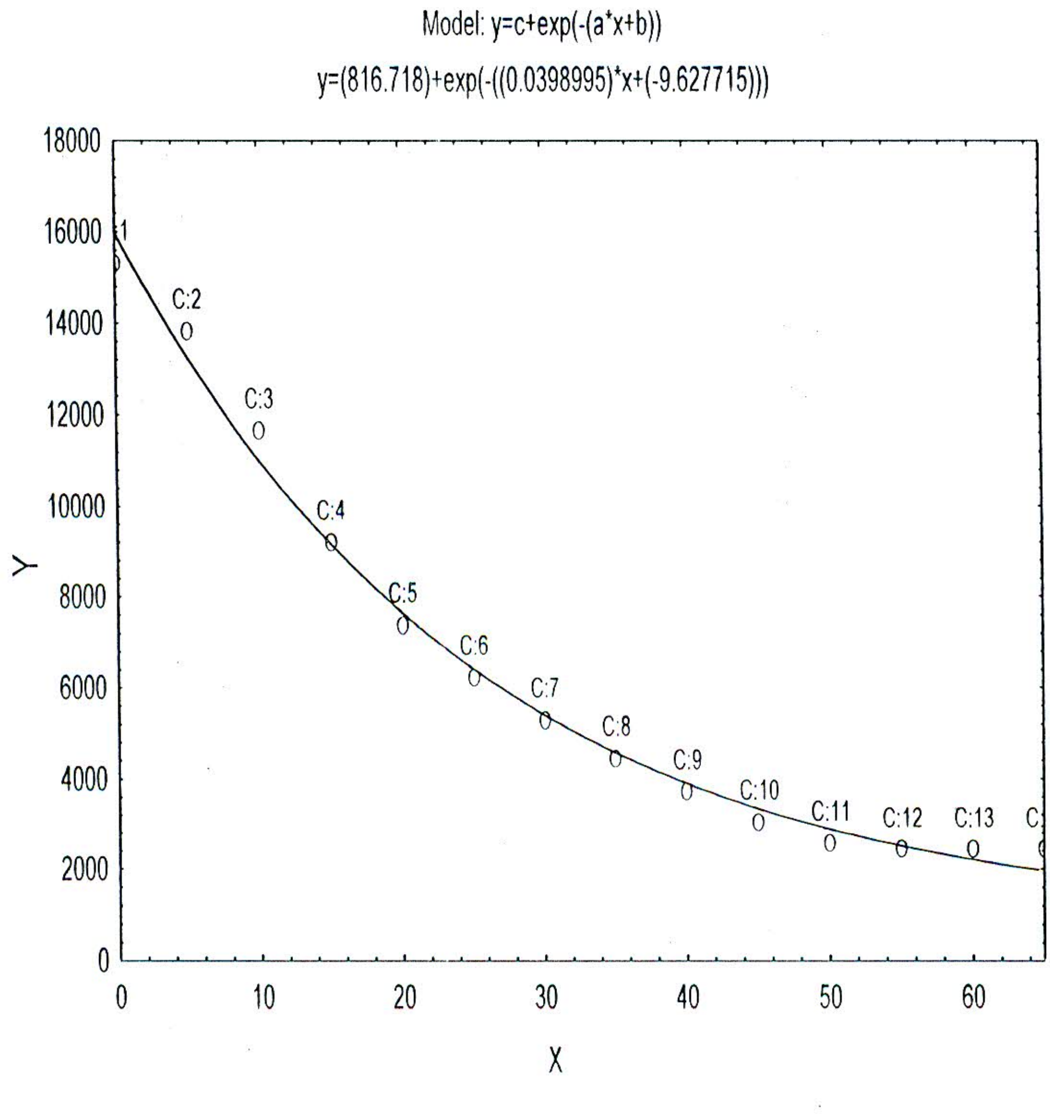


Figure 6.3(c) Observed and Fitted Model for Both Sexes of Age Distribution of Bangladesh in 1981. X: Age Group in Years and Y: Population for Both Sexes.

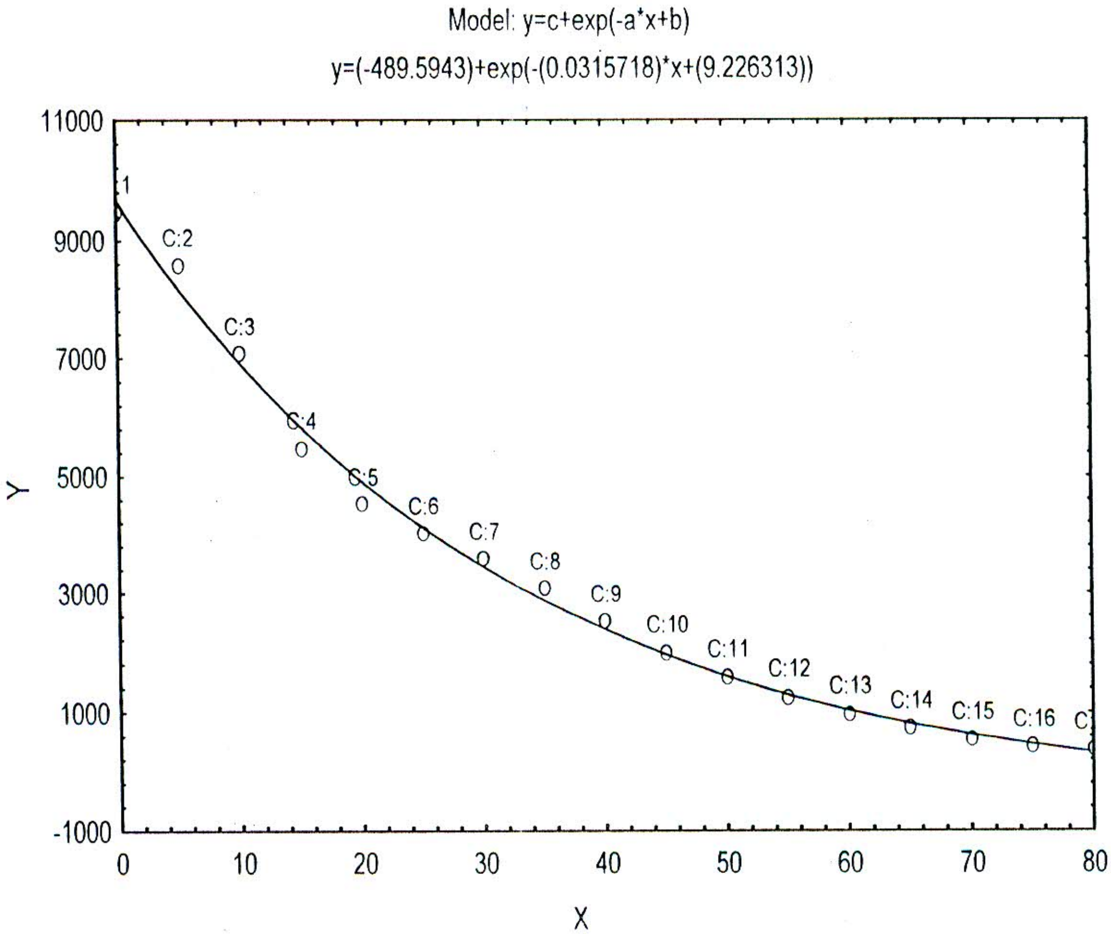


Figure 6.4(a) Observed and Fitted Model for Male Age Distribution of Bangladesh in 1991. X: Age Group in Years and Y: Male Population.

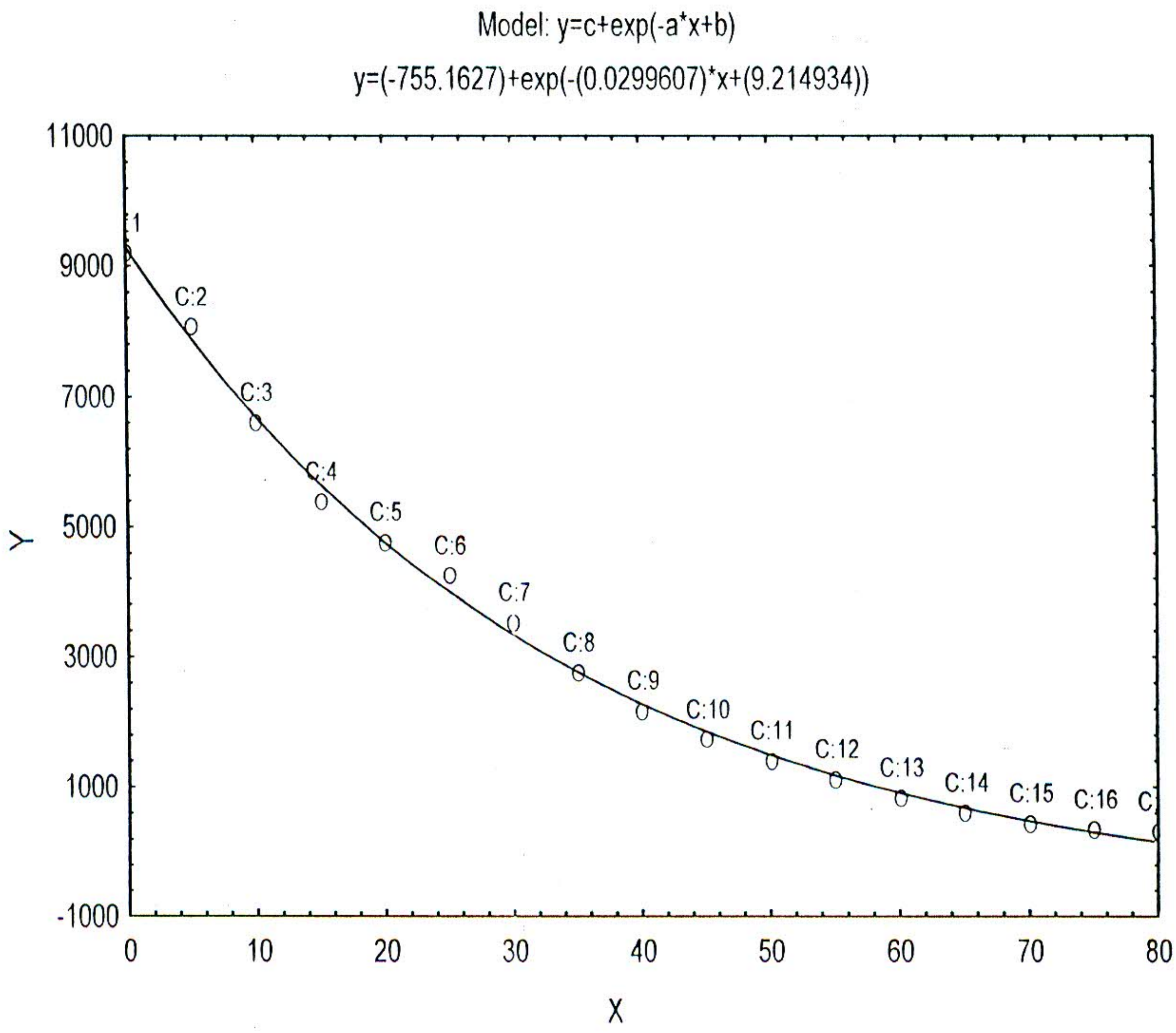


Figure 6.4(b) Observed and Fitted Model for Female Age Distribution of Bangladesh in 1991. X: Age Group in Years and Y: Female Population.

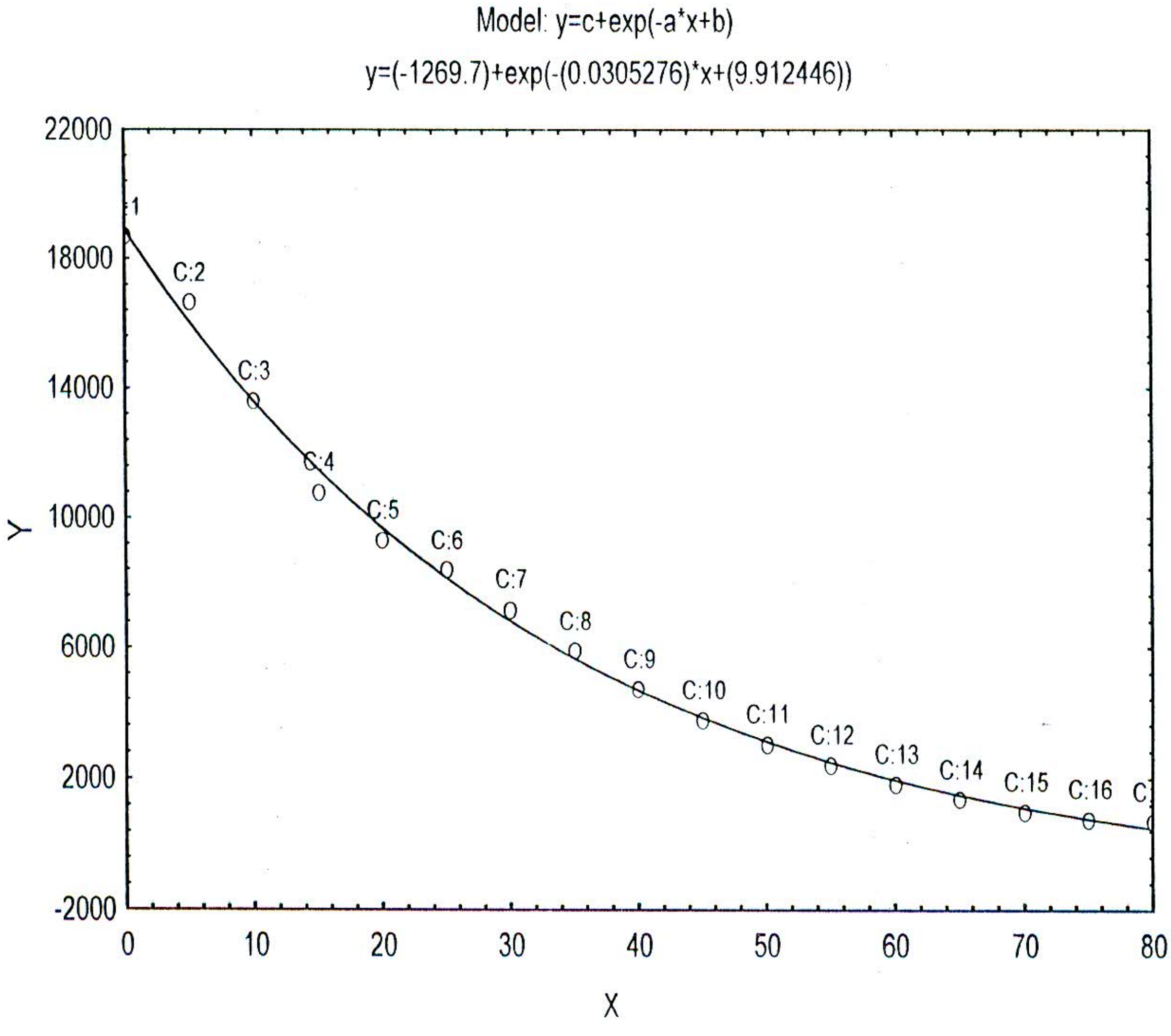


Figure 6.4(c) Observed and Fitted Model for Age Distribution of Both Sexes of Bangladesh in 1991. X: Age Group in Years and Y: Population for Both Sexes.

6.4 Mathematical Modeling of the Number of Surviving Functions (l_x) at an Exact Age x for Male and Female by Age Groups

Using the dotted plot of ages and the number of persons surviving at exact age x (l_x) from the Figure 6.5(a) to Figure 6.8(b), it has been observed that l_x can be fitted by the nth degree polynomial for different ages. Therefore, an nth degree polynomial model is treated and the model of the nth degree polynomial is

$$y = a_0 + \sum_{i=1}^{n} a_i x^i + u$$

where, x is age group in years; y is the number of persons surviving at an exact age x (l_x); a_0 is the constant; a_i is the coefficient of x^i (i=1, 2, 3,......., n) and u is the stochastic error term of the model. Here a suitable n has been selected for which the error sum of square is minimum.

The fitted equations of surviving functions (l_x) at an exact age x of Bangladesh are as follows:

$y = (87765.9)+(-3092.49)x+(144.4443)x^2+(-2.64902)x^3+(0.0147286)x^4$ for male in 1961 (1)

$y = (88381.2)+(-2952.33)x+(128.686)x^2+(-2.261755)x^3+(0.0120718)x^4$ for female in 1961 (2)

$y = (87067.6)+(-2968.473)x+(139.309)x^2+(-2.536965)x^3+(0.0140377)x^4$ for male in 1974 (3)

$y = (87392.8)+(-2831.575)x+(125.604)x^2+(-2.202063)x^3+(0.0117595)x^4$ for female in 1974 (4)

$y = (90119.23)+(-2611.484)x+(124.705)x^2+(-2.30211)x^3+(0.0125538)x^4$ for male in 1981 (5)

$y = (90384.96)+(-2405.437)x+(107.2004)x^2+(-1.88042)x^3+(0.0096983)x^4$ for female in 1981 (6)

$y = (94250.3)+(-2098.563)x+(107.9227)x^2+(-2.06423)x^3+(0.0111536)x^4$ for male in 1991 (7)

$y = (92579.5)+(-1901.99)x+(87.51506)x^2+(-1.53554)x^3+(0.0076038)x^4$ for female in 1991 (8)

The estimated CVPP, ρ^2_{cv}, corresponding to their R^2 is shown in Table 6.3. From the table, it is seen that all the fitted models in equation (1) to equation (8) are cross-validated more than 0.96557 and their shrinkages are also reasonably small quantity. In this case, it is found that six models are 95% stable and others two are more than 93% stable. In all the cases, the stability of R^2 is about more than 98%.

The information on model fitting for l_x has been presented in Table 6.4. From this table, it is observed that all the parameters of the fitted models are highly significant with more than 96% of variance explained.

Table 6.3 Estimated Cross Validity Prediction Power (ρ^2_{cv}) of the Predicted Equations of Number of Persons Surviving (l_x) at Exact Age x for Male and Female of Bangladesh in 1961, 1974, 1981 and 1991

Models	n	k	R^2	ρ^2_{cv}	Shrinkage
Equation 1	21	4	0.97376	0.956475	0.017285
Equation 2	21	4	0.97408	0.957006	0.017074
Equation 3	21	4	0.96557	0.94289	0.02268
Equation 4	21	4	0.9635	0.939456	0.024044
Equation 5	21	4	0.98109	0.968633	0.012457
Equation 6	21	4	0.97835	0.964088	0.014262
Equation 7	21	4	0.98911	0.981936	0.007174
Equation 8	21	4	0.98194	0.970043	0.011897

Table 6.4 Information on Model Fitting of Surviving Functions (l_x) of Bangladesh

Models	Proportion of Variance Explained	Parameters	Significant Probability (p)
Model 1	0.97376		
		a_0	0.00000
		a_1	0.00000
		a_2	0.00050
		a_3	0.00084
		a_4	0.002204
Model 2	0.97408		
		a_0	0.0000
		a_1	0.0000
		a_2	0.0011
		a_3	0.0025
		a_4	0.0075
Model 3	0.96557		
		a_0	0.0000
		a_1	0.0000
		a_2	0.0010
		a_3	0.00176
		a_4	0.004413
Model 4	0.9635		
		a_0	0.0000
		a_1	0.0000
		a_2	0.0022
		a_3	0.0047
		a_4	0.01285
Model 5	0.98109		
		a_0	0.0000
		a_1	0.0000
		a_2	0.0003
		a_3	0.00056
		a_4	0.001775
Model 6	0.97835		
		a_0	0.0000
		a_1	0.0000
		a_2	0.0013
		a_3	0.00283
		a_4	0.010391
Model 7	0.98911		
		a_0	0.0000
		a_1	0.0000
		a_2	0.0001
		a_3	0.00015
		a_4	0.000604
Model 8	0.98194		
		a_0	0.0000
		a_1	0.0000
		a_2	0.00156
		a_3	0.00343
		a_4	0.01529

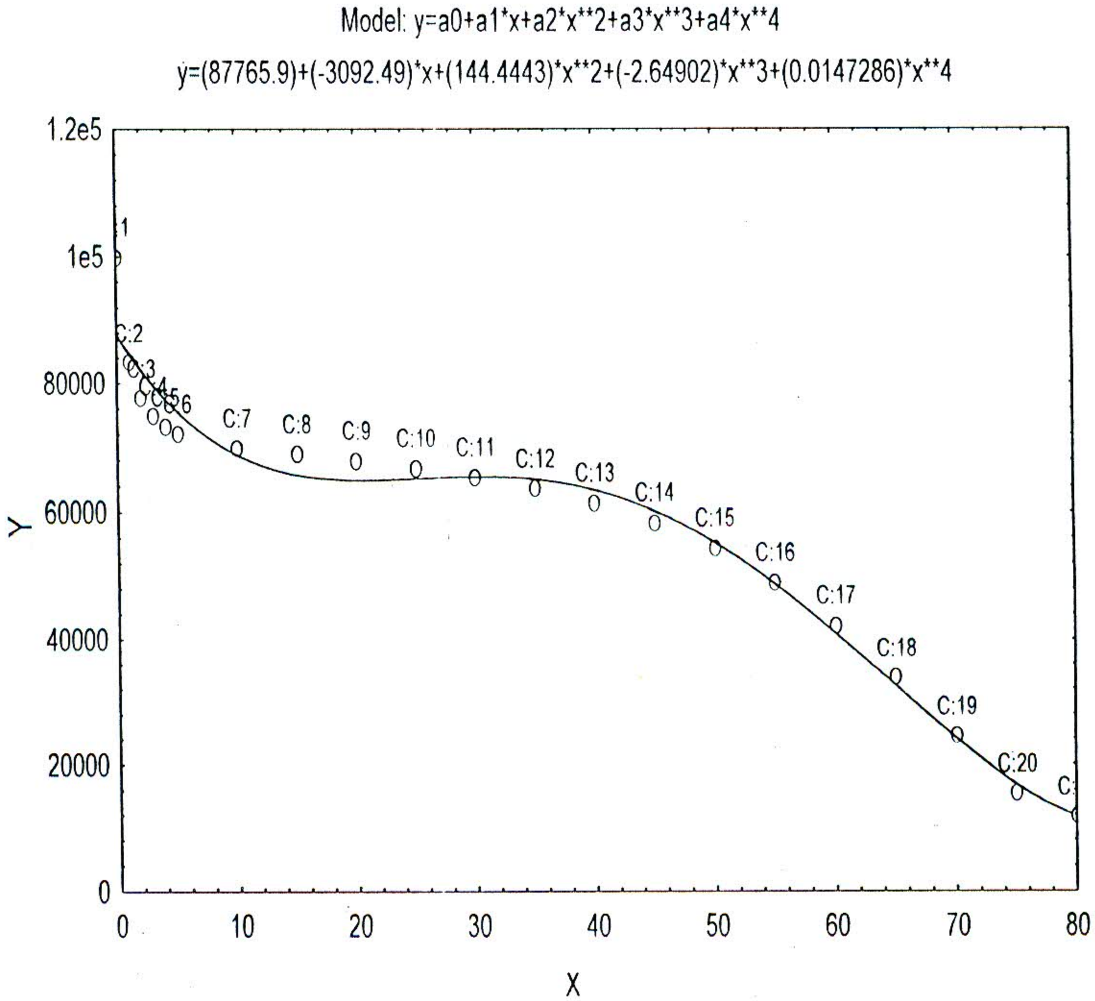


Figure 6.5(a) Observed and Fitted Model of Survival Function (l_x) For Male of Bangladesh in 1961. X: Age Group in Years and Y: Survival Function.

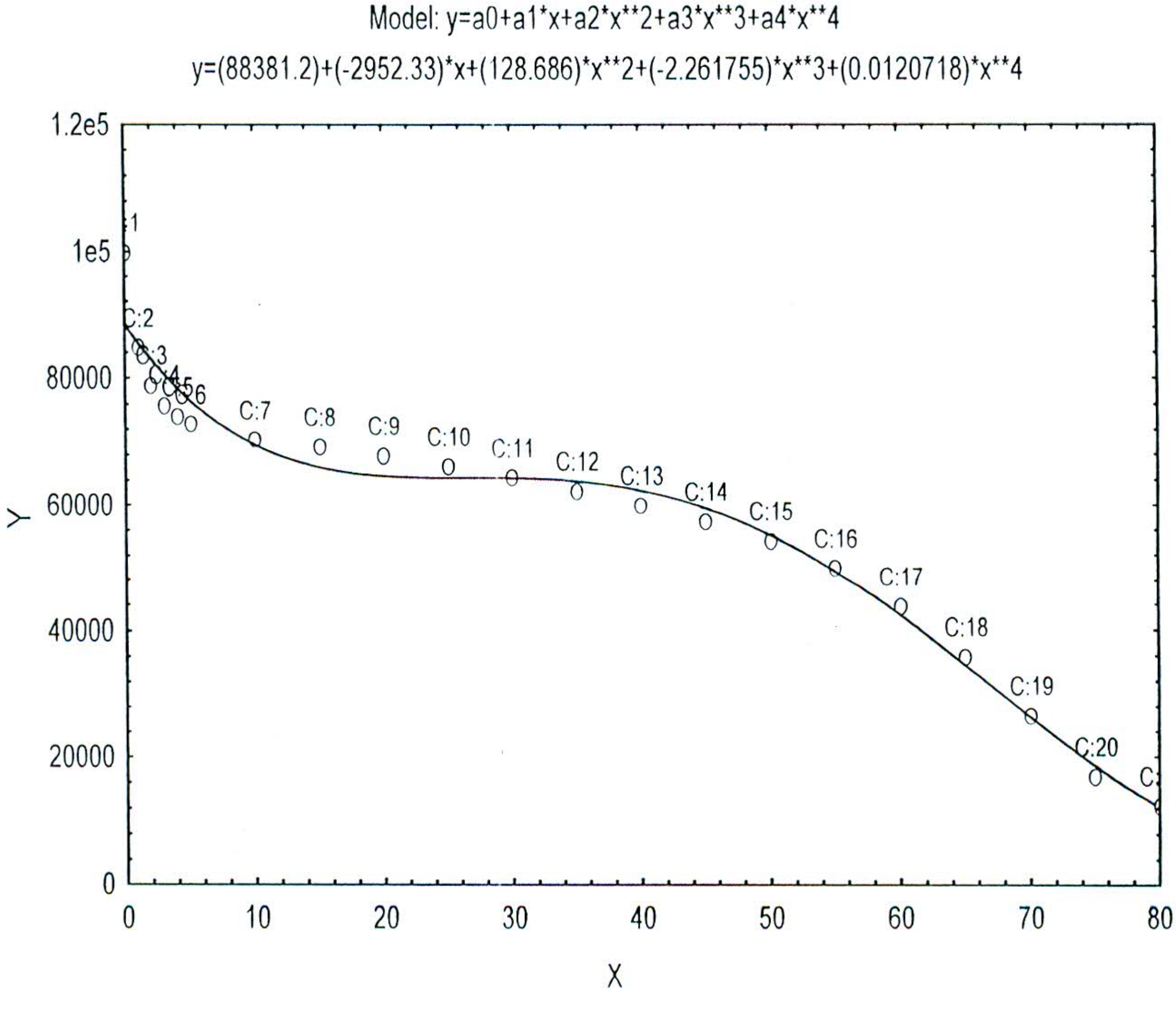


Figure 6.5(b) Observed and Fitted Model of Survival Function (l_x) For Female of Bangladesh in 1961. X: Age Group in Years and Y: Survival Function.

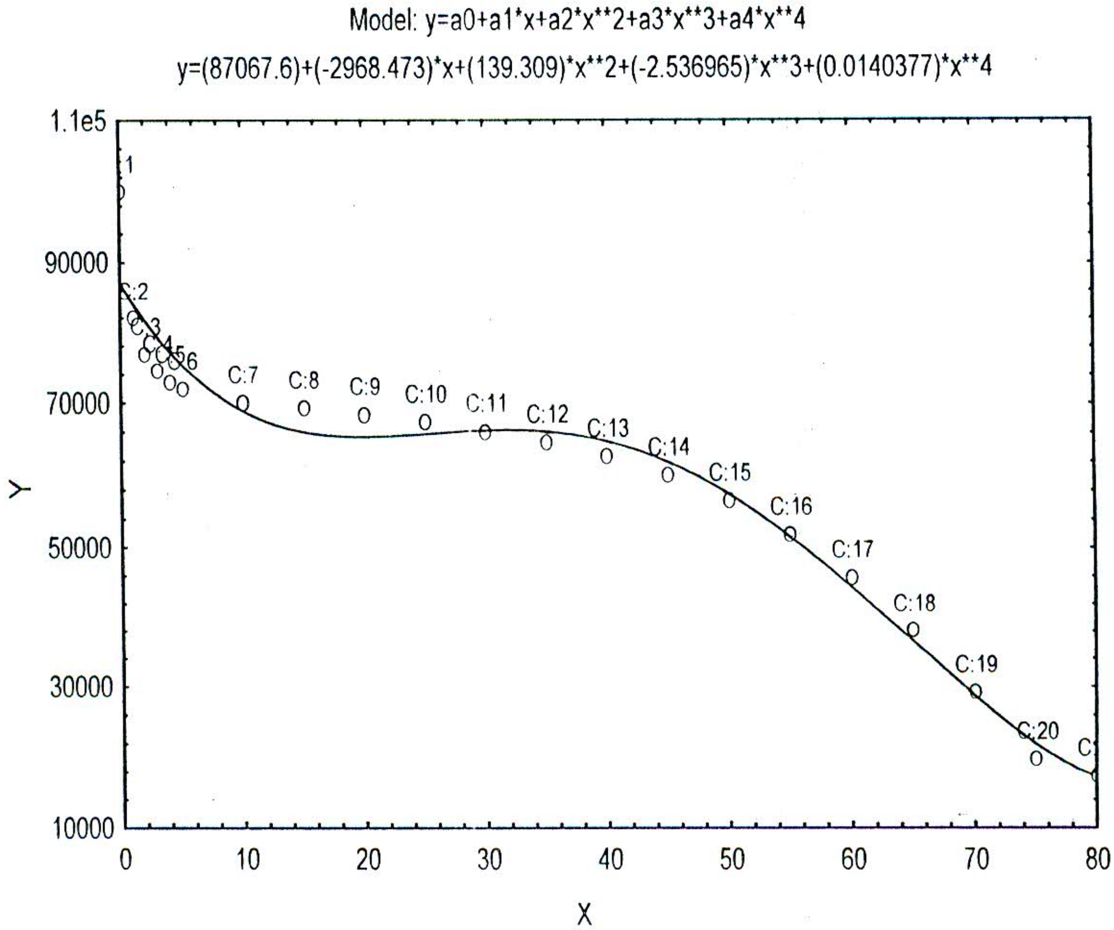


Figure 6.6(a) Observed and Fitted Model of Survival Function (l_x) For Male of Bangladesh in 1974. X: Age Group in Years and Y: Survival Function.

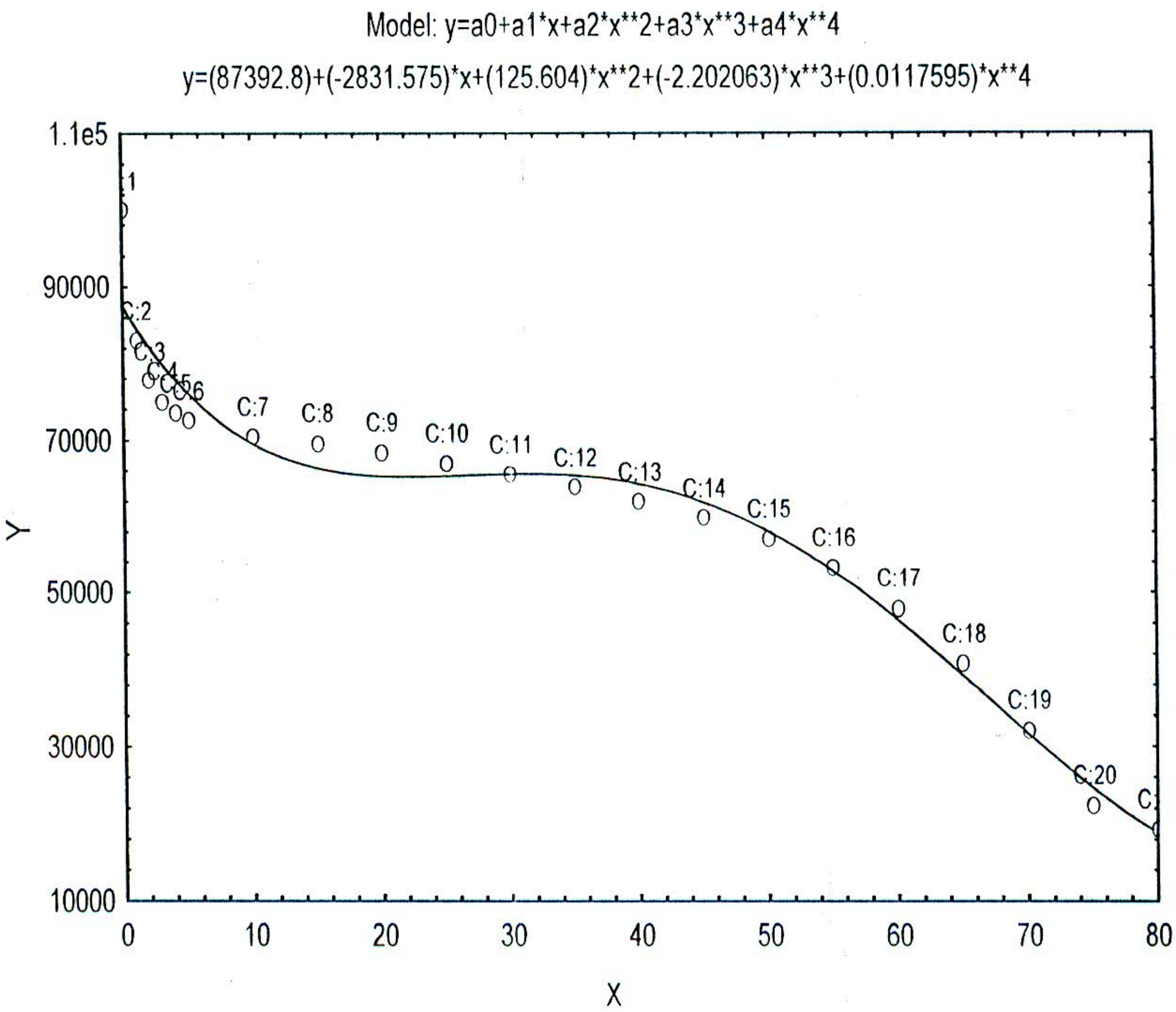


Figure 6.6(b) Observed and Fitted Model of Survival Function (l_x) For Female of Bangladesh in 1974. X: Age Group in Years and Y: Survival Function.

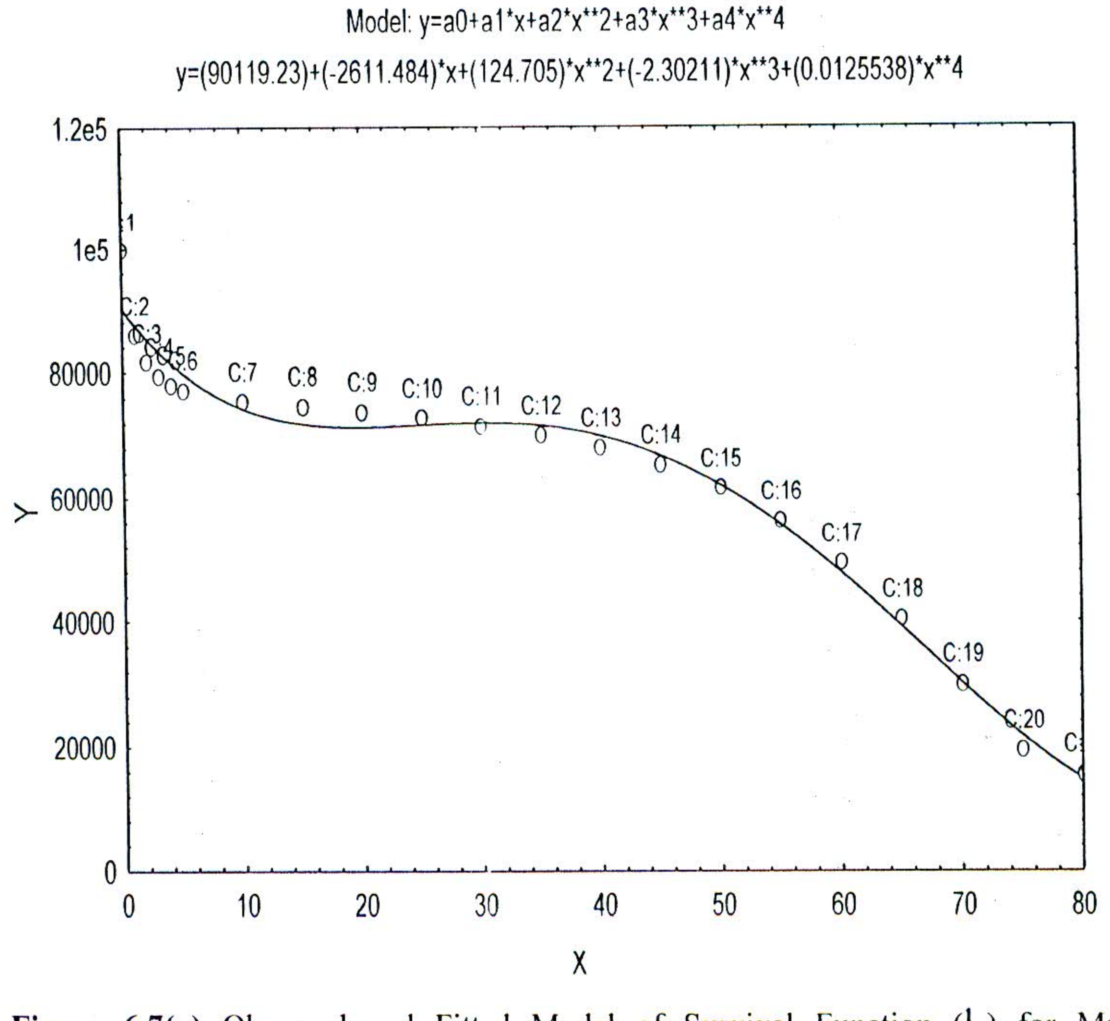


Figure 6.7(a) Observed and Fitted Model of Survival Function (l_x) for Male of Bangladesh in 1981. X: Age Group in Years and Y: Survival Function.

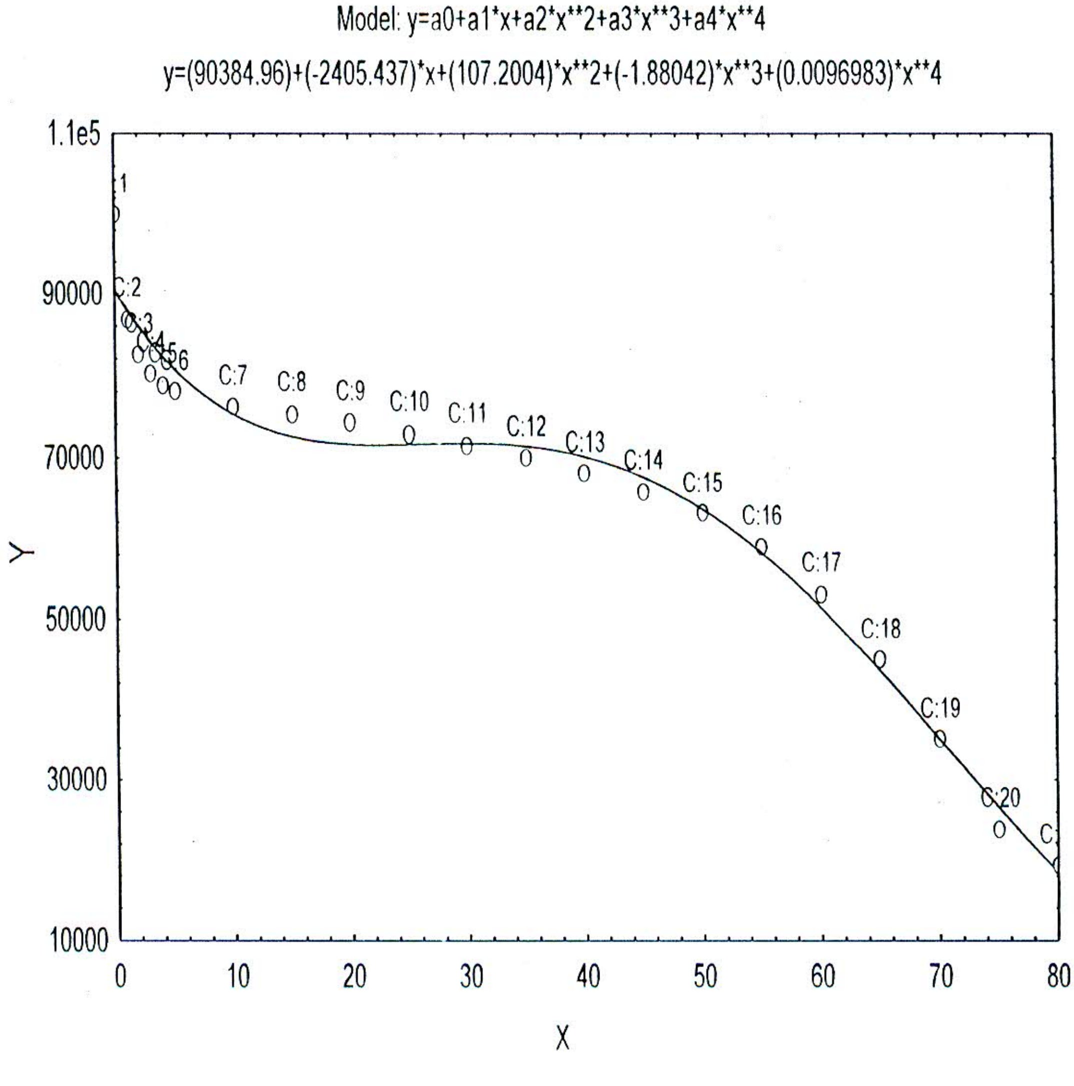


Figure 6.7(b) Observed and Fitted Model of Survival Function (l_x) for Female of Bangladesh in 1981. X: Age Group in Years and Y: Survival Function.

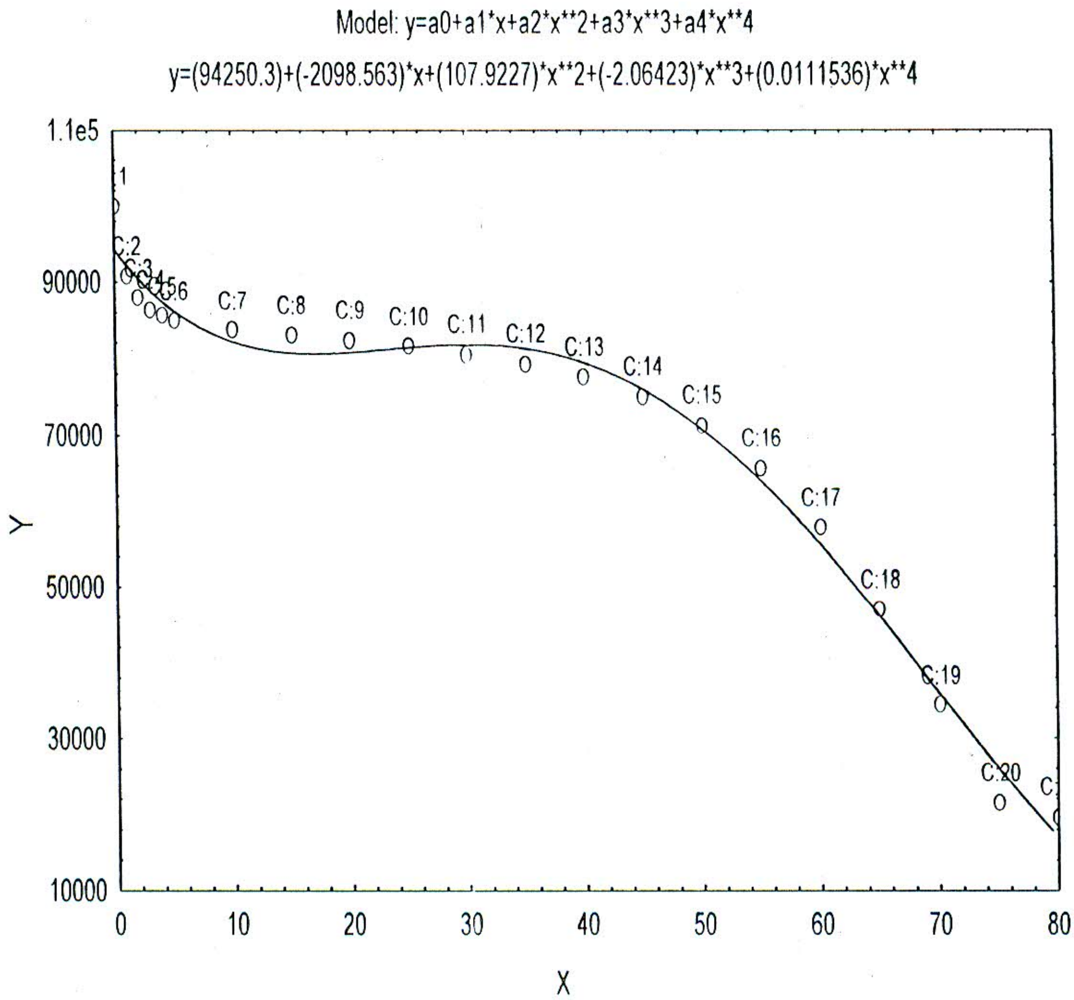


Figure 6.8(a) The Graph of Observed and Fitted Model of Survival Function (l_x) for Male of Bangladesh in 1991. X: Age Group in Years and Y: Survival Function.

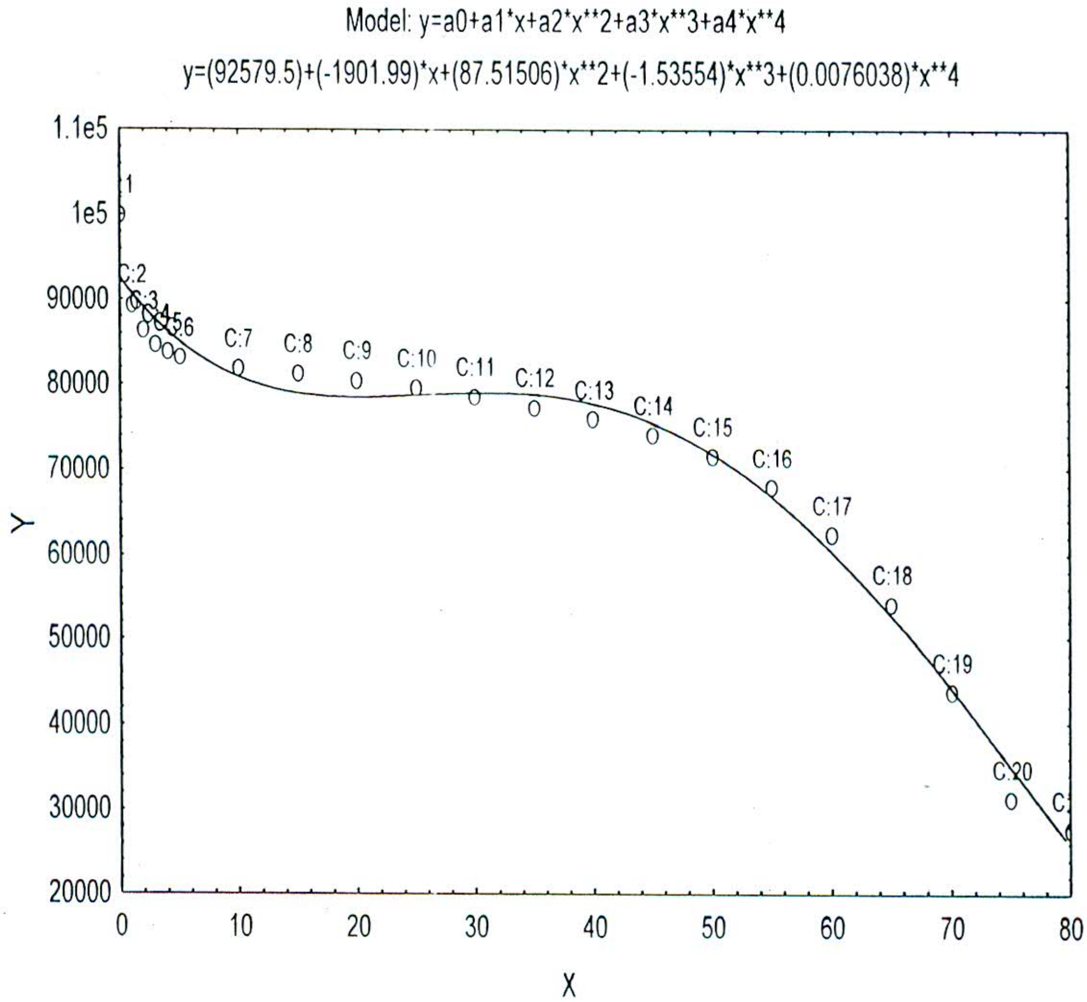


Figure 6.8(b) Observed and Fitted Model of Survival Function (l_x) for Female of Bangladesh in 1991. X: Age Group in Years and Y: Survival Function.

6.5 Mathematical Modeling of Age Specific Death Rates (ASDR) for Male, Female and Both Sexes by Age Groups

Age specific death rates for male, female and both sexes have been presented in Figure 6.9(a) to Figure 6.12(c). From these figures, it has been seen that ASDR can also be fitted by the polynomial function for different ages. In this case, an nth degree polynomial model is treated and the model of the nth degree polynomial is

$$y = a_0 + \sum_{i=1}^{n} a_i x^i + u$$

where, x is independent variable indicating as age group in years; y is dependent variable indicating as ASDR; a_0 is the constant term ; a_i is the coefficient of x^i (i=1, 2, 3,...., n) and u is the stochastic error term of the model. Here we have to choose a suitable n for which the error sum of square is minimum.

The fitted equations of ASDR of Bangladesh are as follows:

$y = (0.111092)+(-0.0209911)x+(0.0010949)x^2+(-0.0000212)x^3+(1.39922e-7)x^4$ for male in 1961 (1)

$y = (0.1038623)+(-0.0194066)x+(0.0010226)x^2+(-0.0000201)x^3+(1.34473e-7)x^4$ for female in 1961 (2)

$y = (0.107459)+(-0.0201947)x+(0.0010585)x^2+(-0.0000206)x^3+(1.371527e-7)x^4$ for both sexes in 1961 (3)

$y = (0.114387)+(-0.0219992)x+(0.001153)x^2+(-0.0000224)x^3+(1.469096e-7)x^4$ for male in 1974 (4)

$y = (0.11137)+(-0.0214477)x+(0.0011365)x^2+(-0.0000223)x^3+(1.48021e-7)x^4$ for female in 1974 (5)

$y = (0.1128867)+(-0.0217294)x+(0.0011453)x^2+(-0.0000223)x^3+(1.47568e-7)x^4$ for both sexes in 1974 (6)

$y = (0.0905521)+(-0.0178697)x+(0.0009607)x^2+(-0.0000191)x^3+(1.286392e-7)x^4$ for male in 1981 (7)

$y = (0.0866549)+(-0.0172177)x+(0.00094)x^2+(-0.000019)x^3+(1.29198e-7)x^4$ for female in 1981 (8)

$y = (0.0888398)+(-0.0175948)x+(0.0009532)x^2+(-0.0000191)x^3+(1.29271e-7)x^4$ for both sexes in 1981 (9)

$y = (0.0587915)+(-0.0122189)x+(0.0006836)x^2+(-0.0000141)x^3+(9.863463e-8)x^4$ for male in 1991 (10)

$y = (0.06804)+(-0.0145161)x+(0.0008269)x^2+(-0.0000172)x^3+(1.199764e-7)x^4$ for female in 1991 (11)

$y = (0.0635039)+(-0.0134227)x+(0.0007597)x^2+(-0.0000157)x^3+(1.100315e-7)x^4$ for both sexes in 1991 (12)

The estimated CVPP, ρ^2_{cv}, corresponding to their R^2 are shown in Table 6.5. From this table it is found that all the fitted models in equation (1) to equation (12) are cross-validated more than 0.92187 and their shrinkages are less than .05. These imply that the fitted models are more than 87% stable. In all the cases, the stability of R^2 is about more than 95%.

The information on model fitting for ASDR has been presented in Table 6.6. From the table, it is seen that all the parameters of the fitted models are highly statistically significant with more than 92% of variance explained.

Table 6.5 Estimated Cross Validity Prediction Power (ρ^2_{cv}) of the Predicted Equations of Age Specific Death Rates (ASDR) for Male, Female and Both Sexes of Bangladesh 1961, 1974, 1981 and 1991

Models	n	k	R^2	ρ^2_{cv}	Shrinkage
Equation 1	21	4	0.93199	0.88719	0.0448
Equation 2	21	4	0.94878	0.91504	0.03374
Equation 3	21	4	0.93431	0.891038	0.043272
Equation 4	21	4	0.93899	0.898801	0.040189
Equation 5	21	4	0.92187	0.870403	0.041467
Equation 6	21	4	0.92898	0.882197	0.036783
Equation 7	21	4	0.95848	0.93113	0.02735
Equation 8	21	4	0.95149	0.919536	0.031955
Equation 9	21	4	0.95415	0.923947	0.030203
Equation 10	21	4	0.96784	0.946655	0.021185
Equation 11	21	4	0.95778	0.929968	0.027812
Equation 12	21	4	0.96267	0.93808	0.02459

Table 6.6. Information on Model Fitting of ASDR of Bangladesh

Models	Proportion of Variance Explained	Parameters	Significant Probability (P)
Model 1	0.93199		
		a_0	0.000002
		a_1	0.00005
		a_2	0.000114
		a_3	0.00012
		a_4	0.000069
Model 2	0.94878		
		a_0	0.000001
		a_1	0.00003
		a_2	0.000058
		a_3	0.00005
		a_4	0.000025
Model 3	0.93431		
		a_0	0.000001
		a_1	0.00004
		a_2	0.000083
		a_3	0.00008
		a_4	0.000043
Model 4	0.93899		
		a_0	0.000004
		a_1	0.00008
		a_2	0.000168
		a_3	0.00017
		a_4	0.000106
Model 5	0.92187		
		a_0	0.000003
		a_1	0.00006
		a_2	0.000114
		a_3	0.0001
		a_4	0.000057
Model 6	0.92898		
		a_0	0.000003
		a_1	0.00007
		a_2	0.000138
		a_3	0.00013
		a_4	0.000077

Model 7	0.95848		
		a_0	0.000002
		a_1	0.00004
		a_2	0.000068
		a_3	0.00005
		a_4	0.000024
Model 8	0.95149		
		a_0	0.000002
		a_1	0.00003
		a_2	0.000039
		a_3	0.00003
		a_4	0.00001
Model 9	0.95415		
		a_0	0.000002
		a_1	0.00003
		a_2	0.000052
		a_3	0.00004
		a_4	0.000016
Model 10	0.96784		
		a_0	0.000003
		a_1	0.00003
		a_2	0.000033
		a_3	0.00002
		a_4	0.000005
Model 11	0.95778		
		a_0	0.000003
		a_1	0.00002
		a_2	0.00002
		a_3	0.00001
		a_4	0.000003
Model 12	0.96267		
		a_0	0.000003
		a_1	0.00002
		a_2	0.000024
		a_3	0.00001
		a_4	0.000003

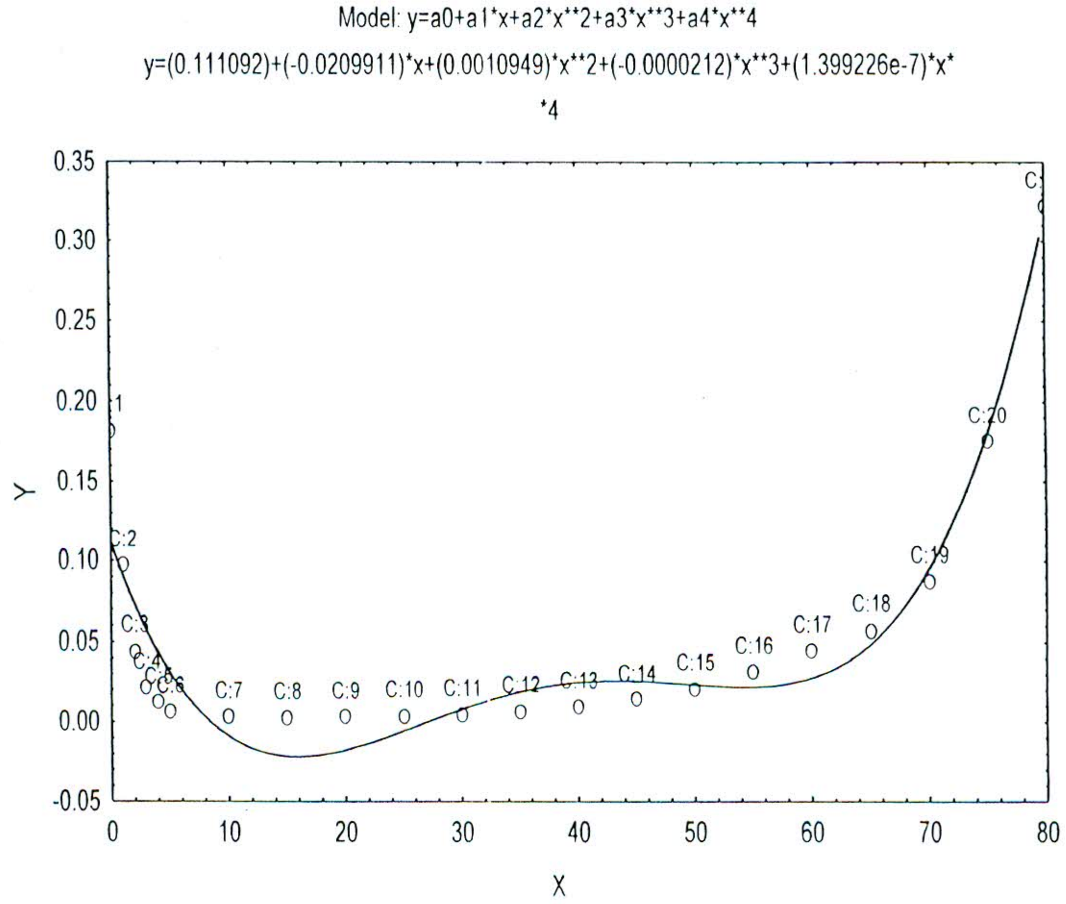


Figure 6.9(a) Observed and Fitted Model for Age Specific Death Rates (ASDR) for Male of Bangladesh in 1961. X: Age Group in Years and Y: Age Specific Death Rates.

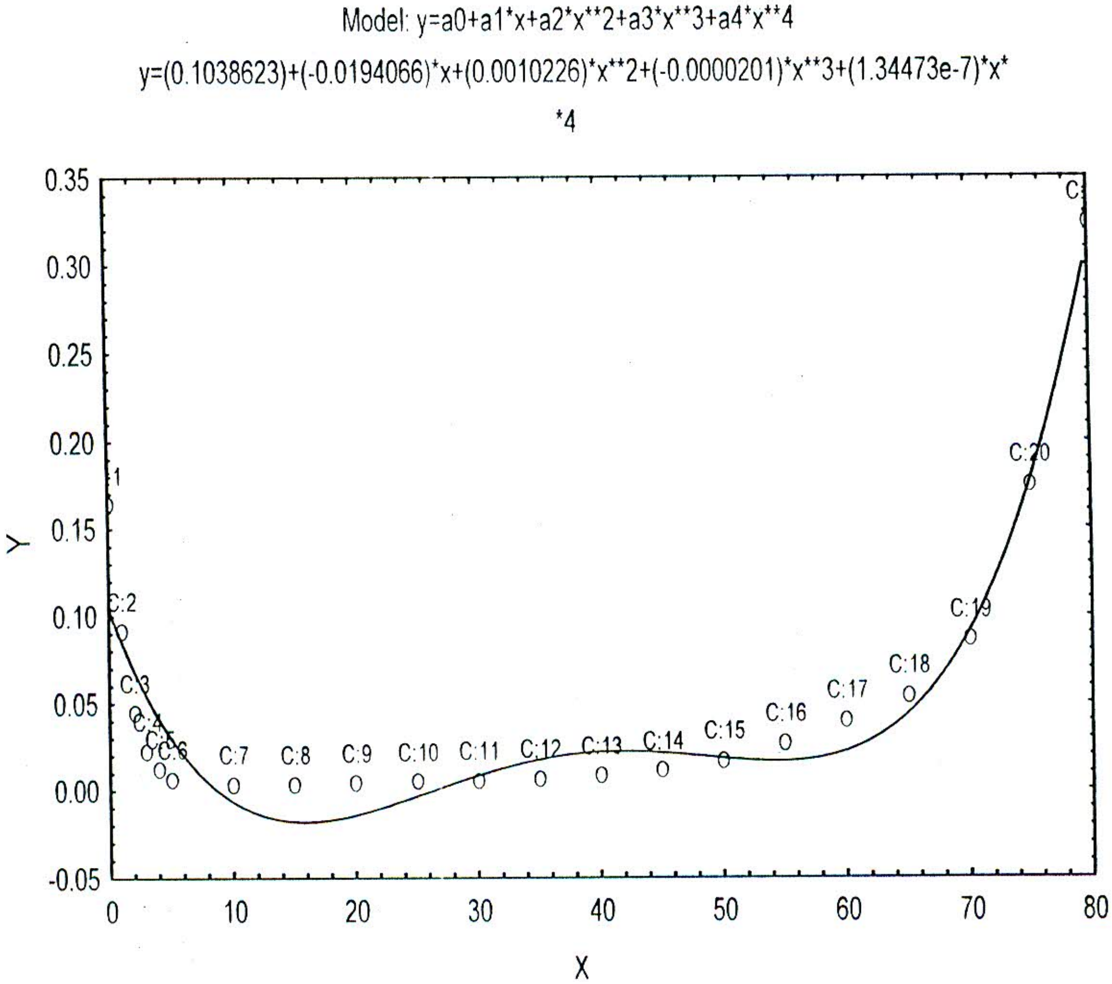


Figure 6.9(b) Observed and Fitted Model for Age Specific Death Rates (ASDR) for Female of Bangladesh in 1961. X: Age Group in Years and Y: Age Specific Death Rates.

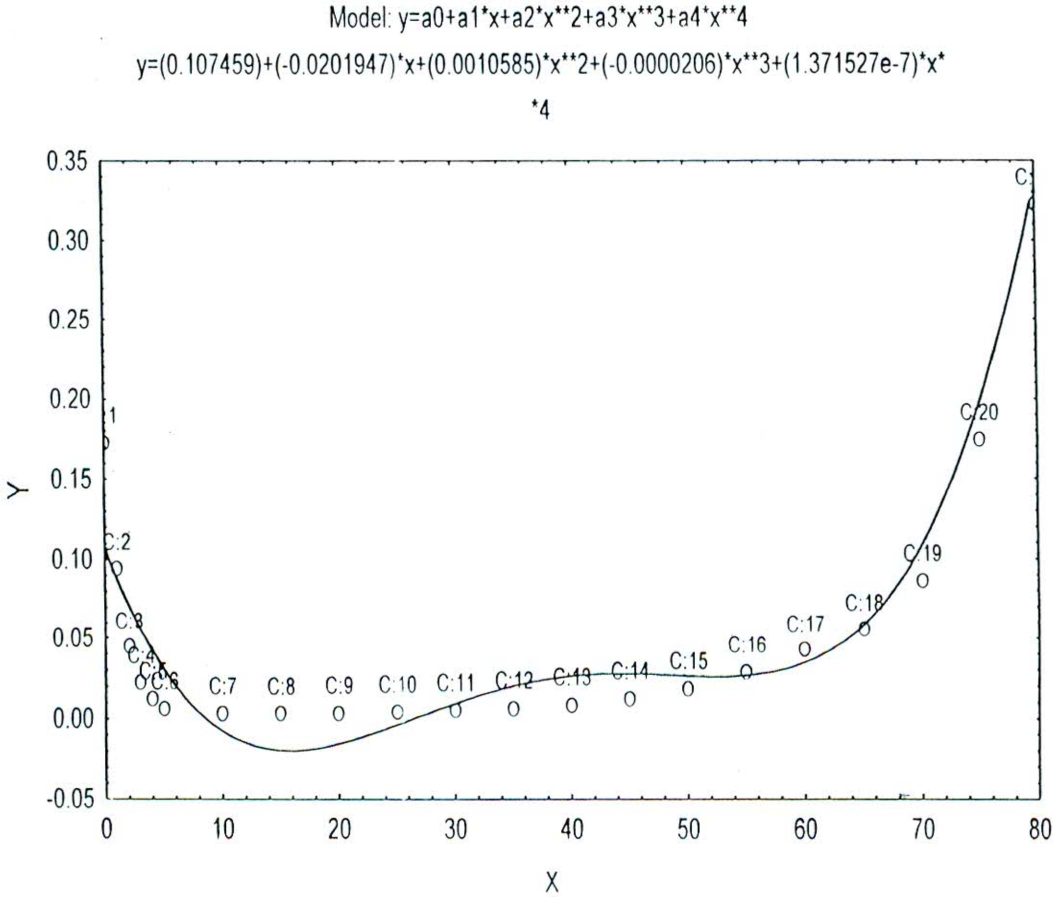


Figure 6.9(c) Observed and Fitted Model for Age Specific Death Rates (ASDR) for Both Sexes of Bangladesh in 1961. X: Age Group in Years and Y: Age Specific Death Rates.

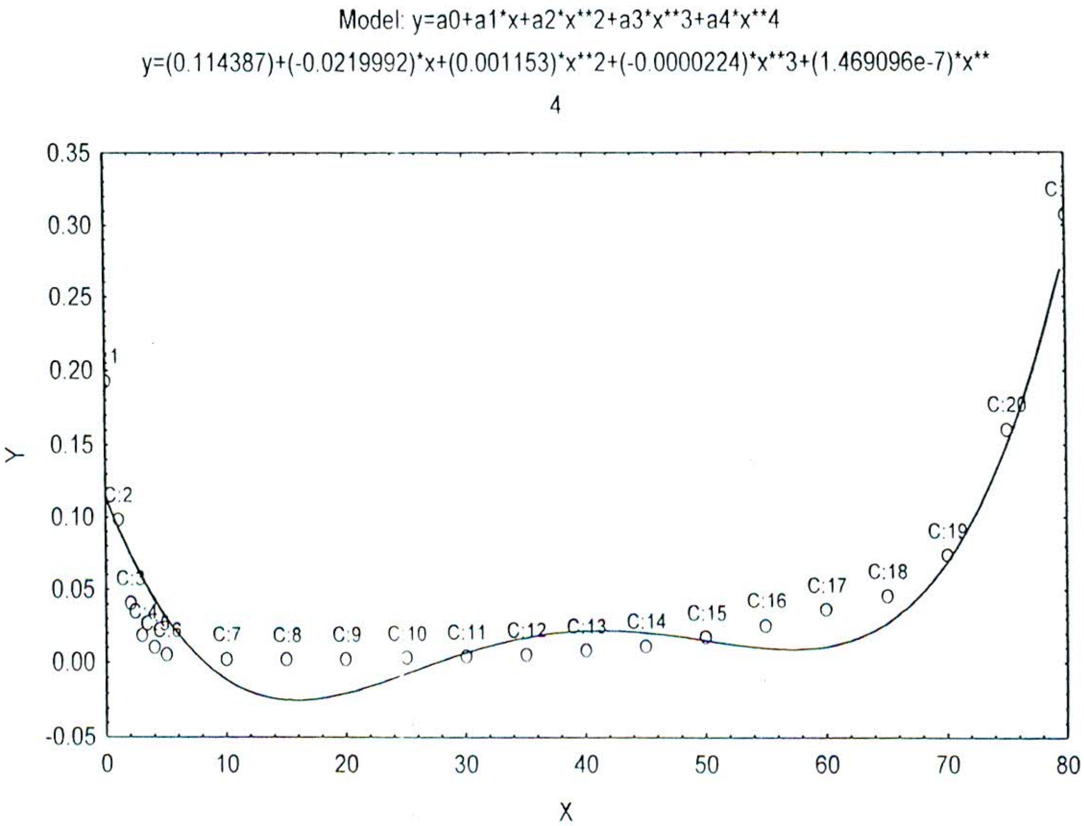


Figure 6.10(a) Observed and Fitted Model for Age Specific Death Rates (ASDR) for Male of Bangladesh in 1974. X: Age Group in Years and Y: Age Specific Death Rates.

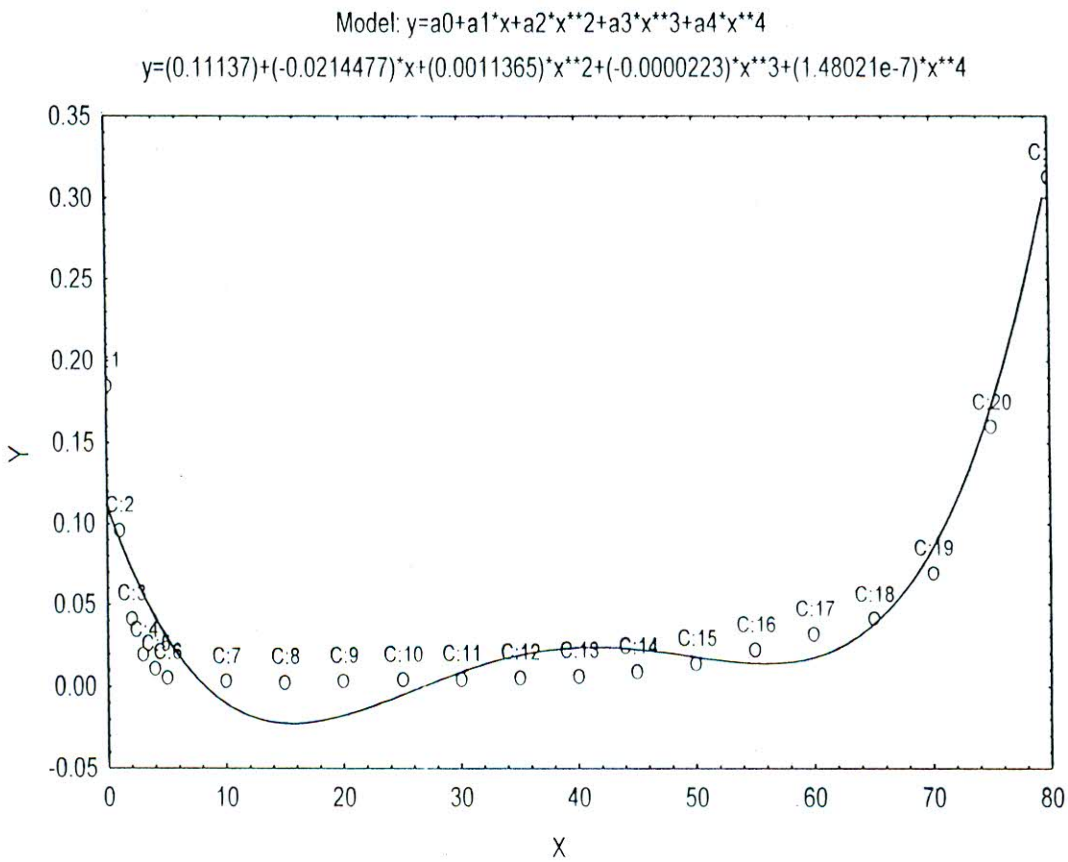


Figure 6.10(b) Observed and Fitted Model for Age Specific Death Rates (ASDR) for Female of Bangladesh in 1974. X: Age Group in Years and Y: Age Specific Death Rates.

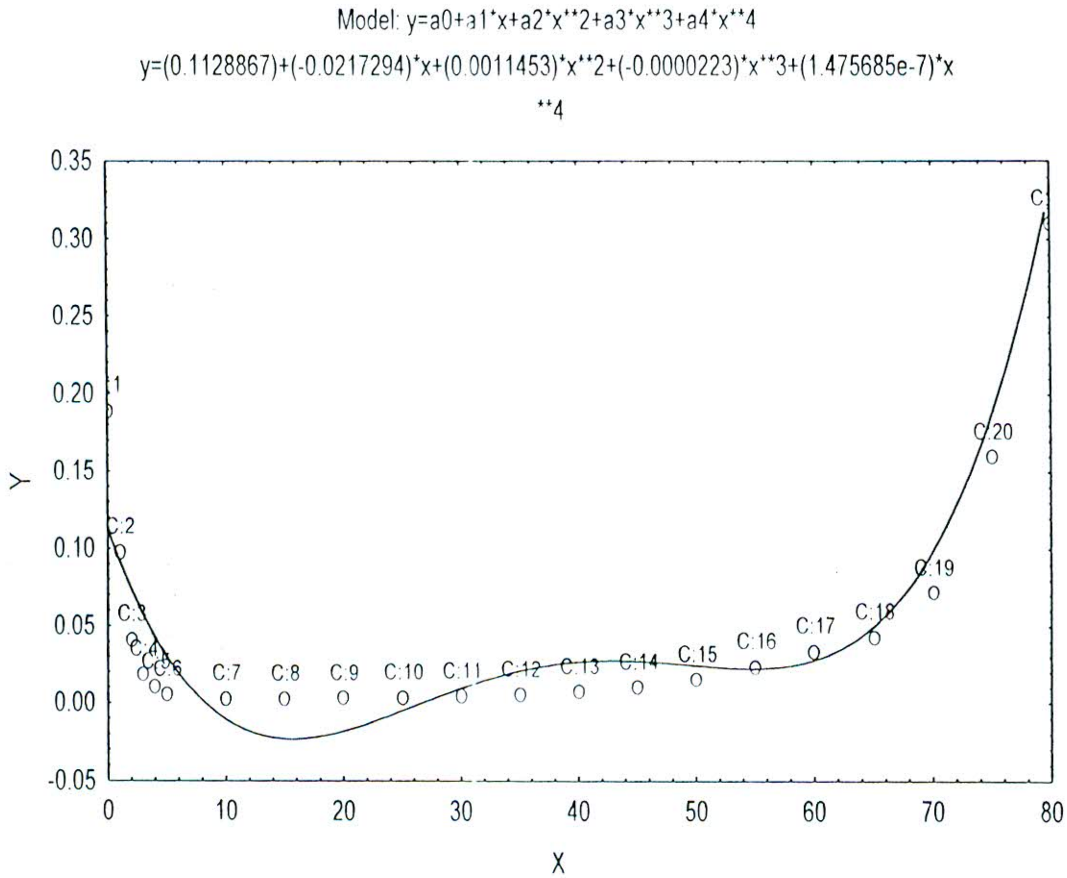


Figure 6.10(c) Observed and Fitted Model for Age Specific Death Rates (ASDR) for Both Sexes of Bangladesh in 1974. X: Age Group in Years and Y: Age Specific Death Rates.

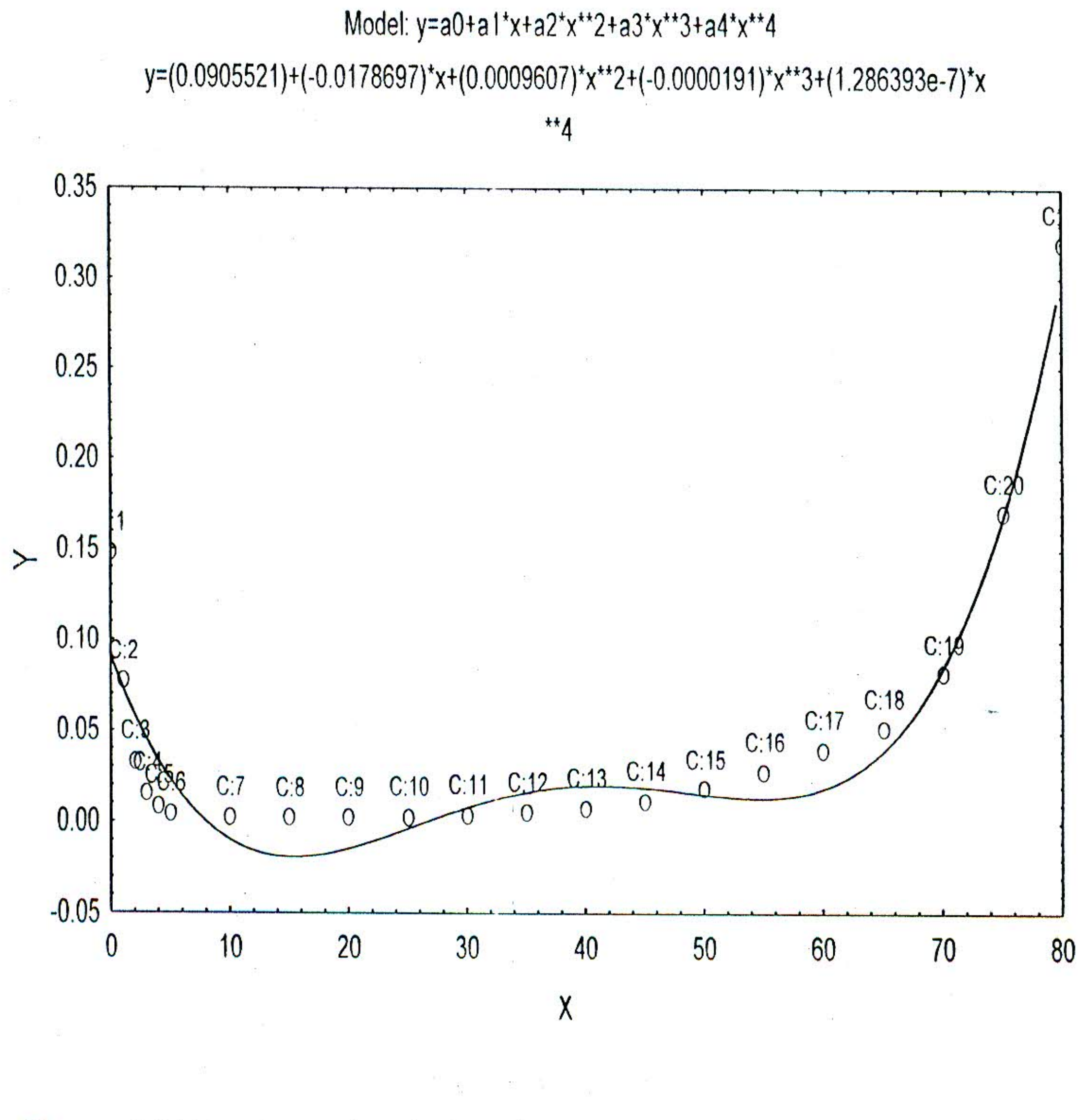


Figure 6.11(a) Observed and Fitted Model for Age Specific Death Rates (ASDR) for Male of Bangladesh in 1981. X: Age Group in Years and Y: Age Specific Death Rates.

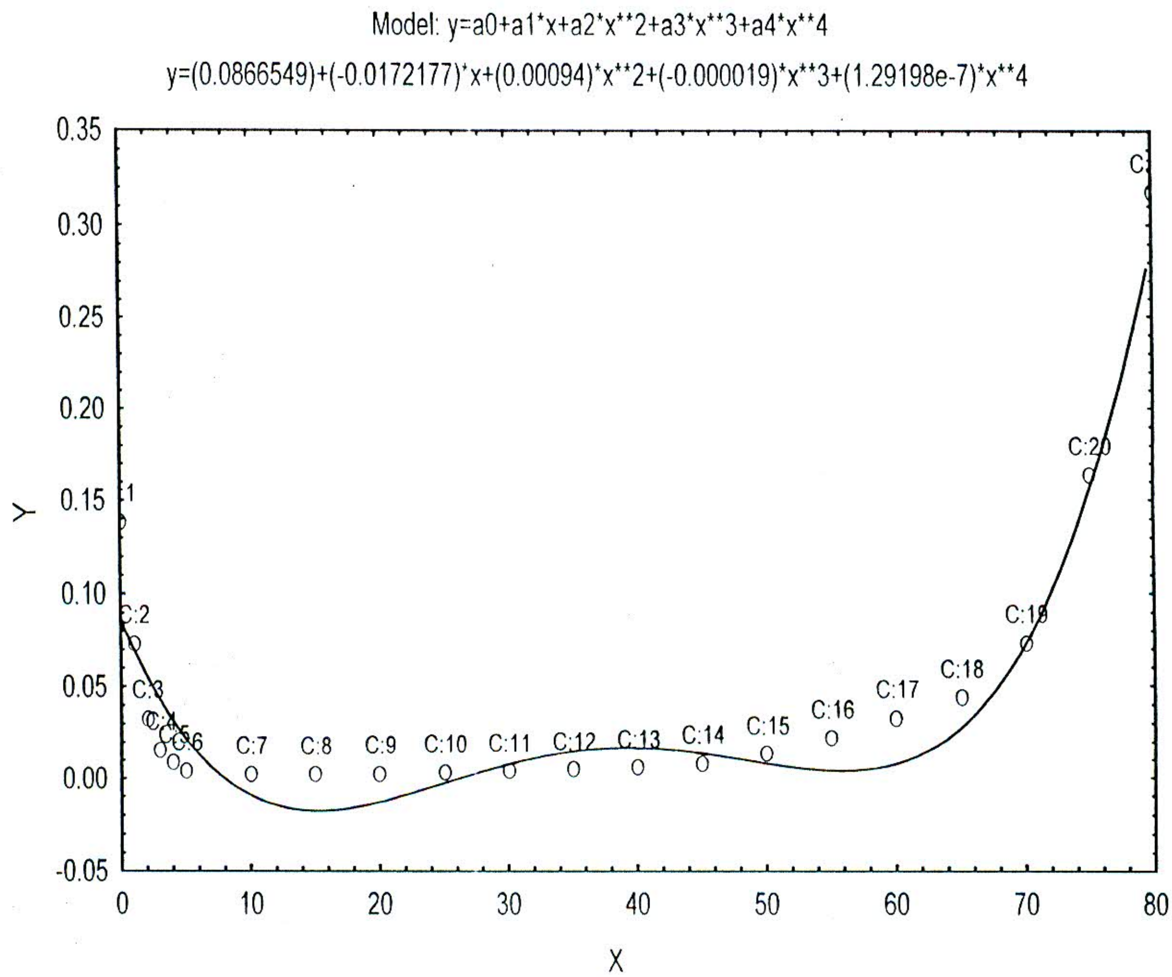


Figure 6.11(b) Observed and Fitted Model for Age Specific Death Rates (ASDR) for Female of Bangladesh in 1981. X: Age Group in Years and Y: Age Specific Death Rates.

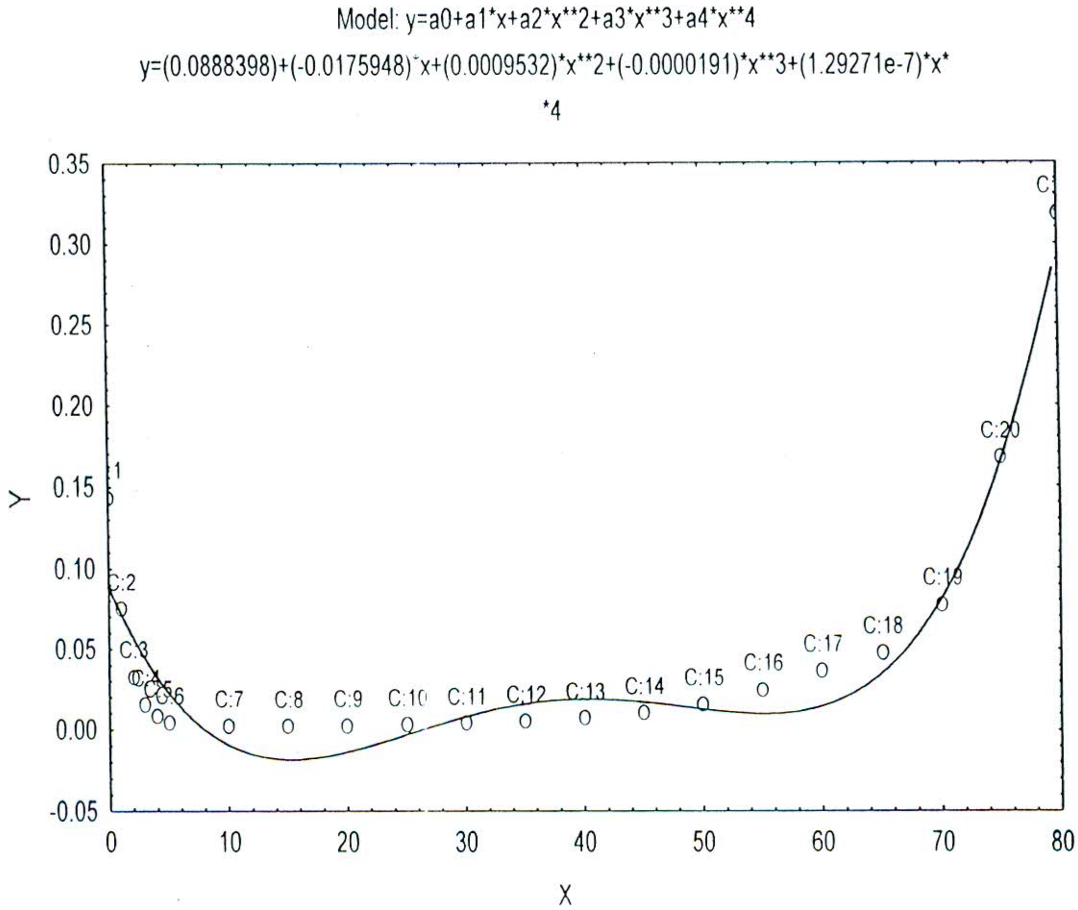


Figure 6.11(c) Observed and Fitted Model for Age Specific Death Rates (ASDR) for Both Sexes of Bangladesh in 1981. X: Age Group in Years and Y: Age Specific Death Rates.

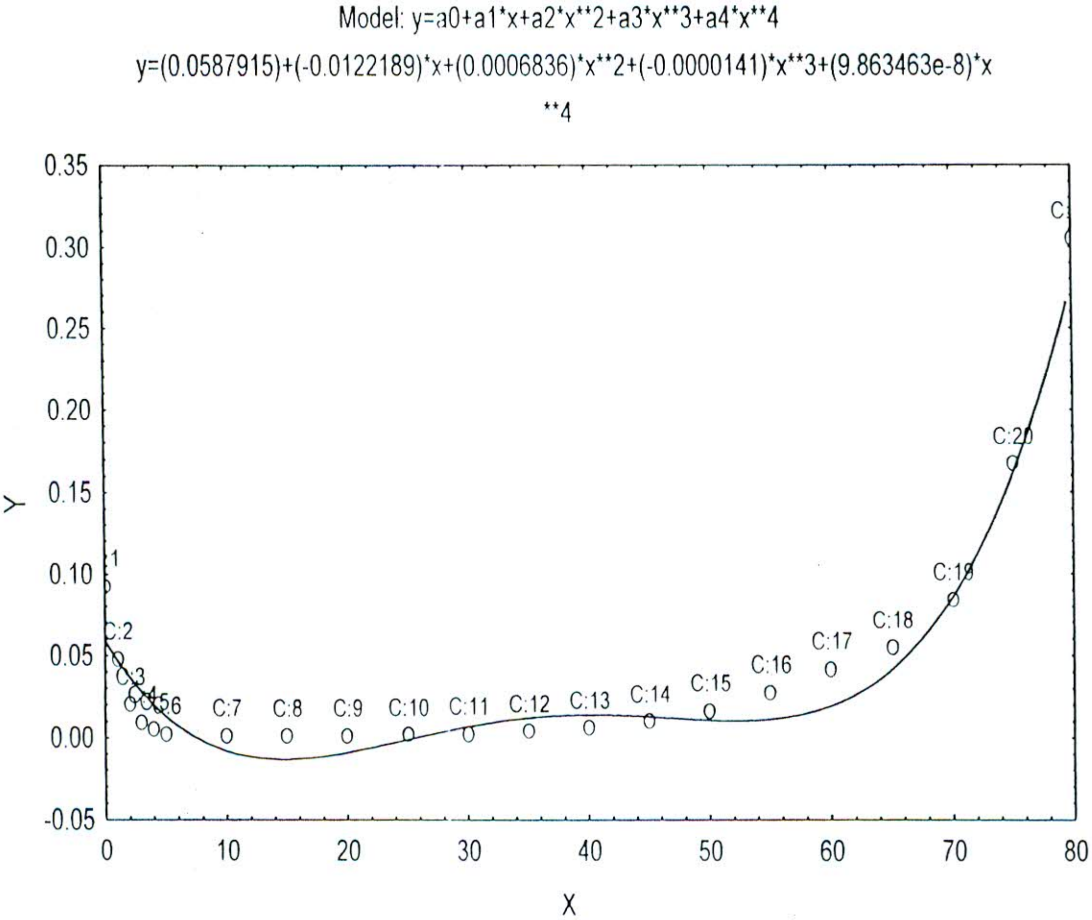


Figure 6.12(a) Observed and Fitted Model for Age Specific Death Rates (ASDR) for Male of Bangladesh in 1991. X: Age Group in Years and Y: Age Specific Death Rates.

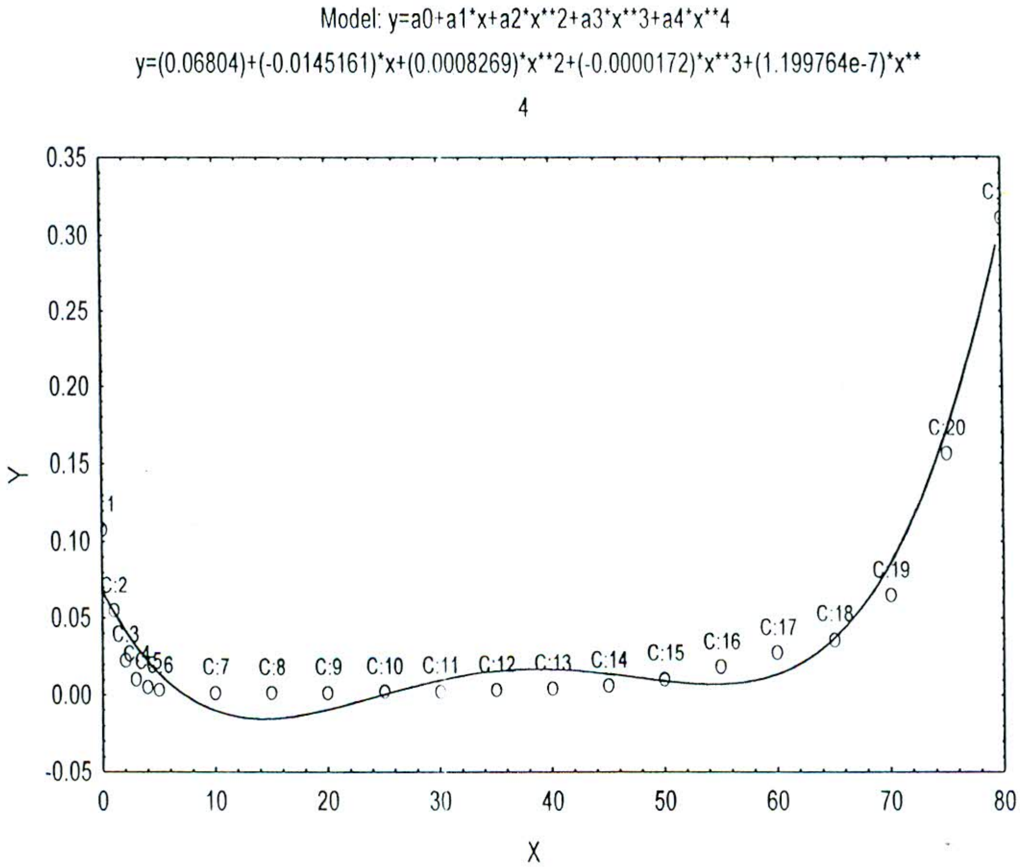


Figure 6.12(b) Observed and Fitted Model for Age Specific Death Rates (ASDR) for Female of Bangladesh in 1991. X: Age Group in Years and Y: Age Specific Death Rates.

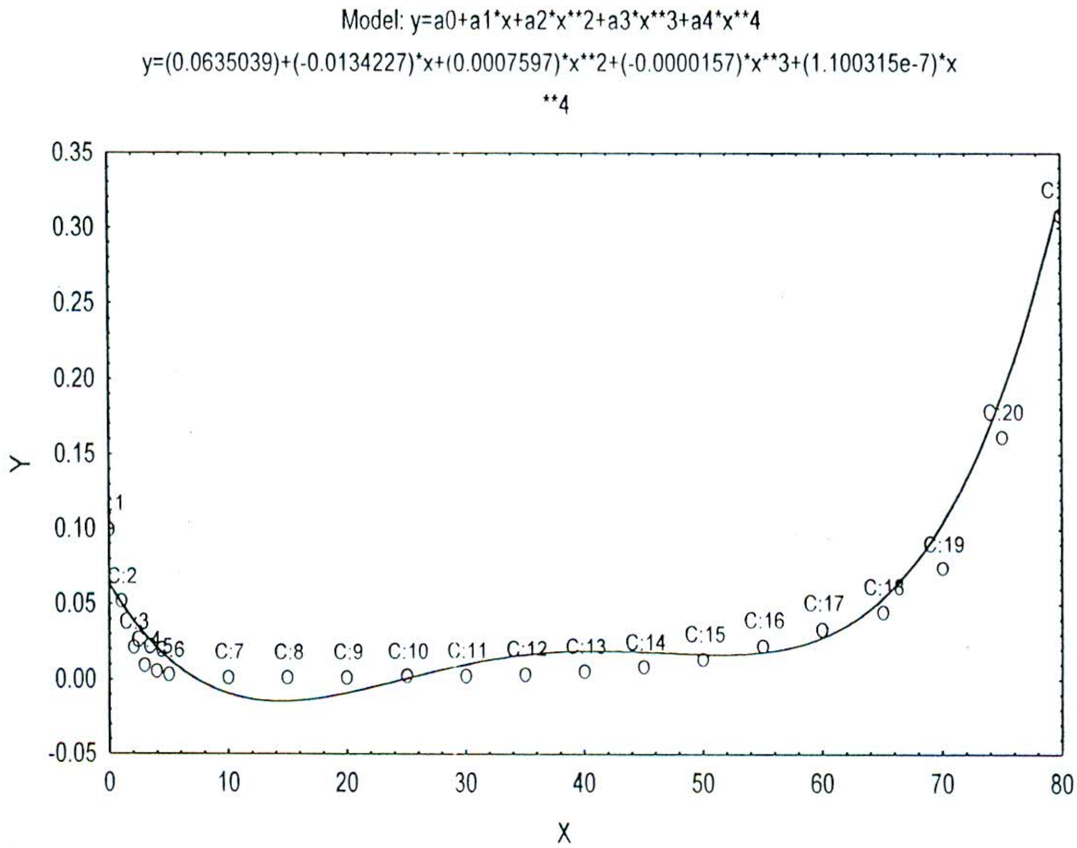


Figure 6.12(c) Observed and Fitted Model for Age Specific Death Rates (ASDR) for Both Sexes of Bangladesh in 1991. X: Age Group in Years and Y: Age Specific Death Rates.

6.6 Mathematical Modeling of Age Specific Fertility Rates (ASFR) by Age Groups

Age specific fertility rates by age groups have been depicted in Figure 6.13 to Figure 6.16. From these figures, it is observed that age specific fertility rates can also be fitted by polynomial model with respect to different ages in years. Therefore, an nth degree polynomial model is considered and the form of the nth degree polynomial model is

$$y = a_0 + \sum_{i=1}^{n} a_i \, x^i + u$$

where, x is age group in years; y is age specific fertility rates; a_0 is the constant; a_i is the coefficient of x^i (i =1, 2, 3, ..., n) and u is the stochastic error term of the model. Here we have also to select a suitable n for which the error sum of square is minimum.

The fitted equations of ASFR of Bangladesh are as follows:

$y = (-2.37412)+(0.2527307)x+(-0.007447)x^2+(0.000067)x^3$ for 1961 (1)

$y = (-2.720567)+(0.2831335)x+(-0.0081544)x^2+(0.0000717)x^3$ for 1974 (2)

$y = (-2.54285)+(0.263671)x+(-0.0075637)x^2+(0.0000662)x^3$ for 1981 (3)

$y = (-1.44125)+(0.1469153)x+(-0.0041452)x^2+(0.0000357)x^3$ for 1991 (4)

The estimated CVPP, ρ^2_{cv}, and corresponding to their R^2 are shown in Table 6.7. From this table, it is seen that all the fitted models in equation (1) to equation (4) are highly cross-validated and their shrinkages are 0.0307, 0.0628, 0.0657 and 0.0875 respectively. These imply that all these models are more than 89% stable. In all the fitted models, the stability of R^2 is about more than 91%.

The information on model fitting for ASFR has been presented in Table 6.8. From this table, it is observed that all the parameters of the fitted models are highly statistically significant with more than 98% of variance explained.

Table 6.7 Estimated Cross Validity Prediction Power (ρ^2_{cv}) of the Predicted Equations of Age Specific Fertility Rates (ASFR) of Bangladesh

Models	n	k	R^2	ρ^2_{cv}
Equation 1	7	3	0.99348	0.962743
Equation 2	7	3	0.98666	0.923771
Equation 3	7	3	0.98605	0.920286
Equation 4	7	3	0.98143	0.893886

Table 6.8 Information on Model Fitting of ASFR of Bangladesh

Models	Proportion of Variance Explained	Parameters	Significant Probability (P)
Model 1	0.99348		
		a_0	0.0021
		a_1	0.00183
		a_2	0.00237
		a_3	0.003438
Model 2	0.98666		
		a_0	0.00584
		a_1	0.005547
		a_2	0.00762
		a_3	0.011647
Model 3	0.98605		
		a_0	0.00625
		a_1	0.005993
		a_2	0.00831
		a_3	0.012841
Model 4	0.98143		
		a_0	0.00900
		a_1	0.009069
		a_2	0.01310
		a_3	0.020886

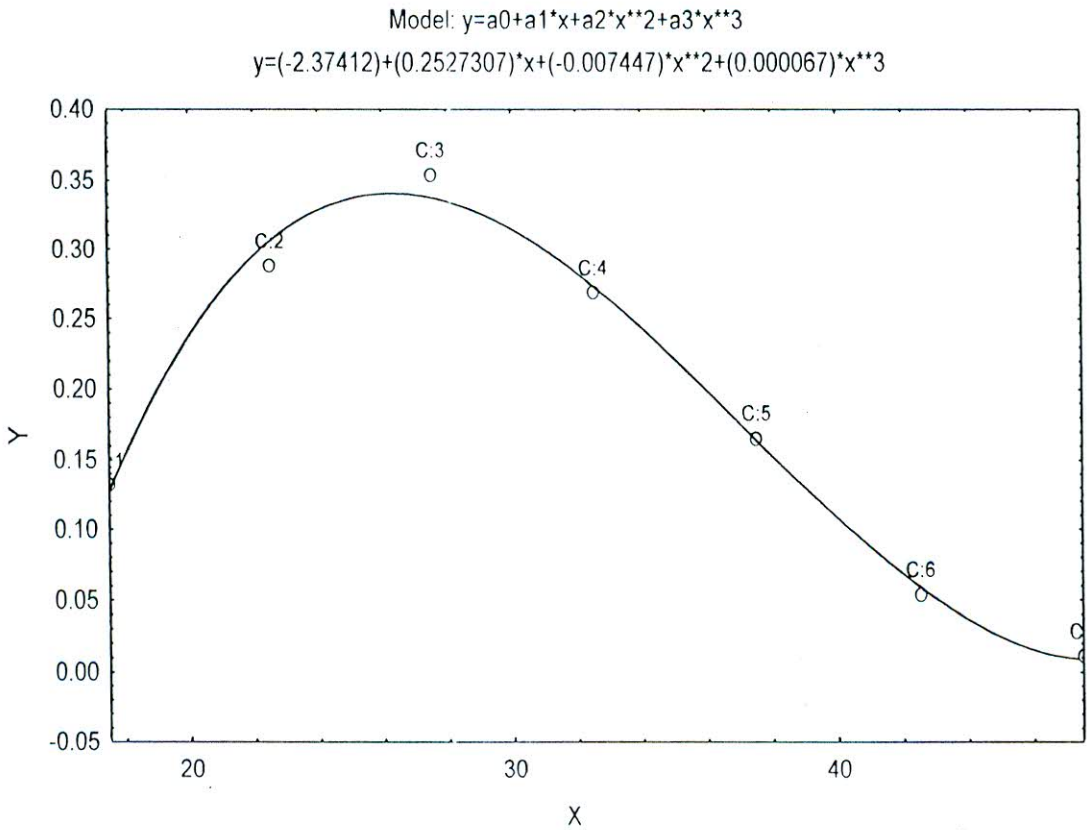


Figure 6.13 Observed and Fitted Age Specific Fertility Rates (ASFR) of Bangladesh in 1961. X: Age Group in Years and Y: Age Specific Fertility Rates.

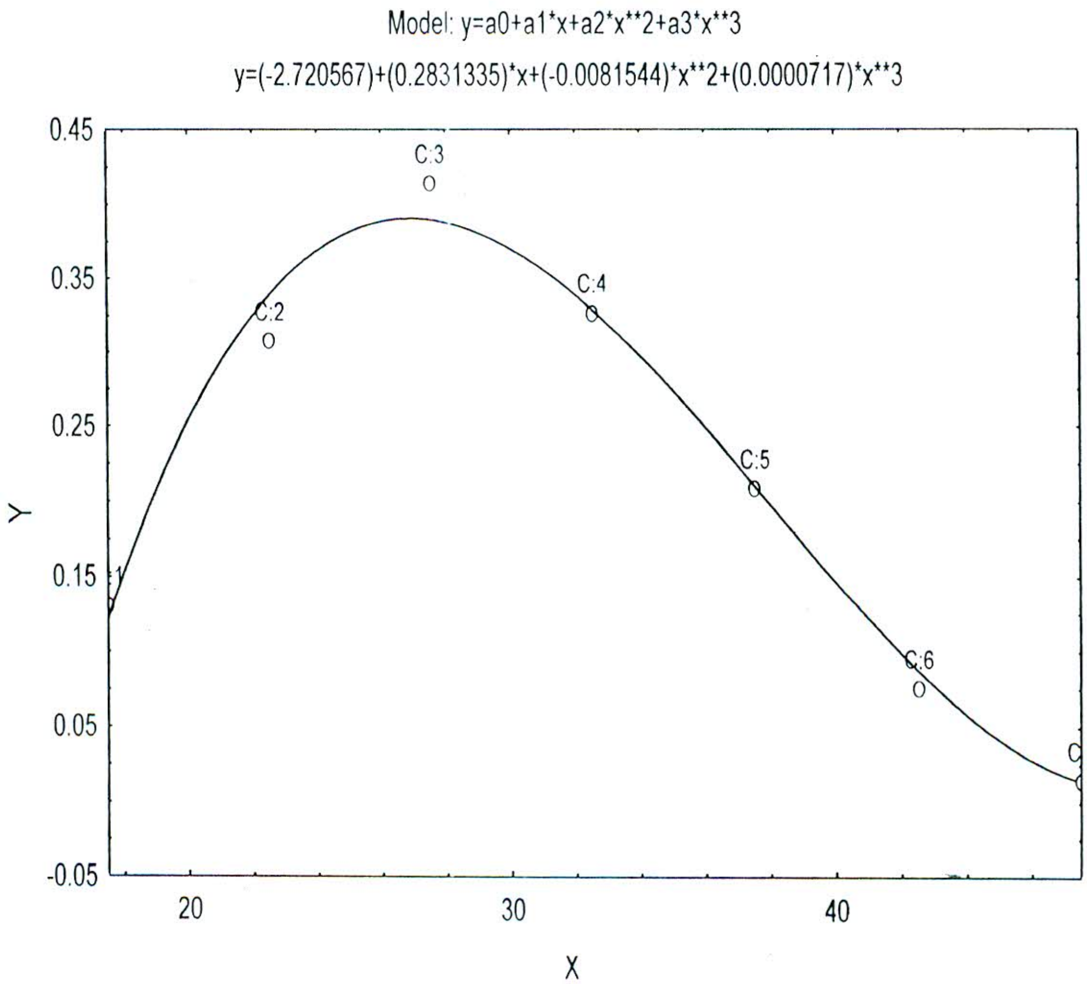


Figure 6.14 Observed and Fitted Age Specific Fertility Rates (ASFR) of Bangladesh in 1974. X: Age Group in Years and Y: Age Specific Fertility Rates.

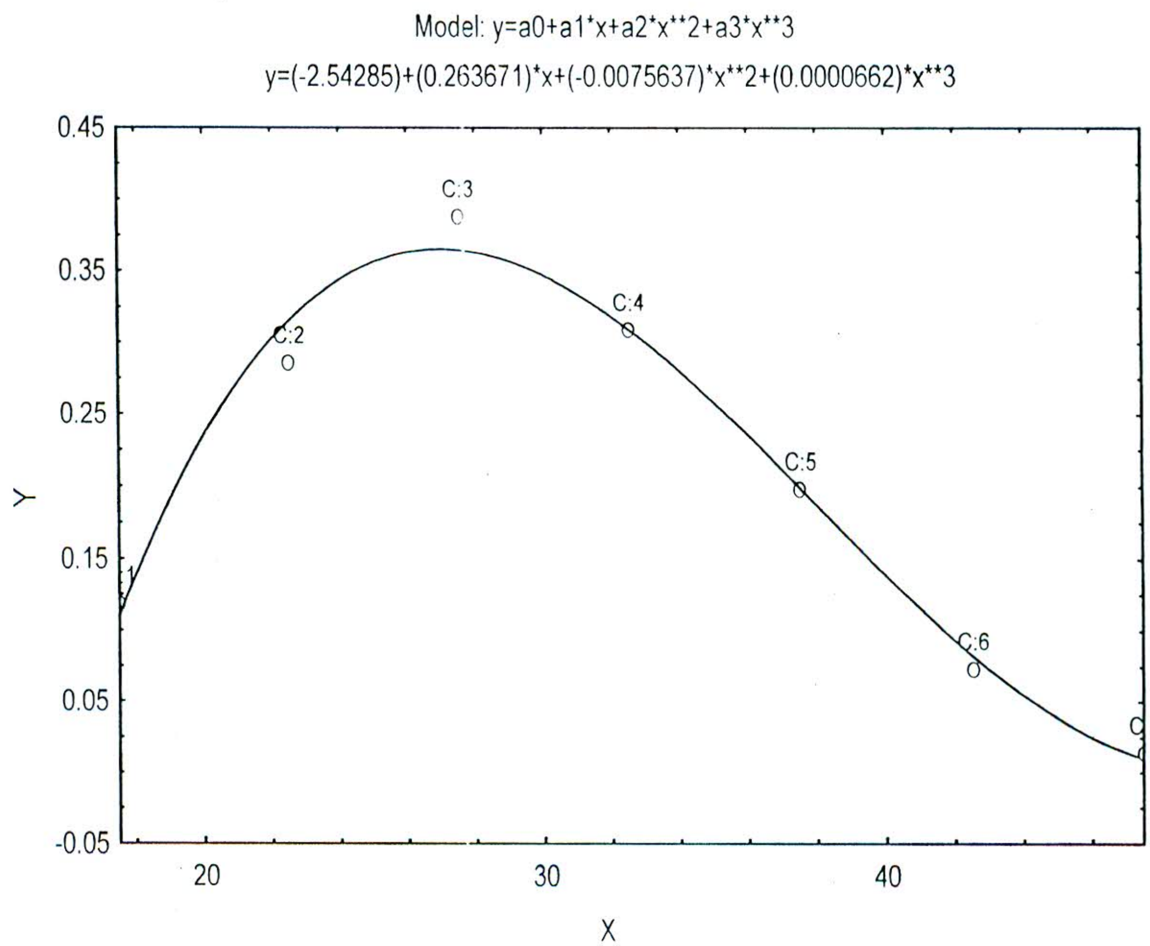


Figure 6.15 Observed and Fitted Age Specific Fertility Rates (ASFR) of Bangladesh in 1981. X: Age Group in Years and Y: Age Specific Fertility Rates.

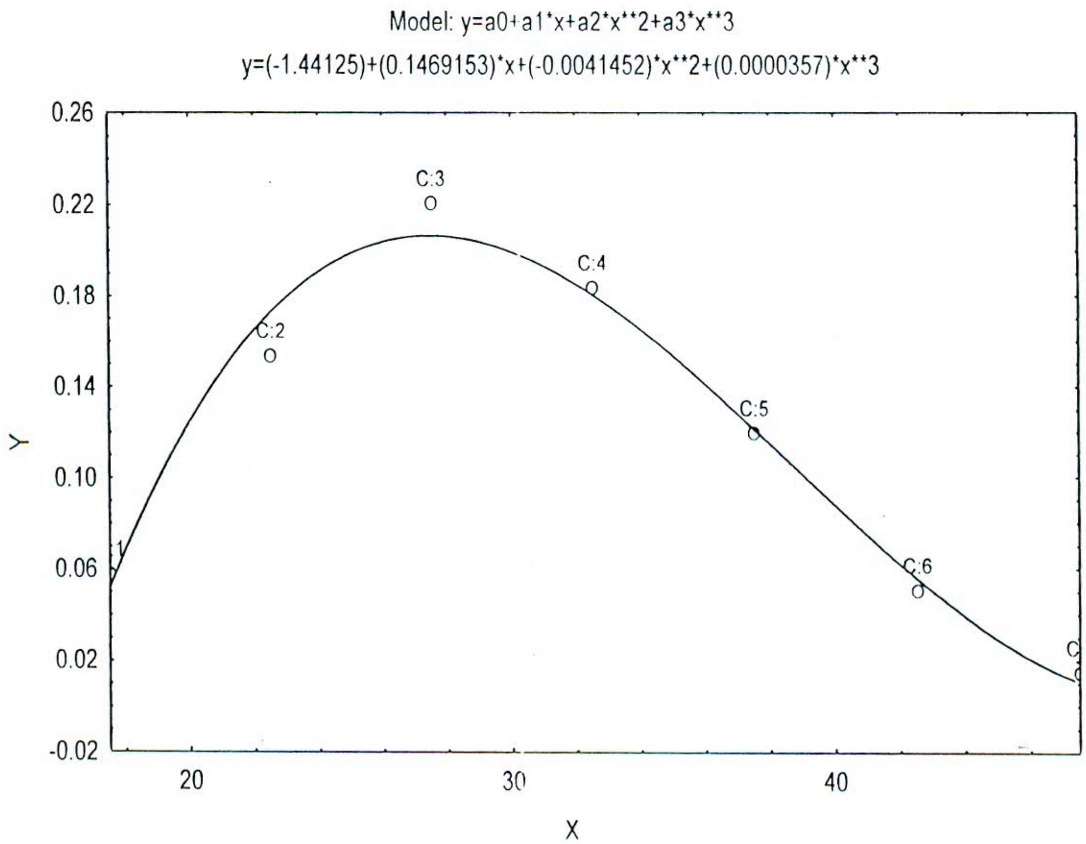


Figure 6.16 Observed and Fitted Age Specific Fertility Rates (ASFR) of Bangladesh in 1991. X: Age Group in Years and Y: Age Specific Fertility Rates.

6.7 Mathematical Modeling of Time Series Data

i) The scattered plot of infant mortality rate (IMR) and years (Fig. 6.17), it seems that infant mortality rate follows semi-log linear model. Therefore, the model is

$$y = a_0 + a_1 x + u$$

where, x represents the years; y is log (IMR); a_0, a_1 are unknown parameters to be estimated and u is the error term of the model.

ii) The crude death rate (CDR) for different years have been plotted in graph paper (Fig. 2.10). It is seen that there are some sort of distortions which is unexpected. Before going to use this data, an adjustment have been made using the Package Minitab Release 12.1 by the latest smoothing method named 4253H, twice. Thus adjusted data has been used for time trend model for CDR. The scattered plot of years and CDR (Fig. 6.18), it seems that CDR follows simple linear regression model. Therefore, a simple linear regression model is fitted and the form of the model is

$$y = a_0 + a_1 x + u$$

where, x represents the years; y represents CDR; a_0, a_1 are unknown parameters to be estimated and u is the error term of the model.

iii) Using the scattered plot of years and life expectancy at birth for male (Fig. 6.19), it seems that life expectancy at birth for male follows simple linear regression model. Therefore, a simple linear regression model is considered and the form of the model is

$$y = a_0 + a_1 x + u$$

where, x represent the years; y represent life expectancy at birth for male; a_0, a_1 are unknown parameters to be estimated and u is the error term of the model.

iv) Using the scattered plot of years and life expectancy at birth for female (Fig. 6.20), it appears that life expectancy at birth for female follows simple linear regression model. Therefore, a simple linear regression model is considered and the form of the model is

$$y = a_0 + a_1 x + u$$

where, x represent the years; y represent life expectancy at birth for female; a_0, a_1 are parameters and u is the stochastic term of the model.

v) From the dotted plot of years and crude birth rate (CBR) (Fig. 6.21), it has been seen that CBR can be fitted by polynomial for different years. In this case, an n^{th} degree polynomial model is treated and the model of the n^{th} degree polynomial is

$$y = a_0 + \sum_{i=1}^{n} a_i x^i + u$$

where, x is years; y is CBR; a_0 is the constant; a_i is the coefficient of x^i (i =1, 2, 3,..., n) and u is the error term of the model. Here we have to choose a suitable n for which the error sum of square is minimum.

vi) Using the scattered plot of years and total fertility rate (TFR) (Fig.6.22), it is observed that TFR can be fitted by simple linear regression model. Therefore, a simple linear regression model is treated and the form of the model is

$$y = a_0 + a_1 x + u$$

where, x represent the years; y represent TFR; a_0, a_1 are parameters and u is the disturbance term of the model.

vii) Using the dotted plot of years and gross reproduction rate (GRR) (Fig. 6.23), it is observed that GRR follows simple linear regression model. Therefore, a simple linear regression model is considered and the form of the model is

$$y = a_0 + a_1 x + u$$

where, x represent the years; y represent GRR; a_0, a_1 are parameters and u is the stochastic term of the model.

viii) Again, from the scattered plot of years and net reproduction rate (NRR) (Fig. 6.24), it has been observed that NRR can also be fitted by simple linear regression model. Therefore, a simple linear regression model is considered and the form of the model is

$$y = a_0 + a_1 x + u$$

where, x represent the years; y represent NRR; a_0, a_1 are parameters and u is the stochastic term of the model.

The fitted equations of time trend models of Bangladesh are as follows:

For IMR, log(y)=(4.920353)+(-0.054526)x. .... ii)

For CDR, y=(13.50731)+(-0.554341)x .... ii)

For life expectancy at birth of male, y=(53.63333)+(0.336842)x .... iii)

For life expectancy at birth of male, y=(53.00719)+(0.335501)x .... iv)

For CBR, $y=(35.89868)+(-0.38937)x^2$ .... v)

For TFR, y=(5.440351)+(-0.118667)x .... vi)

For GRR, y=(2.652281)+(-0.056860)x .... vii)

For NRR, y=(1.952982)+(-0.029035)x .... viii)

The estimated CVPP (ρ^2_{cv}) and corresponding to their R^2 are shown in Table 6.9. From this table, it is seen that all the fitted models in equation (1) to equation (8) are cross-validated and their shrinkage are 0.021, 0.081, 0.025229, 038081, 0.01258, .0099, 0.01256 and 0.01831 respectively. These imply that all these models are more than 84% stable excepting the models (2) and (3) which are more than 77% stable.

The information on model fitting for time trend models has been presented in Table 6.10. From this Table 6.10, it is observed that all the parameters of the fitted models are highly statistically significant with large proportion of variance explained.

The residual analysis of these models has also been shown in Figure 6.17 to Figure 6.22. The forecast values of these models have been presented in Table 6.11 and in Figure 6.23. It should be mention here that all these time trend models have been estimated using the software Econometric Views.

Table 6.9 Estimated Cross Validity Prediction Power (ρ^2_{cv}) of the Predicted Equations of Infant Mortality Rate (IMR), Crude Death Rate (CDR), Life Expectancy at Birth for Male, Life Expectancy at Birth for Female, Crude Birth Rate (CBR), Total Fertility Rate (TFR), Gross Reproduction Rate (GRR) and Net Reproduction Rate (NRR) of Bangladesh

Models	n	k	R^2	ρ^2_{cv}	Shrinkage
Equation 1	15	1	0.954778	0.93349	0.021
Equation 2	13	1	0.859646	0.77123	0.081
Equation 3	18	1	0.871475	0.846246	0.025229
Equation 4	18	1	0.806150	0.77790	0.038081
Equation 5	19	1	0.931707	0.91900	0.01258
Equation 6	19	1	0.946106	0.946106	0.0099
Equation 7	19	1	0.931827	0.919269	0.01256
Equation 8	19	1	0.90598	0.882289	0.01831

Table 6.10 Information on Model Fitting of Time Trend Models of the Parameters of Bangladesh

Models	Proportion of Variance Explained	Parameters	Significant Probability (p)
Model 1	0.954778		
		a_0	0.00000
		a_1	0.00000
Model 2	0.859646		
		a_0	0.0000
		a_1	0.0000
Model 3	0.871475		
		a_0	0.00000
		a_1	0.00000
Model 4	0.806150		
		a_0	0.0000
		a_1	0.0000
Model 5	0.931707		
		a_0	0.0000
		a_2	0.0000
Model 6	0.946106		
		a_0	0.0000
		a_1	0.0000
Model 7	0.931827		
		a_0	0.0000
		a_1	0.0000
Model 8	0.900598		
		a_0	0.0000
		a_1	0.0000

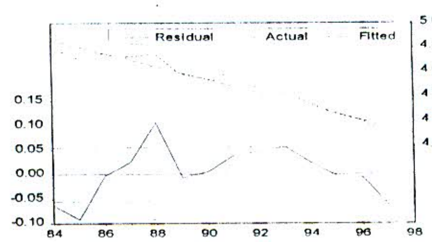


Figure 6.17 Actual, Fitted and Residual of Infant Mortality Rate of Bangladesh

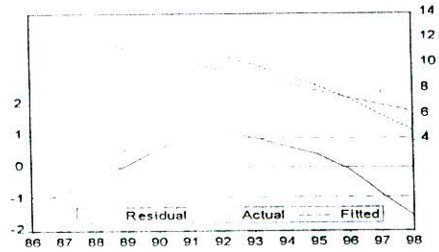


Figure 6.18 Actual, Fitted and Residual of Crude Death Rate of Bangladesh

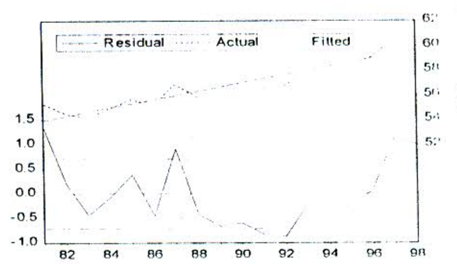


Figure 6.19 Actual, Fitted and Residual of Life Expectancy at Birth for Male of Bangladesh

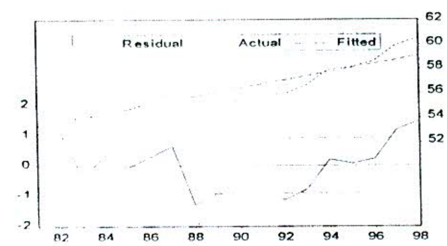


Figure 6.20 Actual, Fitted and Residual of Life Expectancy at Birth for Female of Bangladesh

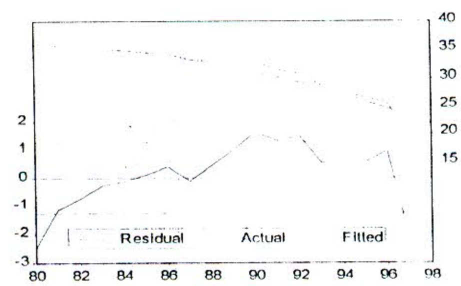


Figure 6.21 Actual, Fitted and Residual of Crude Birth Rate of Bangladesh

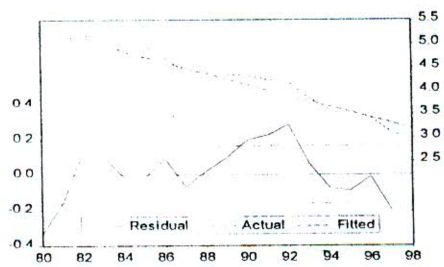


Figure 6.22 Actual, Fitted and Residual of Total Fertility Rate of Bangladesh

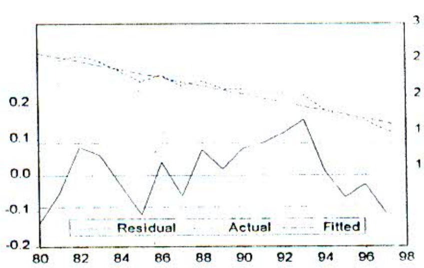


Figure 6.23 Actual, Fitted and Residual of Gross Reproduction Rate of Bangladesh

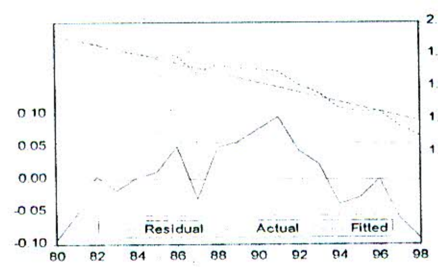


Figure 6.24 Actual, Fitted and Residual of Net Reproduction Rate of Bangladesh

Table 6.11 The Forecasted Values of Infant Mortality Rate (IMR), Crude Death Rate (CDR), Life Expectancy at Birth for Male, Life Expectancy at Birth for Female, Crude Birth Rate (CBR), Total Fertility Rate (TFR), Gross Reproduction Rate (GRR) and Net Reproduction Rate (NRR) of Bangladesh

Years	IMR	CDR	Life Expectancy (Male)	Life Expectancy (Female)	CBR	TFR	GRR	NRR
1999	57.28	5.75	60.03	59.38	20.32	3.07	1.52	1.37
2000	54.24	5.19	60.37	59.72	18.73	2.95	1.46	1.34
2001	51.36	4.64	60.71	60.05	17.05	2.83	1.40	1.31
2002	48.64	4.08	61.04	60.39	15.30	2.71	1.34	1.29
2003	46.06	3.53	61.38	60.72	13.47	2.59	1.29	1.26
2004	43.61	2.97	61.72	61.06	11.56	2.47	1.23	1.23
2005	41.3	2.42	62.05	61.39	9.58	2.36	1.20	1.20

It is observed that age structure for male, female and both sexes of the country have been distributed by negative exponential or modified negative exponential model. It is also found that surviving function and ASDRs follow 4th degree polynomial model where as ASFRs follows cubic polynomial model. Moreover, time trend model of CDR, life expectancy at birth for male and female, TFR, GRR and NRR follow simple linear model. On the other hand, time trend model of IMR and CBR follow semi-log linear model and quadratic polynomial model. These time trend models also provide forecasted values of the parameters. It is to be noted that the forecasted values of life expectancy at birth for male are always greater than that of female. It may be partially due to data error.

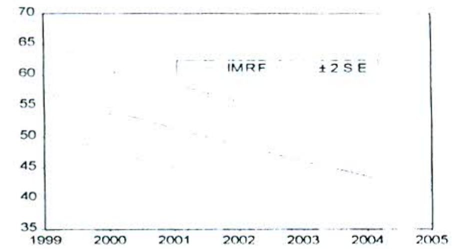


Figure: Forecasted Values of Infant Mortality Rate of Bangladesh

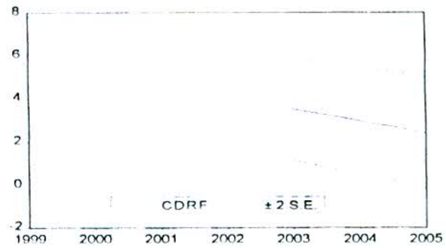


Figure: Forecasted Values of Crude Death Rate of Bangladesh

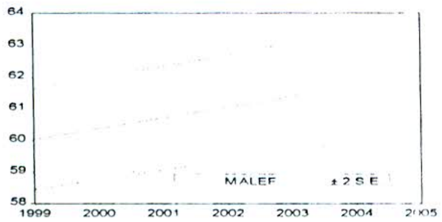


Figure: Forecasted Values of Life Expectancy at Birth for Male of Bangladesh

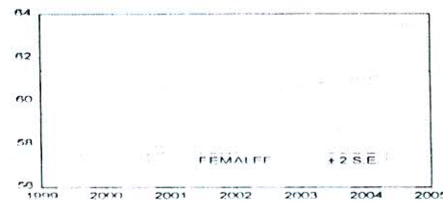


Figure: Forecasted Values of Life Expectancy at Birth for Female of Bangladesh

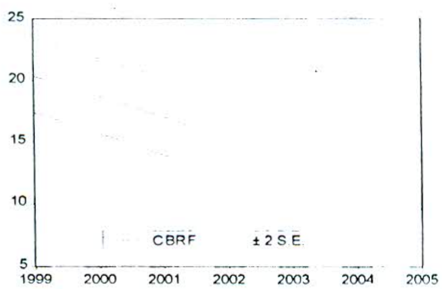


Figure: Forecasted Values of Crude Birth Rate of Bangladesh

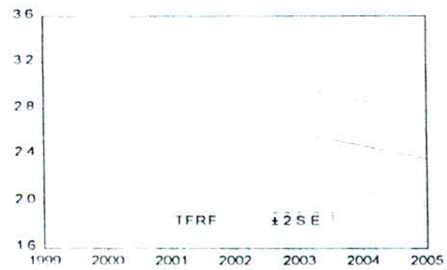


Figure: Forecasted Values of Total Fertility Rate of Bangladesh of Bangladesh

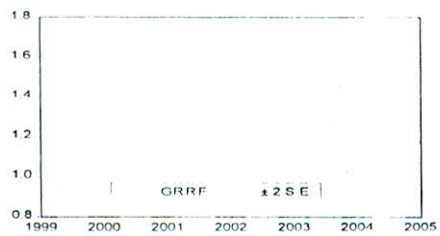


Figure: Forecasted Values of Gross Reproduction Rate of Bangladesh

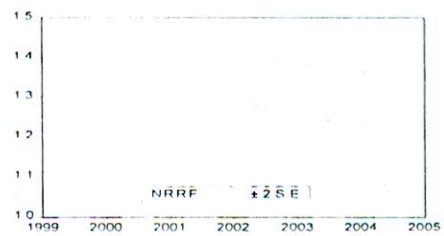


Figure: Forecasted Values of Net Reproduction Rate of Bangladesh

Figure 6.25 Forecasted Values of Infant Mortality Rate, Crude Death Rate, Life Expectancy at Birth for Male, Life Expectancy at Birth for Female, Crude Birth Rate, Total Fertility Rate, Gross Reproduction Rate and Net Reproduction Rate of Bangladesh.

Chapter 7

Conclusion

Chapter 7

Conclusion

Bangladesh is the most densely populated country with limited lands and resources. It is the ninth populous country in the world. The main characteristics of Bangladesh have been presented in the first chapter in section 1.3. Having these sorts of peculiarities, an attempt has been made for Modeling of Demographic Parameters of Bangladesh-An Empirical Forecasting.

Before going to do these, an attempt was made to estimate the demographic parameters (such as IMR, IDR, CDR, growth rate, CBR, ASDR, ASFR, CWR, GFR, TFR, GRR, NRR, RSR, J, VI) of Bangladesh. For this purpose, data on age distribution of population for male, female and both sexes, proportion not widowed for male and female, proportion of married female have been taken from the censuses of 1961, 1974, 1981 and 1991. It is found that census data are highly inaccurate. To remove this inaccuracy, data have been smoothened by latest smoothing method 4253H twice. Smoothing method has been accomplished using the package Minitab Release 12.1. To build time trend models of IMR, CDR, life expectancy at birth for male and female, CBR, TFR, GRR and NRR data have been taken from Statistical Year Book of Bangladesh.

Using the above input data adult mortality have been estimated and then linked with child mortality (l_2) to obtain l_x values using United Nations Model Life Tables for Developing Countries (1982) for male and female separately. Then abridged life tables have been constructed from the estimated l_x values. Demographic parameters of Bangladesh have been estimated to observe the trends and patterns of these estimated parameters. It is indicated that the age distribution of population for male, female and both sexes follow either modified negative exponential model or negative exponential model, surviving function for male and female follow 4th degree polynomial model, ASDR for male, female and both sexes also follow 4th degree polynomial where as ASFR follow 3rd degree polynomial model. Moreover, the study indicates that time trend of CDR, life expectancy at birth for male and female, TFR, GRR, NRR follow simple linear regression models, and IMR and CBR follow semi-log linear model and 2nd degree polynomial model respectively. In this study cross-validity prediction power is applied to check the validity of the models.

7.1 Findings of the Study

The results of this study have already been discussed in the respective section of different chapters. The major findings of this study have been summarized as follows:

Nine modified negative exponential models of age structure for male, female and both sexes for the census years 1961, 1981 and 1991 have been fitted using the software STATISTICA. While three negative exponential models have been fitted for the census year 1974. These models have been shown in section 6.3 of Chapter 6. Cross-validity prediction power and information on model fitting have been shown in Tables 6.1 and 6.2. It is found that all these models are more than 98% stable. It is also observed that all the parameters of the fitted models are highly significant explaining more than 98% of variation of the dependent variable.

Eight life tables, four for male and four for female have been constructed in Chapter 4 for the census years 1961, 1974, 1981 and 1991. It is observed that survival function (l_x) for male and female were showing decreasing trend with respect to ages in all the census years. It is also found that they were showing upward trend over time which are shown in Figures 4.1 and 4.2. The expectation of life at different ages of male and female have been presented in Table 4.1 to Table 4.8 and Figures 4.3 and 4.4. Eight 4th degree polynomial models of surviving functions (l_x) for male and female have been fitted and presented in section 6.4 of Chapter 6. The cross-validity prediction power and information on model fitting have been presented in Tables 6.3 and 6.4. It is found that all the parameters of the fitted models are highly significant explaining more than 96% variation and all the models are 93% stable. Time trend models of life expectancy at birth for male and female have been fitted and presented in section 6.7 of Chapter 6. It is seen that they follow simple linear regression models. The information on model fitting has been shown in Tables 6.9 and 6.10. It is seen that all the parameters of the fitted models are highly significant with more than 80% of variation explained and all the models are more than 76% stable.

Infant mortality rate (IMR) and infant death rate (IDR) have been estimated for the census years 1961, 1974, 1981 and 1991 and presented in Table 5.1 and Figures 5.1, 5.2. It is found that they were showing an increasing trend during 1961-1974 and then, they started to decrease up to 1991. Time trend model of IMR has been fitted and presented in section 6.7 of Chapter 6. It is seen that IMR follows semi-log linear model. The information on model fitting has been shown in Tables 6.9 and 6.10. It is found that all the parameters of the fitted model are highly significant explaining more than 95% variation and the model is more than 93% stable.

Age specific death rates (ASDR) for male, female and both sexes have been estimated and presented in Table 5.2 to Table 5.5 and Figure 5.3 to Figure 5.5. It is seen that they were showing downward trend over time. Twelve 4th degree polynomial models of ASDR for male, female and both sexes have been built and presented in the section 6.5 of Chapter 6. The cross-validity prediction power and information on model fitting have been launched in Tables 6.5 and 6.6. From the tables, it is seen that all the parameters of the fitted models are statistically significant explaining more than 92% variation and all the models are more than 88% stable.

Crude death rate (CDR), growth rate and crude birth rate (CBR) for male, female and both sexes have also been estimated and presented in Table 5.6. Time trend models of CDR and CBR have been fitted and presented in the section 6.7 of Chapter 6. It is seen that CDR and CBR follow simple linear model and 2nd degree polynomial model respectively. The results on model fitting have been shown in Tables 6.9 and 6.10. It is seen that all the parameters of the fitted models are highly significant with more than 85% and 93% of variation explained and the models are more than 77% and 91% stable respectively.

Age specific fertility rates (ASFR) have been estimated and presented in Table 5.7 where an increasing trend during 1961-1974 is observed while during the period 1974-1991 a downward trend is observed. Four 3rd degree polynomial models of ASFR have been fitted and presented in the section 6.6 of Chapter 6. The cross-validity prediction power and information on model fitting have been shown in Tables 6.7 and

6.8. It is observed that all the parameters of the fitted models are significant explaining more than 98% variation. It is also found that all these models are more than 89% stable.

Total fertility rate (TFR), gross reproduction rate (GRR) and net reproduction rate (NRR) have been estimated and presented in Table 5.8. Time trend models of TFR, GRR and NRR have been fitted and presented in the section 6.7 of Chapter 6. It is seen that they follow simple linear regression model. The results on model fitting have been shown in Tables 6.9 and 6.10. It is seen that all the parameters of the fitted models are highly significant explaining more than 90% variation and all the models are more than 88% stable. The forecasted values of these time trend models have been shown in Table 6.11.

General fertility rate (GFR) and child woman ratio (CWR), reproduction survival ratio (RSR), replacement index (J) and vital index (VI) have also been estimated and presented in Table 5.8.

7.2 Policy Implications and Further Study

In this study we have developed stationary models for age structure (C_x), number of surviving function (l_x), age specific death rates (ASDR) and age-specific fertility rates (ASFR) of Bangladesh for the census years 1961, 1974, 1981 and 1991. Interested researchers, Government and non-Government organization can use these models according to their needs. So, if one is interested to compute age structure (C_x values) he/she can use either negative exponential or modified negative exponential model. We also suggest for obtaining l_x values and ASDR one can use 4^{th} degree polynomial model in both cases. Again if any body is interested to get ASFR he/she can also use 3^{rd} degree polynomial model of ages.

We have also developed time trend models for infant mortality rate (IMR), crude death rate (CDR), life expectancy at birth for male and female, crude birth rate (CBR), total fertility rate (TFR), gross reproduction rate (GRR) and net reproduction rate (NRR) of Bangladesh. Researcher, planner and policy-maker can use these models according to their needs and they could also obtain forecasted values for future plan of action. Using simple linear regression model CDR, life expectancy at birth for male and female, TFR, GRR and NRR can be computed. We also suggest that to obtain IMR and CBR one can use semi-log linear model and quadratic polynomial model. Moreover, to obtain forecasted values it can easily help us these time trend models.

However, an attempt has been taken to develop various models for demographic parameters in this study in the context of Bangladesh. It is hoped that similar models can also be fitted to estimate the demographic parameters in relation to many other developing and developed countries. These models can be compared at various situations. Considering the similarities and dissimilarities of models for a parameter in the developing and developed countries, we may recommend some unique model or functional form for the certain parameter.

7.3 Limitations of the Study

Almost all the parameters have been estimated using indirect techniques and trends and patterns have been shown on the basis of those parameters and various models have been constructed using these parameters. It is obvious that the results depend on the estimated parameters that are dependent on the method of estimation. At the time of using these results a researcher and /or a reader should bear it in mind that the results are based on the parameters that are estimated using indirect techniques.

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Abbreviations

ASAR	Age Specific Accession Rates
ASDR	Age Specific Death Rates
ASFR	Age Specific Fertility Rates
ASSR	Age Specific Separation Rates
BBS	Bangladesh Bureau of Statistics
BFS	Bangladesh Fertility Survey
BRSFM	Bangladesh Retrospective Survey of Fertility and Mortality
CBR	Crude Birth Rate
CDR	Crude Death Rate
CPR	Contraceptive Prevalence Rate
CVPP	Cross Validity Prediction Power
CWR	Child Woman Ratio
GDP	Gross Domestic Product
GFR	General Fertility Rate
GNP	Gross National Product
GRR	Gross Reproduction Rate
IDR	Infant Death Rate
IMR	Infant Mortality Rate
NRR	Net Reproduction Rate
PEC	Post Enumeration Check
RSR	Reproduction Survival Ratio
SMAM	Singulate Mean Age at Marriage
TFR	Total Fertility Rate
UN	United Nations
VI	Vital Index
WFS	World Fertility Survey
WSMAM	Weighted Singulate Mean Age at Marriage

Symbols

$\hat{\pi}_x$	corrected proportion not widowed
π_x	proportion not widowed
l^f_{x-5} / $l^f_{17.5}$	survivorship probabilities for female
l^m_{x+5} / $l^m_{22.5}$	survivorship probabilities for male
l_2	child mortality
Y_x	logit survival function of l_x
Y^s_x	logit survival function of l^s_x
l^s_x	survival function of standard life table at an exact age x
l_x	survival function at an exact age x
s	sex ratio at birth
$\mathbf{e_x}$	life expectancy at age x
e^0_0	life expectancy at birth
${}_nd_x$	number of deaths in the age interval x to x+n in a life table
${}_nq_x$	probability of dying in the age group x to x+n in a life table
${}_np_x$	probability of surviving in the age group x to x+n in a life table
${}_nL_x$	person-years lived in the age interval x to x+n in a life table
T_x	total person-years lived in the age x and onward in a life table
p	proportion of remarriage
x	age or age group
B	number of live births during a calendar year
D	number of deaths during a calendar year
P	mid-year population
r	growth rate

J	replacement index
D_0	number of death of infants during a calendar year
P_0	mid-year population under age 1 year
C_x	age structure
p_{t_1}	initial population at time t_1
p_{t_2}	terminal population at time t_2
P_a^f	mid-year female population in the age group a
$\frac{B^F}{B^T}$	proportion of all births which are female
f_a	age specific fertility rates at age a
p(a)	probability of surviving from birth to age a
P_{0-4}	mid-year children under age 5 years
P_{15-49}^f	mid-year female population in the reproductive ages 15 to 49 years
L_{0-4}^f	life table female population under age 5 years
L_{0-4}^m	life table male population under age 5 years
L_{15-49}^f	life table female population in the reproductive ages 15 to 49 years
e/exp	exponential
R^2	coefficient of determination
ρ_{cv}^2	cross-validity prediction power