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Theoretical Investigations of Turbulence and Magneto-Hydrodynamic Turbulence in Incompressible Fluid

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THEORETICAL INVESTIGATIONS OF TURBULENCE AND MAGNETO-HYDRODYNAMIC TURBULENCE IN INCOMPRESSIBLE FLUID

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BY

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BANGLADESH

Dedicated to my Parents

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and

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Certified that the Thesis entitled "***Theoretical investigations of turbulence and Magneto-hydrodynamic turbulence in incompressible fluid***" submitted by Mr. **Md. Anowarul Islam** in fulfillment of the requirement for the degree of Doctor of Philosophy in Mathematics, University of Rajshahi, Rajshahi-6205, Bangladesh has been completed under my supervision. I believe that this research work is an original one and it has not been submitted elsewhere for any degree.

(M. Shamsul Alam Sarker)
Supervisor.

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PREFACE

The thesis entitled "**Theoretical investigations of turbulence and Magneto-hydrodynamic turbulence in incompressible fluid**" is being presented for the award of the degree of Doctor of Philosophy in Mathematics. It is the outcome of my researches conducted in the Department of Mathematics, Rajshahi University, Rajshahi, Bangladesh under the guidance of **Dr. M. Shamsul Alam Sarker,** Professor, Department of mathematics, Rajshahi University, Rajshahi-6205, Bangladesh.

The thesis has been divided into **six chapters**. The first is a general introductory chapter and gives the general idea of turbulence and Magneto-hydrodynamics turbulence and its principal concepts. Some results and theories, which are needed in the subsequent chapter, have been included in this chapter. A brief review of the past researches related to this thesis has also been given.

In the **second chapter** we have discussed the decay of temperature fluctuation in homogeneous turbulence before the final period for the case of multi-point and multi-time. Two-point, two-time and three-point, three-time Fourier-transformed temperature equations is made determinate by neglecting the fourth-order correlation terms. Finally, we have obtained the decay law of temperature fluctuation energy before the final period.

In the **third chapter**, the decay of MHD turbulence at times before the final period for the case of multi-point and multi-time has been studied. In this chapter, first we have obtained the two-point, two-time and the three-point, three-time correlation equations. Then the correlation equations are converted into spectral form by taking their Fourier transforms and then the magnetic energy decay law before the final period for the case of multi-point and multi-time has been obtained.

In the **fourth chapter** we have studied the magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in MHD turbulence at times before the final period. Here we have obtained multi-point and single-time correlation equations after neglecting the quadruple correlation terms in comparison with lower-order correlation terms applicable at times before the final period. These equations are converted into spectral form by taking their Fourier transformed. Finally, the decay law has been obtained.

The **fifth chapter** is divided into two parts. In **part-A** of this chapter, we have studied the decay of dusty fluid turbulence before the final period in a rotating system. In this problem we have considered the two- and three-point correlation equations and solved these equations after neglecting the quadruple correrelation terms applicable at times before the final period. Finally the energy decay law of fluctuating velocity is obtained.

In **part-B**, of the fifth chapter, the problem of **part-A** is extended for the case of MHD turbulence.

The **chapter six** is also divided into two parts. In **part-A** of this chapter we have defined distribution functions for simultaneous velocity and concentration of dilute contaminant undergoing a first order chemical reaction. Some properties of the constructed distribution functions have been discussed. Equation for the evolution of one- and two-point distribution function for velocity and concentration fields have been derived.

In **part-B** of the chapter we have considered the distribution function for velocity, magnetic and concentration fields of reacting (first order) fluid. Here, **part-B** is the extension work of **part-A** of the chapter in MHD turbulence.

In the **last chapter (chapter VII)**, we have discussed the effect of strong uniform magnetic field on acceleration covariance in MHD turbulence of dusty fluid in a rotating system. An expression for acceleration covariance is obtained in terms of the defining scalars and it is assumed that the whole system is rotating with a uniform angular velocity. The regions are considered where the inhomogeneity due to rotation plays no important role.

The following research papers, which are extracted from this thesis, have been accepted for publication or presented in different international mathematical conference or communicated in different national and international journals.

(1) Decay of MHD turbulence before the final period for the case of multi-point and multi-time. (Accepted for publication, "Indian Journal of Pure and Applied Mathematics").

(2) Decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time. (Presented in the "International conference on Geometry, Analysis and Applications" Department of Mathematics, Faculty of Science, Banaras Hindu University, 21^{st} – 24^{th} August, 2000 and communicated for publication).

(3) First order reactant in MHD turbulence before the final period of decay. (Presented in the "25^{th} International Nathiagalis Summer College on Physics and Contemporary needs", 26^{th} Jun – 15^{th} July, Pakistan and communicated for publication).

(4) Decay of dusty fluid turbulence before the final period in a rotating system. (Communicated for publication).

(5) Decay of dusty fluid MHD turbulence before the final period in a rotating system. (Communicated for publication).

(6) Distribution functions in the statistical theory for velocity and concentration undergoing a first order reaction. (Communicated for publication).

(7) Effect of very strong magnetic field on acceleration covariance in MHD turbulence of dusty fluid in a rotating system. (Communicated for publication).

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GENERAL INTRODUCTION

1.1 DEFINATION AND STATISTICAL NATURE OF TURBULENCE

Turbulence is the most common, important and complicated kind of fluid motion. From the beginning of the study of fluid dynamics, the turbulent flow is an unsolved problem. Since the Navier-stokes equation is a non-linear partial differential equation, the nonlinear terms make the solution of turbulent flow more and more difficult. Turbulent flows are very common in nature, especially in atmosphere, rivers, seas and oceans, that is almost everywhere.

In turbulent flow, the steady motion of the fluid is only steady in so far as the temporal mean values of the velocities and the pressure are concerned whereas actually both the velocities and the pressures are irregularly fluctuating. The velocity and the pressure distributions in turbulent flows as well as the energy losses are determined mainly by the turbulent fluctuations. The essential characteristic of turbulent flow is that the turbulent fluctuations are random in nature. In 1937, Taylor and Von Karman [106] gave the following definition:

"Turbulence is an irregular motion which in general makes its appearance in fluids, gaseous or liquid, when the flow past solid surfaces or even when neighboring streams of the same fluid flow past over one another".

According to this definition, the flow has to satisfy the condition of irregularity. Indeed, this irregularity is a very important feature. Because of irregularity, it is impossible to describe the motion in all details as a function of time and space co-ordinates. But, fortunately, turbulent motion is irregular in the sense that it is possible to describe it by laws of probability. It appears possible to indicate distinct average values of various quantities, such as velocity, pressure, temperature etc, and this is very important. Therefore, it is not sufficient just to say that turbulence is an irregular motion. Yet we do not have a clear-cut definition of turbulence. This is rather difficult. Hinze [34] suggests in his book turbulence:

"Turbulent fluid motion is an irregular condition of flow in which various quantities show random variation with time and space coordinates, so that statistically distinct average values can be discerned".

Turbulence is a continuum phenomenon governed by the Navier-Stokes equation and the continuity equation. Its small-scale structure is assumed to be large compared with molecular length scale. Thus the continuum approximation seems to be valid as long as the minimum eddy size is much larger than the mean free path. The consequence of very small-scale structure is the enhancement of transport processes. The most important property of the turbulent motion is it greatly increased rates of momentum, mass and energy transport by irregular small-scale motions. These rates are extremely larger than the corresponding rates due to molecular diffusion.

In view of random fluctuating motions of a fluid having statistical properties, it has often raised the question how the Navier-Stokes equations can really describe such random motions, since a given set of initial conditions determine the motions for all subsequent times. This question has not yet been answered completely. However, it has been demonstrated both theoretically and experimentally that the Navier-Stokes equations have tremendous simplifying power under suitable conditions. On the other hand, if the

Navier-Stokes equations are in fact, inadequate then there is a definite need for new formulation of proper equations. Until such proper equations are developed, its seems reasonable to accept the Navier-Stokes equations for the study of turbulence.

Another difficulty arises from the strong non-linearity of the Navier-Stokes equations. This non-linearity leads to an infinite number of equations for all possible moments of the velocity field. This system of equations is very complicated, and any sub-system of this system is always non-closed in the sense that it contains more unknowns than the number of equations in the given system. For instance, the dynamical equation for second order moments involves third order moments, that for third order moments involves fourth order moments and so on. This is so called the closure problem in the statistical theory of turbulence. This is perhaps the most difficult and formidable problem in turbulence theory.

Turbulent flow always occurs from instabilities of laminar motions at very high Reynolds numbers. These instabilities are closely associated with direct interaction of the non-linear inertia term and the viscous terms in the Navier-Stokes equation. Instability to small perturbation is also another feature of turbulent flows.

Turbulent motion is three dimensional and rotational. It is also characterized by the random distribution of vorticity in which there is no unique relation between the frequency and the wave number of the Fourier modes. It is essentially diffusive and dissipative. The vorticity dynamics plays an important role in the statistical description of turbulence.

Based upon averaging procedures, considerable theoretical and experimental studies have been made of the statistical properties of ensembles of turbulent flows under macroscopically identical external conditions. So far, these studies are based on suitable mathematical simplification, physically plausible assumptions and on model equations. Unfortunately, from mathematical and physical point of view, neither the classical nor the modern theory of turbulence is entirely satisfactory. Indeed turbulence is still one of the most poorly or partially understood phenomena in all of fluid mechanics.

It is now generally recognized that turbulent motion is the more natural state of fluid motion. Therefore, its study is extremely important from theoretical as well as practical point of view.

1.2 SHORT EARLY HISTRY OF TURBULENCE

The history of turbulence began with the pioneering works of Reynolds [85,86] and Reyleigh [84]. It was prandtl [82] who first advanced a semi-empirical momentum transfer theory of turbulence based on the concept of mixing length (the mean distance through which a fluid mass in a turbulent flow conserves its momentum). Prandtl's theory was then successfully applied to the turbulent flow of a liquid in a circular pipe and also to the meteorological problem of wind distribution in the layer of air adjacent to the ground. However, his theory has had a serious weakness in the sense that it requires some adhoc assumption on the mixing length. On the other hand, G. I. Taylor [104,105] first recognized the random fluctuation of turbulent flows and formulated a theory of turbulence based on the concept of vorticity transfer. Although in certain simple cases the vorticity transfer theory predicted as good result as the momentum transfer, still the former theory on the whole was less successful than the latter. At the same time, he first formulated a statistical theory of isotropic turbulence. In fact, advances in the early development of the semi-empirical approach to the theory of turbulence were made notable by Taylor, Prandtl and Von Karman. In his famous papers Taylor [107,108] made further significant contributions to the understanding of the physical nature of turbulence based upon the Navier-Stokes equations. He formulated another method of investigation in which the turbulent elements are assumed to consist of small eddies of different macroscopic lengths, and the energy of turbulent motions is supposed to be distribution among these eddies. His analysis reveals the existence and usefulness of velocity correlation tensor, and the Fourier transform of the correlation between two velocities, which leads to the concept of energy spectrum function. The central problem of investigation is then the energy spectrum function of wave number and time which describes the distribution of kinetic energy over the various Fourier wave number components of turbulence. It has also become clear that the nonlinear inertia terms of the Navier-Stokes equations play a significant role in the statistical description of turbulence. The important consequences of the non-linearity are the existence of an interaction between the turbulent elements of different length scales, and the skewness of the probability distribution of the difference between the velocities at two points of the turbulent field. This pioneering work of Taylor has served as a basis of all subsequent developments of the theory of turbulence. Simultaneously, Von Karman [112,113] alone and in collaboration with Howarth [114] made some further progress on the turbulence theory based on the idea of self preservation of the shape of velocity product during decay process. The combined works of Taylor, Van Karman and others constitute a significant progress towards the early classical theory of isotropic and homogeneous turbulence. However, the model of isotropic and homogeneous turbulence is a special case of turbulent flow, and is, in general, unsuitable for the description of any real turbulent flows because the assumption of isotropy and homogeneity are not fulfilled for the real flows.

In the following, instead of giving a detailed account of the historical development of the subject, we shall confine to mere concepts and method of turbulence together with a few theories of turbulence, which have been used in subsequent chapters.

1.3. THE NAVIER–STOKES AND THE CONTINUITY EQUATIONS

The Navier-Stokes and the continuity equations for an incompressible viscous fluid flow are

$$\frac{\partial \hat{u}}{\partial t} + (u \cdot \nabla)\, \hat{u} = -\frac{1}{\rho} \nabla p + \nu\, \nabla^2 \hat{u} \qquad (1.3.1)$$

$$\nabla \cdot \hat{u} = 0 \qquad (1.3.2)$$

where $\hat{u} = \hat{u}\,(\hat{r}, t\,)$ represent the velocity field, p is the pressure, ρ is the constant density and ν is the kinematic viscosity. The Reynolds number (the ratio of inertial and viscous terms in (1.3.1)) is UL/ν where L is the characteristic length scale in which the velocity varies in magnitude U .

The use of the Navier-Stokes equations for the study of turbulence is perhaps justified since the each number of an incompressible turbulence flow is small. However, there is still a controversy for the following additional reasons. First, the mathematical theory of the Navier-Stokes equations is incomplete in the sense that there are no general existence and uniqueness theorem which ensure the well posedness of the system (1.3.1) – (1.3.1). Second the closure problem of the Navier-Stokes equations is inconclusive. In view of these inherent difficulties, Ladyzhenskaya [54] and others suggest to abandon the application of the Navier-Stokes equations, especially for the study of turbulence. According to Ladyzhenskaya, if a biharmonic damping term $-\lambda\,\nabla^4\,\hat{u}$ is included in the right hand side of the Navier-Stokes equations (1.3.1), the existence and the uniqueness of solutions can be established for all $\lambda \rangle 0$. She also formulated new equations for the description of the motion of the motion of an incompressible viscous fluid and explained the advantages of her new equations relative to the Navier-Stokes equations.

It is important to make an observation from (1.3.1) – (1.3.2). We first take the divergence of (1.3.1) and use (1.3.2) to obtain

$$\nabla^2 p_1 = -\frac{\partial^2 u_i\, u_j}{\partial x_i\, \partial x_j} \tag{1.3.3}$$

where $p_1 = p/\rho$ is often referred to as the kinematic pressure.

It follows from (1.3.3) that the pressure field is determined by the velocity distribution, and satisfies the Poisson equation.

1.4 REYNOLDS RULES OF AVERAGES

Reynolds [85] was the first to introduce elementary statistical motion into the consideration of turbulent flow. In the theoretical investigations of turbulence, he assumed the physical quantities in the flow field as

$$u_i = \overline{u_i} + u_i', \qquad p = \overline{p} + p', \qquad \rho = \overline{\rho} + \rho', \qquad T = \overline{T} + T'$$

Here the quantities with bar denote the mean values and those with primes are fluctuations. Furthermore, $\overline{u_i'} = \overline{p'} = \overline{T'} = 0$.

In the study of turbulence we often have to carry out an averaging procedure not only on single quantities but also on products of quantities.

Consider three arbitrary statistically dependent physical quantities A, B and C, each consisting of a mean and a fluctuating part, i.e.,

$A = \overline{A} + a$, $\quad B = \overline{B} + b$, and $C = \overline{C} + c$ then , $\overline{A} = \overline{\overline{A} + a} = \overline{\overline{A}} + \overline{a} = \overline{A}$, whence $\overline{a} = 0$

In the above relations we used the properties that the average of the sum is equal to the sum of the average, and the average of a constant time B is equal to the constant times the average of B.

Next, $\overline{A\,B} = \overline{(\overline{A} + a)(\overline{B} + b)} = \overline{\overline{A}\;\overline{B} + \overline{A}\,b + \overline{B}\,a + a\,b} = \overline{\overline{A}\;\overline{B}} + \overline{\overline{A}b} + \overline{\overline{B}a} + \overline{ab}$

$$= \overline{\overline{A}}\;\overline{\overline{B}} + \overline{\overline{A}}\;\overline{b} + \overline{\overline{B}}\;\overline{a} + \overline{a\,b} = \overline{A}\;\overline{B} + \overline{a\,b}$$

Consequently, $\quad \overline{\overline{A}\;\overline{B}} = \overline{\overline{A}}\;\overline{\overline{B}} = \overline{A}\;\overline{B}$

Note that the average of a product is not equal to the product of the averages. Terms such as $\overline{a\,b}$ are called correlations. For the product of three quantities, we have

$$\overline{A\,B\,C} = \overline{(\bar{A}+b)(\bar{B}+b)(\bar{C}+c)} = \bar{A}\,\bar{B}\,\bar{C} + \bar{A}\,\overline{bc} + \bar{B}\,\overline{ac} + \bar{C}\,\overline{ab} + \overline{a\,b\,c}$$

1.5 AVERAGING METHOD AND EQUATION OF TURBULENCE ENERGY SPECTRUM

Method of averaging is indispensable for the statistical formulation of the theory of turbulence. There are three different kinds of averaging procedures that are found to be useful for the study of turbulent flows. These include the time average, space average and the ensemble average. The time average is very useful for statistically steady turbulence, in which time scales are much larger than the time scale of turbulent fluctuations. The space average has a definite advantage for homogeneous turbulence. On the other hand, the ensemble average (or the statistical average over a large number of identical system) is more general than the time and space averages and very useful for the study of inhomogeneous, non stationary turbulent flow. This type of averaging can be applied to any flow. Most of the modern theories have used the ensemble averaging procedure for describing the statistical properties of turbulence. However, like the time and the space averages, the physical interpretation of the ensemble average is not so simple.

In general any turbulent field is completely determined by the hierarchy of correlations.

$$\langle u_i(r,t)\rangle, \quad \langle u_i(r,t)\, u_j(r',t)\rangle, \quad \langle u_i(r,t)\, u_j(r',t)\, u_m(r'',t)\rangle \qquad (1.5.1)$$

where $\langle \quad \rangle$ denotes the ensemble average defined in Leslie's Book [55].

In homogeneous isotropic turbulence the first correlation represents the mean velocity, and is simply zero. The pair correlation $\langle u_i(r), u_j(r')\rangle$ is often considered to be a sufficient description of turbulent flows. The higher order correlations are assumed to give less and less information so that only a finite number of correlations are required to determine the statistical properties of turbulence. This is a possible method of reducing the infinite hierarchy of equations into a closed set.

The double correlation tensor $R_{ij}(\hat{r},\hat{x};t)$ for two points separated by the space vector r is defined by

$$R_{ij}(\hat{r},\hat{x},t) = \langle u_i(\hat{x} - \frac{1}{2}\hat{r},t)\, u_j(\hat{x} + \frac{1}{2}\hat{r},t)\rangle \quad . \tag{1.5.2}$$

Similarly, the triple correlation tensor T_{ijk} or higher correlation tensors can be introduced.

The Fourier transform of R_{ij} with respect to $\hat{r}$ defined by

$$\phi_{ij}(\hat{k},\hat{x},t) = \frac{1}{(2\pi)^3} \int\int\int_{-\infty}^{\infty} e^{-i(\hat{k}.\hat{r})} R_{ij}(\hat{r},\hat{x},t)\, d\hat{r} \tag{1.5.3}$$

represents the energy spectrum function $E(\hat{k};t)$ in the sense that it describes the distribution of kinetic energy over the various wave number components of turbulent flows. The Fourier transform defined above can be treated as generalized functions or distributions in the sense of Lighthill [56]. It follows from the inverse Fourier transform that

$$\frac{1}{2}\langle u^2\rangle = \frac{1}{2}\langle u_i(\hat{x})\, u_i(\hat{x})\rangle = \frac{1}{2} R_{ii}(0,\hat{x},t) = \int_0^\infty E(\hat{k},t)\, d\hat{k} \tag{1.5.4}$$

so that $E(\hat{k},t)$ represents the density of contributions to the kinetic energy in the wave number space k, thus the investigation of the energy spectrum function $E(\hat{k},t)$ is the central problem of the dynamics of turbulence. After some algebraic manipulation of the Navier-Stokes equations at two points combined with averaging process and the Fourier transform, it can be shown that $E(k,t)$ satisfies the dynamical equation.

$$\frac{\partial E(\hat{k},t)}{\partial t} = T(\hat{k},t) - 2k^2 \nu E(\hat{k},t) \tag{1.5.5}$$

where the terms of this equation represent contributions of the Navier-Stokes equations, and in particular , $T(\hat{k},t)$ represents the contributions due to transfer of energy from other wave numbers.

It follows from the condition of incompressibility that the pressure term in (1.3.1) does not contribute any term to equation (1.5.5). This implies that the net effect of pressure forces is to conserve the total energy in the wave number space. On the other hand the non-linear inertia terms in (1.3.1) also conserve the energy and the net effect of inertia forces is to spread energy over all wave number. In other words, the inertia forces can only transfer energy from one range of wave number to another in the energy spectrum on the wave number space, and this spectral energy transfer is in fact, an important consequence of the Navier-Stokes equations. However, the direction of the energy transfer has not yet been established, but the conjecture is that the transfer is from the smaller towards the larger wave numbers that is, from large to smaller eddies.

The last term of (1.5.5) represents the dissipation of energy by molecular viscosity. The action of viscosity leads to a decrease in the kinetic energy of disturbances with the wave number, which is proportional to the intensity of the disturbances multiplied by $2\nu k^2$.

It also follows from the conservation of energy by the non-linear inertia terms that

$$\int_0^\infty T(k)\,dk = 0 \tag{1.5.6}$$

so that (1.5.5) yields

$$\frac{d}{dt}\left(\frac{1}{2}\langle u_i\, u_i\rangle\right) = \frac{\partial}{\partial t}\int_0^\infty E(\hat{k},t)\,d\hat{k} = -\in(t)\ . \tag{1.5.7}$$

Where from (1.5.5) it follows that

$$\in(t) = 2\nu\int_0^\infty k^2 E(\hat{k},t)\,d\hat{k}\ . \tag{1.5.8}$$

This clearly represent the rate of energy dissipation and shows that small scale or high wave number components are dissipated more rapidly by viscosity than large scale or low wave number components.

The summary of the above discussion is that the pressure and the nonlinear inertial forces separately conserve the total energy of turbulence, where as the viscous forces dissipate it.

1.6 ISOTROPIC AND HOMOGENEOUS TURBULENCE

The turbulence is called isotropic if its statistical features have no preference for any specific direction and minimum number of quantities and relations are required to describe its structure and behavior.

Since turbulence is a very complicated problem, in order to bring out the essential features of the turbulence problem we have to study the simplest type of turbulence. In isotropic turbulence the mean value of any function of velocity components and their space derivatives are unaltered by any rotation or reflection of axes of references. Thus, in particular

$$\overline{u_1^2} = \overline{u_2^2} = \overline{u_3^2} \quad \text{and} \quad \overline{u_1 u_2} = \overline{u_2 u_3} = \overline{u_3 u_1} .$$

Isotrophy introduces a great simplicity into the calculations. The study of isotropic turbulence may also be of practical importance, since far from solid boundaries it has been observed that $\overline{u_1^2}$, $\overline{u_2^2}$, $\overline{u_3^2}$ tend to become equal to one another , e.g. in the natural winds at a sufficient height above the ground and in a pipe flow near the axis.

Another simplest type of turbulence is homogeneous turbulence. It is defined as the turbulence having quantitatively the same structure in all parts of the flow field. In a homogeneous turbulent flow field the statistical characteristics are invariant for any translation in the space occupied by the fluid.

Most of the theoretical works in turbulence and MHD turbulence concern homogeneous and isotropic field in an incompressible fluid at rest. Throughout the present work, we have also assumed the homogeneity and isotropy of the turbulent flow field.

1.7 MAGNETOHYDRODYNAMICS AND TURBULENCE

Magnetohydrodynamics (MHD) is an important branch of Fluid Dynamics. MHD is the science, which deals with the motion of highly conduction fluids in the presence of a magnetic field. The motion of the conducting fluid across the magnetic field generates electric currents which change the magnetic field, and the action of the magnetic field on these currents gives rise to mechanical force which modifies the flow of the field.

There are two basic approaches to the problem, the macroscopic fluid continuum model known as MHD, and microscopic statistical model known as plasma dynamics. We shall be concerned here only with the MHD, that is electrically conducting fluids, and study the problems about MHD turbulent flow.

The magnetohydrodynamic turbulence is the study of the interaction between a magnetic field and the turbulent motions of an electrically conduction fluid. The interaction between the velocity and magnetic fields results in a transfer of energy between the kinetic and magnetic spectra (or modes).

Modern applications of magnetohydrodynamics in the fields of propulsion, nuclear fission, and electrical power generation make the problem of magneto-hydrodynamic turbulence one of considerable interest to engineers since turbulent phenomena seem to be inherent in almost all types of flow problems.

The fundamental equations of magnetohydrodynamics for an incompressible fluid are:

$$\frac{\partial \hat{u}}{\partial t} + (\hat{u}.\nabla)\hat{u} = -\frac{1}{\rho}\nabla p + \frac{\rho_e}{\rho}\hat{E} + \frac{\mu_e}{\rho}\hat{J} \times \hat{H} + \nu\nabla^2\hat{u} + \hat{F} \qquad (1.7.1)$$

$$\nabla.\hat{u} = 0 \qquad (1.7.2)$$

$$\frac{K}{c}\frac{\partial \hat{E}}{\partial t} = curl\ \hat{H} - 4\pi\hat{j} \qquad (1.7.3)$$

$$\frac{\mu_e}{c}\frac{\partial \hat{H}}{\partial t} = -curl\ \hat{E} \qquad (1.7.4)$$

$$\nabla.\hat{H} = 0 \qquad (1.7.5)$$

$$\hat{J} = \sigma(c\hat{E} + \mu_e\hat{u} \times \hat{H}) + \rho_e\frac{\hat{u}}{c} \qquad (1.7.6)$$

where $\hat{u}$, the velocity vector; $\hat{F}$, the body force; p, the pressure; ρ, the density of the fluid which is constant; ρ_e, the excess electric charge; $\hat{E}$, the electric field strength; μ_e, the magnetic permeability; $\hat{J}$, the electric current density; $\hat{H}$, the magnetic field strength; ν, the coefficient of kinematic viscosity; k, the dielectric constant; c, the speed of Light; σ, the electrical conductivity; ∇, the gradient operator, and t is the time.

When conductivity σ of the fluid tends to infinity the electric field strength $\hat{E}$, at each point must tends to the value $\frac{\mu_e\hat{u} \times \hat{H}}{c}$, otherwise the current $\hat{J}$ given by equation

(1.10.6) will become very large even when the slightest mass motions are present. Hence when σ is large we may assume that,

$$\hat{E} = -\mu_e \frac{\hat{u} \times \hat{H}}{c} \tag{1.7.7}$$

a relation which will be increasingly valid as $\sigma \to \infty$.

An important consequence of relation (1.10.7) is that under the circumstance in which this is a good approximation the energy in the electric field is of the order of $|\hat{u}|^2 / c^2$ of the energy in the magnetic field and can, therefore, be neglected. Consequently in this approximation which is known as the approximation of magnetohydrodynamics. We have to consider only the interaction between the two fields $\hat{u}$ and $\hat{H}$.

In the magnetohydrodynamics approximation, Maxwell equation (1. 7. 3) becomes,

$$\hat{J} = \frac{1}{4\pi} curl\, \hat{H} \ . \tag{1.7.8}$$

In the framework of the approximations (1.7.7) and (1.7.8) the Navier-Stokes equations are modified to take into account the electromagnetic body force (assuming that there is no body force $\hat{F}$) and equation (1.7.1) becomes

$$\frac{\partial \hat{u}}{\partial t} + (\hat{u}.\nabla)\hat{u} = \frac{\mu_e}{4\pi\rho} curl\, \hat{H} \times \hat{H} - \frac{1}{\rho}\nabla p + \nu\nabla^2 \hat{u} \ . \tag{1.7.9}$$

Again, in the approximation (1.7.7), Maxwell equation (1.7.4) becomes

$$\frac{\partial \hat{E}}{\partial t} = curl\, (\hat{u} \times \hat{H}) \ . \tag{1.7.10}$$

In a higher approximation in which the loss of energy by joule heat is allowed for equation (1.7.10) is modified to [4].

$$\frac{\partial \hat{H}}{\partial t} - curl\,(\hat{u} \times \hat{H}) = \lambda \nabla^2 \hat{H} \qquad (1.7.11)$$

where $\lambda = (\,4\pi\mu_e\sigma)^{-1}$ is the magnetic diffussivity.

The magnetic field intensity $\hat{H}$ is a solenoidal vector, and in an incompressible fluid the velocity $\hat{u}$ is also a solenoidal vector. When we use this property of $\hat{u}$ and $\hat{H}$ equations, (1.7.9) and (1.7.11) can be written in the form [13] as

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_k}{\partial x_k} - \frac{\mu_e}{4\pi\rho}\frac{\partial}{\partial x_k} H_i H_k = -\frac{1}{\rho}\frac{\partial}{\partial x_k}(p + \mu_e \frac{|\hat{H}|^2}{8\pi}) + \nu\nabla^2 u_i \qquad (1.7.12)$$

and , $$\frac{\partial H_i}{\partial t} + \frac{\partial}{\partial x_k}(H_i u_k - u_i H_k) = \lambda\nabla^2 H_i \qquad (1.7.13)$$

where, here and in the sequel, summation over the repeated indices is to be understood. Equation (1.7.12.) and (1. 7. 13) form the basis of Batchelor's discussion [4].

Chandrasekhar [13] extended the invariant theory of turbulence to the case of magnetohydrodynamics. He introduced the new variable

$$\hat{h} = \sqrt{\frac{\mu_e}{4\pi\rho}}\,\hat{H} \qquad (1.7.14)$$

Which has the dimension of a velocity (known as Alfeven's velocity).

In terms of $\hat{h}$, equations, (1.7.12) and (1.7.13) can be written as

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial P_n}{\partial x_i} + \nu\nabla^2 u_i \qquad (1.7.15)$$

and , $$\frac{\partial h_i}{\partial t} + \frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \nu\nabla^2 h_i \qquad (1.7.16)$$

where, $P_n = \frac{p}{\rho} + \frac{1}{2}\left|\hat{h}\right|^2$ is the total MHD pressure.

Chandrasekhar [13,14] in his theory, considered the correlation's between $\hat{u}$ and $\hat{h}$ at two points P and P' in the field of isotropic turbulence in the same manner as in ordinary turbulence. Here, we have the double correlation, $\overline{u_i u'_j}$, $\overline{h_i h'_j}$ and $\overline{u_i h'_j}$, and triple correlation, $\overline{u_i u_j u'_k}$, $\overline{h_i h_j u'_k}$, $\overline{u_i u_j h'_k}$, $\overline{h_i h_j h'_k}$, $\overline{(h_i u_j - u_i h_j) h'_k}$ and $\overline{(h'_j u'_k - h'_k u'_j) u_i}$, where the subscripts refer to the components of the vectors $i, j, k = 1, 2, 3$.

Each of these double and triple correlation depends on one scalar function in the case of isotropic turbulence because the divergence of both $\hat{u}$ and $\hat{h}$ is zero.

One of the results of chandrasekhar's theory [13,14] shows that the kinetic energy is dissipated into heat by viscosity and transformed into magnetic energy by stretching the lines of magnetic force. He has also shown that the magnetic energy is gained from the stretching of magnetic force and dissipated into heat. The gain in magnetic energy is equal to the loss by stretching of the lines of magnetic forces.

1.8 RATE OF REACTION AND ORDER OF REACTION

The rate of change of concentration as a function of time and may be expressed either in the form of disappearance of reactants or the appearance of new products.

According to Bansal [1] the general reaction equation in which A and B are transformed to give P

$$aA + bB \rightarrow cP \quad , \qquad (1.8.1)$$

the reaction rate can be written as

$$-\frac{1}{a}\frac{d[A]}{dt}, \qquad -\frac{1}{b}\frac{d[B]}{dt}, \qquad +\frac{1}{c}\frac{d[P]}{dt}$$

and the rate law may be written in the form of equation

$$-\frac{1}{a}\frac{d[A]}{dt} = k[A]^n[B]^m \qquad (1.8.2)$$

where $[A]$, $[B]$ and $[P]$ denote the active concentrations in moles/litre of species A, B and P, t represent the time, n and m are integers, k is the proportionality constant referred to as the reaction rate constant or specific rate constant and a, b, c are the stoichiometric coefficients.

Since the concentrations of A and B are diminishing, $-\frac{1}{a}\frac{d[A]}{dt}, -\frac{1}{b}\frac{d[B]}{dt}$ are negative number while $\frac{1}{c}\frac{d[P]}{dt}$ is positive. Any of these derivatives may be used to express the rate of the reaction.

The order of a reaction is the algebraic sum of the exponents of all the concentration terms, which appear in the rate law (1.8.2). For the reaction given in equation (1.8.1) the rate law may be expressed as

$$-\frac{1}{a}\frac{d[A]}{dt} = k[A]^n[B]^m$$

where n is the order of the reaction with respect to A, and m is the order of the reaction with respect to B. The over all order of the reaction is given by the sum $(n+m)$.

A reaction is said to be of the first order if the rate of the reaction is proportional to the concentration of only one of the reacting substances. Let us consider a reaction in which A is being transformed to product P, ($A \rightarrow P$). If C is the concentration of A, then the differential rate law can be written as

$$-\frac{dC}{dt} = k_1[C] \tag{1.8.3}$$

where k_1 is the first order rate constant and t the time . This can be rearranged to

$$-\frac{dC}{C} = k_1 dt$$

(1.8.4)

Integrate both sides of the above equation to obtain

$-\ln C = k_1 t + \theta$, where θ is a constant of integration.

1.9. DISTRIBUTION FUNCTIONS IN TURBULENCE

Probability distribution functions have been described in the various classic text books in the past, but the dynamical equations describing the time evolution of the finite dimensional probability distributions in turbulence were first proposed by Lundgren [59] and Monin [68,69]. Lundgren [59] considered a large ensemble of identical fluid system in turbulent state. In his consideration each member of the ensemble is an incompressible fluid in an infinite space with velocity $\hat{u}(\hat{r},t)$, satisfying the continuity and Navier-Stokes equations. The only difference in the members of ensemble is the initial conditions that vary from member to member. He considered a function $G(\hat{u}(\hat{r}_1,t), \hat{u}(\hat{r}_2,t),\cdots)$ whose ensemble is given as $\langle G(\hat{u}(\hat{r}_1,t), \hat{u}(\hat{r}_2,t),\cdots)\rangle$ and defined one point distribution function $f_1(\hat{r}_1,\hat{v}_1,t)$ such that $\int f_1(\hat{r}_1,\hat{v}_1,t)d\hat{v}_1$ is the probability that the velocity at a point $\hat{r}_1$ at time t is in element $d\hat{v}_1$ about $\hat{v}_1$ and is given by

$$f_1(\hat{r}_1,\hat{v}_1,t) = \langle \delta(\hat{u}(\hat{r}_1,t) - \hat{v}_1)\rangle .$$

And two points' distribution function is given by

$$f_2(\hat{r}_1,\hat{v}_1,\hat{r}_2,\hat{v}_2,t) = \langle \delta(\hat{u}(\hat{r}_1,t) - \hat{v}_1)\delta(\hat{u}(\hat{r}_2,t) - \hat{v}_2)\rangle .$$

In short one and two point distribution functions are denoted as $f_1^{(1)}$ and $f_2^{(1,2)}$. Here δ is the dirac-delta function, which is defined as

$$\int \delta(\hat{u} - \hat{v})dv = \begin{cases} 1 & \text{at the point } \hat{u} = \hat{v} \\ 0 & \text{elsewhere} \end{cases} ,$$

and $\langle \quad \rangle$ denote the ensemble average.

1.10. A BRIEF DESCRIPTION OF PAST RESEARCHES RELEVENT TO THE THESIS WORK

The essential characteristic of turbulent flows is that turbulent fluctuations are random in nature and therefore, by the application of statistical laws, it has been possible to give the idea of turbulent fluctuations. The turbulent flows, in the absence of external agencies always decay. Millionshtchikov [65], Batchelor and Townsend [2], Proudman and Reid

[83], Tatsumi [102], Deissler [21,22], and Ghosh [30,31] had given various analytical theories for the decay process of turbulence so far.

Batchelor and townsend [2] studied the decay of turbulence in the final period. They said that the final period of a turbulent motion occurs when the effects of the inertia force in the momentum equation are negligible. Deisler [21,22] studied the decay of turbulence at times before the final period. Also Loeffer and Deissler [57], discussed the decay of temperature fluctuation in homogeneous turbulence before the final period. In their approach they considered the two and three point correlation equations and solved these equations after neglecting the fourth and higher order correlation terms in comparison to the lower order correlation terms. Using Deissler's theory Kumar and Patel [52] studied the concentration fluctuation of dilute Contaminants undergoing a first order chemical reaction before the final period of decay for the case of multi-point and single-time. Kumar and Patel [53] also extended their problem of [52] for the case of multi-point and multi-time.

Likewise the hydrodynamic turbulence, MHD turbulent fluctuations are random in nature. The statistical laws can also be applied in MHD turbulence. Sarker and Kishore [91] studied the decay of MHD turbulence. Kishore and Upathdyay [49], also studied the decay of MHD turbulence in rotating system. In both the cases they obtained the decay law for the case of multi-point and single time before the final period.

By considering the above theories, we have studied the **Chapter II, Chapter III and Chapter IV.**

In **chapter II**, we have studied the decay of temperature fluctuation in homogeneous turbulence before the final period for the case of multi-point and multi-time.

In **chapter III**, we have considered the MHD turbulence and derived the decay law for magnetic field fluctuation before the final period for the case of multi-point and multi-time.

In **chapter IV**, we have derived a decay law for the magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in MHD turbulence at times before the final period. In this case two and three- point correlation equations are made determinate by neglecting the quadruple correlation in comparison with lower order correlation applicable at times before the final period.

In geophysical flows, the system is usually rotating with a constant angular velocity. Such large Scale flows are generally turbulent. When the motion is referred to axes, which rotate steadily with the bulk of the fluid, the Coriolis and centrifugal force must be supposed to act on the fluid. The coriolis force due to rotation plays an important role in a rotating system of turbulent flow, while the centrifugal force with the potential is incorporated into the pressure.

Funada, Tutiya and Ohji [29] considered the effect of coriolis force on turbulent motion in presence of strong magnetic field. Kishore and Dixit [43], Kishore and singh [40], Dixit and Upadhyay [24], Kishore and Golsefied [45] and Kishore and Sarker [48] discussed the effect of coriolis force on acceleration and vorticity covariance in ordinary and MHD turbulent flow. Shimomura and Yoshizawa [97], Shimomura [98,99] discussed the statistical analysis of turbulent viscosity, turbulent scalar flux and turbulent shear flows respectively in a rotation system by two-Scale Direct- Interaction approach.

Saffman [89] derived an equation that described the motion of a fluid containing small dust particles which in applicable to laminar flows as well as turbulent flow. Using the equations given by Saffman, Michael and Miller [64] discussed the motion of dusty gas occupying the Semi-infinite space above a rigid plane boundary. Sinha [100], Sarker [92], and Sarker and Rahman [93], considered dust particle on their own works.

In **part-A of Chapter V**, we have studied the decay of dusty fluid turbulence before the final period in a rotating system, using the Deissler's [21] approach.

In **part-B of chapter V**, the problem of **part-A** of the chapter has been extended for the case of MHD turbulence.

Various analytical theories in the statistical theory of turbulence have been given in the past by Hopt [35], Kraichanan [51], Edward [26] and Herring [33] but the dynamical equations describing the time evolution of the finite dimensional probability distribution of turbulent quantities were first derived by Lundgren [58]. He derived the dynamical equations for one and two-point probability distribution functions of velocity fluctuation and compared with the BBGKY heirarchy of equation in the kinetic theory of gases. Further Lundgren [59] considered a similar problem for non-homogeneous turbulence.

The basic difficulty is that the above theories faced to closure problem. Lyubimov and Ulinch [61,62] made some general approaches to closure problem for multidimensional probability density equations. Two other closure hypotheses for the probability distribution equation of single time values were investigated by Fox [27], Lundgren [60] and Bray and Moss [11]. They considered the probability density function of a progress variable in an idealized premised turbulent flow. Bigler [10] gave the hypothesis that in turbulent flow, the thermochemical quantities can be related locally to few Scalars. Further Janicka et al. [38] and Pope [79] gave a more suitable model for the probability density functions of scalars in turbulent reacting flows.

Recently pope [81] derived the transport equation for the joint probability density function of velocity and scalars in turbulent flows and obtained the solution by using the Moutocarlo method. More recently Kollman and Janickal [50] obtained the transport equation for the probability density function of a scalar in turbulent shear flow and considered closure model based on the gradient flux model. Kishore [39] derived the equations for the evolution of one- and two-point distribution functions for MHD turbulent flow. Sarker and Kishore [90] also studied the distribution function in the statistical theory of convective MHD turbulence.

The above theories give the basic ideas for the **chapter VI** in which we have considered the distribution functions for simultaneous velocity and concentration of a dilute contaminant undergoing a first order reaction in turbulent flow. The **chapter VI** is divided into two parts. In **part-A and part-B**, we have considered ordinary and MHD turbulent flow respectively.

Tailor [100] pointed out that the equation of motion of turbulence relates the pressure gradient and acceleration of the fluid particles and the mean square acceleration can be determined from the observation of the diffusion of the marked fluid particles. The behavior of dust particles in a turbulent flow depends on the concentration and size of the particles with respect to scale of turbulent fluid. A good deal of theoretical studies of MHD turbulent has been made during last fifteen years. Ohji [1964] presented a first order theory of turbulence of an electrically conducting fluid in the presence of a uniform magnetic field which is so strong that the non-linear mechanism as well as the dissipation terms are of minor important when comparing with the external coupling terms. Ohji [1978], discussed the effect of a very strong uniform magnetic field on incompressible MHD turbulence in presence of a constant angular velocity and Hall effect. Kishore and Dixit [1982] studied the effect of a uniform magnetic field on acceleration covariance in MHD turbulence. Dixit [1989] discussed the effect of uniform magnetic field on acceleration covariance in MHD dusty fluid turbulence.

In the **chapter VII**, we have discussed the effect of a strong magnetic field on acceleration covariance in MHD turbulence of dusty fluid in a rotating system.

CHAPTER - II

DECAY OF TEMPERATURE FLUCTUATIONS IN HOMOGENEOUS TURBULENCE BEFORE THE FINAL PERIOD FOR THE CASE OF MULTI-POINT AND MULTI-TIME.

2.1. INTRODUCTION

Corrsin [18,19] made an analytical discussion on the problem of turbulent temperature fluctuations using the approaches employed in the statistical theory of turbulence. His result pertains to the final period of decay and, for the case of appreciable convective effects, to the 'energy' spectral form in specific wave number ranges. Further work along this same line had been done by Oruga [73].

Deissler [21] developed a theory for homogeneous turbulence, which was valid for times before the final period. Using Deissler's theory Loeffer and Deissler [57] studied the temperature fluctuation in homogeneous turbulence. In their study, they presented the theory which is valid during the period for which the fourth- and higher- order correlation terms are negligible compared to the second- and third-order correlation terms. By considering the Deissler's same theory, Kumar and Patel [52] studied the first order reactant in homogeneous turbulence before the final period for the case of multi-point and single-time consideration. Kumar and Patel's problem [52] is extended to the case of multi-point and multi-time concentration correlation by the same authors [53] and the numerical results of [53] carried-out by Patel [78].

In this work the method of [21] is used to study the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time.

2.2. CORRELATION AND SPECTRAL EQUATIONS

For an incompressible fluid with constant properties and for negligible frictional heating, the energy equation may be written as

$$\frac{\partial \widetilde{T}}{\partial t}+\widetilde{u}_i\frac{\partial \widetilde{T}}{\partial x_i}=\gamma\frac{\partial^2 \widetilde{T}}{\partial x_i \partial x_i} \tag{2.2.1}$$

where $\widetilde{T}$ and $\widetilde{u}_i$ are instantaneous values of temperature and velocity; $\gamma=\frac{k}{\rho c_p}$, thermal diffusivity ; k, thermal conductivity ; ρ, fluid density; c_p, heat capacity at constant pressure; x_i, space co-ordinate; t, time; and the repeated subscripts are summed from 1 to 3.

Breaking these instantaneous values into time average and fluctuating components as $\widetilde{T}=\langle T\rangle+T$ and $\widetilde{u}_i=\langle u_i\rangle+u_i$, and using the conditions of homogeneity ($\langle T\rangle=0$, $\frac{\partial\langle T\rangle}{\partial x_i}=0$, $\langle u_i\rangle=0$) allows equation (2.2.1) to be written

$$\frac{\partial T}{\partial t}+u_i\frac{\partial T}{\partial x_i}=\left(\frac{\nu}{p_r}\right)\frac{\partial^2 T}{\partial x_i \partial x_i} \tag{2.2.2}$$

where $p_r = \frac{\nu}{\gamma}$, prandtle number; ν, kinematic viscosity.

Equation (2,2.2) is assumed to hold at the arbitrary point P. For the point P' the corresponding equation can be written

$$\frac{\partial T'}{\partial t} + u'_i \frac{\partial T'}{\partial x'_i} = (\frac{\nu}{p_r}) \frac{\partial^2 T'}{\partial x'_i \partial x'_i}. \tag{2.2.3}$$

Multiplying equation (2.2.2) by T', equation (2.2.3) by T, and taking ensemble average, result in

$$\frac{\partial \langle TT' \rangle}{\partial t} + \frac{\partial \langle TT'u_i \rangle}{\partial x_i} + \frac{\partial \langle TT'u'_i \rangle}{\partial x'_i} = (\frac{\nu}{p_r}) \cdot \frac{\partial^2 \langle TT' \rangle}{\partial x_i \partial x_i}, \tag{2.2.4}$$

$$\frac{\partial \langle TT' \rangle}{\partial t} + \frac{\partial \langle TT'u'_i \rangle}{\partial x'_i} + \frac{\partial \langle TT'u'_i \rangle}{\partial x'_i} = (\frac{\nu}{p_r}) \frac{\partial^2 \langle TT' \rangle}{\partial x'_i \partial x'_i} \tag{2.2.5}$$

with the continuity equation

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial u'_i}{\partial x'_i} = 0. \tag{2.2.6}$$

Angular bracket $\langle \cdots\cdots \rangle$, which is used to denote an ensemble average.

Using the transformations

$$\frac{\partial}{\partial x_i} = -\frac{\partial}{\partial x_i} = \frac{\partial}{\partial x'_i}, \qquad (\frac{\partial}{\partial t})_{t'} = (\frac{\partial}{\partial t})_{\Delta t} - \frac{\partial}{\partial \Delta t}, \qquad \frac{\partial}{\partial t'} = \frac{\partial}{\partial \Delta t}$$

into equations, (2.2.4) and (2.2.5), one obtains

$$\frac{\partial \langle TT' \rangle}{\partial t} - \frac{\partial \langle u_i TT' \rangle}{\partial r_i}(-\hat{r}, -\Delta t, t+\Delta t) + \frac{\partial \langle u'_i TT' \rangle}{\partial r_i}(\hat{r}, \Delta t, t) = 2(\frac{\nu}{p_r}) \frac{\partial^2 \langle TT' \rangle}{\partial r_i \partial r_i} \tag{2.2.7}$$

and

$$\frac{\partial\langle TT'\rangle}{\partial\Delta t} - \frac{\partial\langle u_i TT'\rangle}{\partial r_i}(-\hat{r},-\Delta t, t+\Delta t) = (\frac{\nu}{p_r})\frac{\partial^2\langle TT'\rangle}{\partial r_i \partial r_i}. \qquad (2.2.8)$$

It is convenient to write this equation in spectral form by use of the following three-dimensional Fourier transforms

$$\langle TT'(\hat{r},\Delta t,t)\rangle = \int_{-\infty}^{\infty}\langle \tau\tau'(\hat{K},\Delta t,t)\rangle \exp[\hat{i}(\hat{K}.\hat{r})]d\hat{K}, \qquad (2.2.9)$$

$$\langle u_i TT'(\hat{r},\Delta t,t)\rangle = \int_{-\infty}^{\infty}\langle \phi_i \tau\tau'(\hat{K})\rangle \exp[\hat{i}(\hat{K}.\hat{r})]d\hat{K} \qquad (2.2.10)$$

and

$$\langle u_i' TT'(\hat{r},\Delta t,t)\rangle = \langle u_i TT'(-\hat{r},-\Delta t,t+\Delta t)\rangle = \int_{-\infty}^{\infty}\langle \phi_i \tau\tau'(-\hat{K},-\Delta t,t+\Delta t)\rangle \exp[\hat{i}(\hat{K}.\hat{r})]d\hat{K} \qquad (2.2.11)$$

(Interchange are made between the points P and P')

where $\hat{K}$ is known as a wave number vector and the magnitude of $\hat{K}$ has the dimension 1/length and can be considered to be the reciprocal of an eddy size.

Substitution of equations, (2.2.9) - (2.2.11) into equations, (2.2.7) and (2.2.8) leads to the spectral equation

$$\frac{d\langle\tau\tau'\rangle}{dt} + 2\frac{\nu}{p_r}k^2\langle\tau\tau'\rangle = ik_i[\ \langle\phi_i\tau\tau'(\hat{K},\Delta t,t)\rangle - \langle\phi_i\tau\tau'(-\hat{K},-\Delta t,t+\Delta t)\ ], \qquad (2.2.12)$$

$$\frac{d\langle\tau\tau'\rangle}{dt} + 2\frac{\nu}{p_r}k^2\langle\tau\tau'\rangle = -ik_i\langle\phi_i\tau\tau'(-\hat{K},-\Delta t,t+\Delta t). \qquad (2.2.13)$$

In equations, (2.2.12) and (2.2.13), the quantity $\tau\tau'(\hat{K})$ may be interpreted as a temperature fluctuation 'energy' contribution of thermal eddies of size 1/k. The time derivative of this 'energy' as a function of the convective transfer to other wave numbers and the 'dissipation' due to the action of thermal conductivity.

2.3. THREE-POINT, THREE-TIME CORRELATION AND SPECTRAL EQUATIONS

In order to obtain the three-point three-time correlation and spectral equation, we write the Navier-Stokes equation at the point P, energy equations at the points P' and P'' separated by the vectors $\hat{r}$ and $\hat{r}'$

$$\frac{\partial u_j}{\partial t} + \frac{\partial}{\partial x_i}(u_j u_i) = -\frac{1}{\rho}\frac{\partial p}{\partial x_j} + \nu\frac{\partial^2 u_j}{\partial x_i \partial x_i}, \tag{2.3.1}$$

$$\frac{\partial T'}{\partial t} + u_i'\frac{\partial T'}{\partial x_i'} = \left(\nu/P_r\right)\frac{\partial^2 T'}{\partial x_i' \partial x_i'} \tag{2.3.2}$$

and

$$\frac{\partial T''}{\partial t} + u_i''\frac{\partial T''}{\partial x_i''} = \left(\nu/P_r\right)\frac{\partial^2 T''}{\partial x_i'' \partial x_i''}. \tag{2.3.3}$$

Multiplying equations, (2.3.1)-(2.3.3) by $T'T''$ $u_i T''$ and $u_j T'$ respectively and then taking ensemble average, we obtained

$$\frac{\partial \langle u_j T'T''\rangle}{\partial t} + \frac{\partial \langle u_j T'T'' u_i\rangle}{\partial x_i} = -\frac{1}{\rho}\frac{\partial \langle p T'T''\rangle}{\partial x_j} + \nu\frac{\partial^2 \langle u_j T'T''\rangle}{\partial x_i \partial x_i}, \tag{2.3.4}$$

$$\frac{\partial \langle T' u_j T''\rangle}{\partial t'} + \frac{\partial \langle u_i' T' u_j T''\rangle}{\partial x_i'} = \frac{\nu}{P_r}\frac{\partial^2 \langle T' u_j T''\rangle}{\partial x_i' \partial x_i'} \tag{2.3.5}$$

and

$$\frac{\partial\langle T''u_jT'\rangle}{\partial t''}+\frac{\partial\langle u_i''T''u_jT'\rangle}{\partial x_i''}=\frac{\nu}{p_r}\frac{\partial^2\langle T''u_jT'\rangle}{\partial x_i''\partial x_i''}. \tag{2.3.6}$$

Using the transformation

$$\frac{\partial}{\partial x_i}=-\left(\frac{\partial}{\partial r_i}+\frac{\partial}{\partial r_i'}\right),\quad \frac{\partial}{\partial x_i'}=\frac{\partial}{\partial r_i},\quad \frac{\partial}{\partial x_i''}=\frac{\partial}{\partial r_i'},$$

$$\left(\frac{\partial}{\partial t}\right)_{t',t''}=\left(\frac{\partial}{\partial t}\right)_{\Delta t,\Delta t'}-\frac{\partial}{\partial\Delta t}-\frac{\partial}{\partial\Delta t'},\quad \frac{\partial}{\partial t'}=\frac{\partial}{\partial\Delta t},\quad \frac{\partial}{\partial t''}=\frac{\partial}{\partial\Delta t'},$$

the equations, (2.3.4)-(2.3.6) can be written as

$$\frac{\partial\langle u_jT'T''\rangle}{\partial t}-\left(\frac{\partial}{\partial r_i}+\frac{\partial}{\partial r_i'}\right)\langle u_jT'T''u_i\rangle+\frac{\partial\langle u_iT'u_jT''\rangle}{\partial r_i}+\frac{\partial\langle u_i''T''u_jT'\rangle}{\partial r_i'}=-\frac{1}{\rho}\left[\frac{\partial\langle pT'T''\rangle}{\partial r_j}+\frac{\partial\langle pT'T''\rangle}{\partial r_j'}\right]$$

$$+\nu\left(\frac{\partial}{\partial r_i}+\frac{\partial}{\partial r_i'}\right)^2\langle u_jT'T''\rangle+\frac{\nu}{p_r}\left(\frac{\partial^2}{\partial r_i\partial r_i}+\frac{\partial}{\partial r_i'\partial r_i'}\right)\langle u_jT'T''\rangle, \tag{2.3.7}$$

$$\frac{\partial\langle u_jT'T''\rangle}{\partial\Delta t}+\frac{\partial\langle u_i'T'u_jT''\rangle}{\partial r_i}=\frac{\nu}{p_r}\frac{\partial^2\langle u_jT'T''\rangle}{\partial r_i\partial r_i}, \tag{2.3.8}$$

$$\frac{\partial\langle u_jT'T''\rangle}{\partial\Delta t'}+\frac{\partial\langle u_i'T'u_jT''\rangle}{\partial r_i'}=\frac{\nu}{p_r}\frac{\partial^2\langle u_jT'T''\rangle}{\partial r_i'\partial r_i'}. \tag{2.3.9}$$

The six-dimensional Fourier transforms for quantities in the equations, (2.3.7)-(2.3.9) may be defined as

$$\langle u_jT'T''\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\beta_j\theta'\theta''\rangle\exp[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}d\hat{K}', \tag{2.3.10}$$

$$\langle u_i u_j T'T'' \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \beta_i \beta_j \theta'\theta'' \rangle \exp[\hat{i}(\hat{K}.\hat{r} + \hat{K}'.\hat{r}')] d\hat{K} d\hat{K}', \tag{2.3.11}$$

$$\langle pT'T'' \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \alpha \theta'\theta'' \rangle \exp[\hat{i}(\hat{K}.\hat{r} + \hat{K}'.\hat{r}')] d\hat{K} d\hat{K}', \tag{2.3.12}$$

Interchanging the points P' and P'', shows that $\langle u_j u_i'' T''T' \rangle = \langle u_j u_i' T'T'' \rangle$.

By use of this fact and equations, (2.3.10)-(2.3.12), the equations (2.3.7)-(2.3.9) may be transformed as

$$\frac{\partial \langle \beta_j \theta'\theta'' \rangle}{\partial t} + \frac{\nu}{p_r}\left[(1+p_r)k^2 + 2p_r k_i k_i' + (1+p_r)k'^2 \right] \langle \beta_j \theta'\theta'' \rangle$$

$$= \frac{1}{\rho} i(k_j + k_j') \langle \alpha \theta'\theta'' \rangle , \tag{2.3.13}$$

$$\frac{\partial \langle \beta_j \theta'\theta'' \rangle}{\partial \Delta t} + \frac{\nu}{p_r} k^2 \langle \beta_j \theta'\theta'' \rangle , \tag{2.3.14}$$

$$\frac{\partial \langle \beta_j \theta'\theta'' \rangle}{\partial \Delta t'} + \frac{\nu}{p_r} k'^2 \langle \beta_j \theta'\theta'' \rangle \tag{2.3.15}$$

with the assumption that the quadruple correlation terms are neglected because they decay faster than the lower order correlation terms.

If the derivative with respect to x_j is taken of the momentum equation (2.3.1) for point P, the equation multiplied through by $T'T''$ and taken the ensemble averages, the resulting equation is

$$\frac{\partial^2\langle u_j u_i T'T''\rangle}{\partial x_j \partial x_i} = -\frac{1}{\rho}\frac{\partial^2\langle pT'T''\rangle}{\partial x_j \partial x_j} \quad . \tag{2.3.16}$$

In terms of the displacement vectors $\hat{r}$ and $\hat{r}'$, equation (2.3.16) becomes

$$\left[\frac{\partial^2}{\partial r'_j \partial r'_i} + 2\frac{\partial^2}{\partial r'_j \partial r_i} + \frac{\partial^2}{\partial r_j \partial r_i} \right]\langle u_j u_i T'T''\rangle =$$

$$-\frac{1}{\rho}\left[\frac{\partial^2}{\partial r'_j \partial r'_j} + 2\frac{\partial^2}{\partial r'_j \partial r_j} + \frac{\partial^2}{\partial r_j \partial r_j} \right]\langle pT'T''\rangle \tag{2.3.17}$$

which in Fourier-space can be written as

$$\langle \alpha\theta'\theta''\rangle = -\frac{\rho\left(k'_j k'_i + 2k'_j k_i + k_j k_i\right)}{\left(k'_j k'_j + 2k'_j k_j + k_j k_j\right)}\langle \beta_j \beta_i \theta'\theta''\rangle \quad , \tag{2.3.18}$$

Equation (2.3.18) can be used to eliminate $\langle \alpha\theta'\theta''\rangle$ from the equation (2.3.13).

2.4. SOLUTION FOR TIMES BEFORE THE FINAL PERIOD.

To obtain the equation for final period of decay the third-order fluctuation terms are neglected compared to the second-order terms. Analogously, it would be anticipated that for times before but sufficiently near to the final period the fourth-order fluctuation terms should be negligible in comparison with the third-order terms. If this assumption is made the equation (3.3.14) shows that the term $\langle \alpha\theta'\theta''\rangle$, associated with the pressure fluctuations should also be neglected. Thus the equation (3.3.10) simples to

$$\frac{d\langle \beta_j \theta'\theta''\rangle}{dt} + \frac{\nu}{p_r}\left[(1+p_r)k^2 + 2p_r k_i k'_i + (1+p_r)k'^2 \right]\langle \beta_j \theta'\theta''\rangle = 0 \quad . \tag{2.4.1}$$

Inner multiplication of equations, (2.4.1), (2.3.14) and (2.3.15) by k_j, and integration between t_0 and t to give

$$\langle k_j \beta_j \theta'\theta'' \rangle = f_j \exp\{-\frac{\nu}{p_r}[(1+p_r)k^2 + 2p_r kk'\cos\xi + (1+p_r)k'^2](t-t_0)\}, \tag{2.4.2}$$

$$\langle k_j \beta_j \theta'\theta'' \rangle = g_j \exp(-\frac{\nu}{p_r} k^2 \Delta t) \tag{2.4.3}$$

and

$$\langle k_j \beta_j \theta'\theta'' \rangle = q_j \exp(-\frac{\nu}{p_r} k'^2) \tag{2.4.4}$$

For above relation to be consistent, we have

$$\langle k_j \beta_j \theta'\theta'' \rangle = \langle k_j \beta_j \theta'\theta'' \rangle_0 \exp[-\frac{\nu}{p_r}\{k^2[(1+p_r)(t-t_0)+\Delta t] + 2p_r kk'\cos\xi(t-t_0)$$

$$+ k'^2[(1+p_r)(t-t_0)+\Delta t'] \tag{2.4.5}$$

where $\langle k_j \beta \beta' \theta'' \rangle_0$ is the value of $\langle k_j \beta \beta' \theta'' \rangle$ at $t = t_0$ and ξ is the angle between k and k'.

Letting $r' = 0$ in equation (2.3.10) and comparing the result with the equation (2.2.10), shows that

$$\langle k_j \phi_j \tau\tau'(\hat{K}, \Delta t, t) \rangle = \int_{\infty}^{\infty} \langle k_j \beta_j \theta'\theta''(\hat{K}, \hat{K}', \Delta t, 0, t) \rangle d\hat{K}' . \tag{2.4.6}$$

Substituting the equation (2.4.5) and (2.4.6) into the equation (2.2.12), we obtain

$$\frac{\partial E}{\partial t} + 2 \cdot \frac{\nu}{p_r} k^2 E = W \tag{2.4.7}$$

where $E = 2\pi k^2 \langle \tau\tau' \rangle$, the energy spectrum function and

$$W = \int_0^\infty ik_j [\ \langle \beta_j \theta'\theta'' \rangle (\hat{K}, K') - \langle \beta_j \theta'\theta'' \rangle (-\hat{K}, -\hat{K}')\ ]_0 (2\pi)^2 k^2 k'^2$$

$$\exp[-\frac{\nu}{p_r}\{(1+p_r)(k^2+k'^2)(t-t_0)+k^2\Delta t+2p_r kk'(t-t_0)\cos\xi d(\cos\xi)\}dk'. \qquad (2.4.8)$$

Here $dK'(= dK_1' dK_2' dK_2')$ is written in terms of k' and ξ (cf. Deissler[22]).

In order to find the solution completely and following Loeffler and deissler [57], we assume that

$$ik_j[\langle \beta_j \theta'\theta'' \rangle (\hat{K}, \hat{K}') - \langle \beta_j \theta'\theta'' \rangle (-\hat{K}, -\hat{K}')]_0 = -\frac{\delta_0}{(2\pi)^2}(k^2 k'^4 - k^4 k'^2) \qquad (2.4.9)$$

where, δ_0 is a constant depending on the initial condition. The negative sign is placed in front of δ_0 in order to make the transfer of energy from small to large wave numbers of positive values of δ_0.

Substituting equation (2.4.9) into equation (2.4.8) and completing the integration with respect to $\cos\xi$ and k', one obtains

$$W' = -\frac{\delta_0 \sqrt{\pi}\, p_r^{\frac{5}{2}}}{4\nu^{\frac{3}{2}}(t-t_0)^{\frac{3}{2}}(1+p_r)^{\frac{5}{2}}} \exp\left[-k^2\nu \frac{1+2p_r}{p_r(1+p_r)}\left(t-t_0+\frac{1+p_r}{1+2p_r}\Delta t\right)\right]$$

$$\times\left[\frac{15 p_r k^4}{4\nu^2(t-t_0)^2(1+p_r)} + \frac{1}{(t-t_0)}\left\{\frac{5 p_r^2}{\nu(1+p_r)^2} - \frac{3}{2\nu}\right\}k^6 + \left\{\frac{p_r^3}{(1+p_r)^3} - \frac{p_r}{1+p_r}\right\}k^8\right]$$

$$-\frac{\delta_0\sqrt{\pi}\,p_r^{\frac{5}{2}}}{4\nu^{\frac{3}{2}}(t-t_0+\Delta t)^{\frac{3}{2}}(1+p_r)^{\frac{5}{2}}}\exp\left[-k^2\nu\frac{1+2p_r}{p_r(1+p_r)}(t-t_0+\frac{p_r}{1+p_r}\Delta t)\right]$$

$$\times\left[\frac{15p_rk^4}{4\nu^2(t-t_0+\Delta t)^2(1+p_r)}+\left\{\frac{5p_r^2}{\nu(1+p_r)^2}-\frac{3}{2\nu}\right\}\frac{k^6}{(t-t_0+\Delta t)}+\left\{\frac{p_r^3}{(1+p_r)^3}-\frac{p_r}{1+p_r}\right\}k^8\right].$$

(2.4.10)

The series of equation (2.4.10) contain only even power of k. It can be shown that

$$\int_0^\infty W dk = 0 \qquad (2.4.11)$$

which indicates that the conditions of continuity and homogeneity are maintains.

The linear equation (2.4.7) can be solved to give

$$E=\exp\left[-2\frac{\nu}{p_r}k^2(t-t_0+\frac{\Delta t}{2})\right]\int W\exp\left[2\frac{\nu}{p_r}k^2(t-t_0+\frac{\Delta t}{2})\right]dt$$

$$+J(k)\exp\left[-2\frac{\nu}{p_r}k^2\left(t-t_0+\frac{\Delta t}{2}\right)\right] \qquad (2.4.12)$$

where $J(k)=\frac{N_0k^2}{\pi}$ is a constant of integration and can be obtained as by Corrsin [18].

Substituting the values of W and $J(k)$ into the equation (3.4.11), and integrating with respect to t, we get

$$E=\frac{N_0k^2}{\pi}\exp\left[-2\frac{\nu}{p_r}k^2(t-t_0+\frac{\Delta t}{2})\right]+\frac{\delta_0\sqrt{\pi}\,p_r^{\frac{5}{2}}}{4\nu^{\frac{3}{2}}(1+p_r)^{\frac{7}{2}}}$$

$$\times \exp\left[-\nu k^2 \frac{1+2p_r}{p_r(1+p_r)}\left(t-t_0+\frac{1+p_r}{1+2p_r}\Delta t\right]\right[\; \frac{3p_r k^4}{2\nu^2(t-t_0)^{5/2}}+\frac{p_r(7p_r-6)k^6}{3\nu(1+p_r)(t-t_0)^{3/2}}$$

$$-\frac{4(3p_r^2-2p_r+3)k^8}{3(1+p_r)^{5/2}(t-t_0)^{1/2}}+\frac{8\nu^{1/2}(3p_r^2-2p_r+3)k^9}{3(1+p_r)^{5/2}p_r^{1/2}}F(\eta)\;\Big]$$

$$+\frac{\delta_0\sqrt{\pi}\,p_r^{5/2}}{4\nu^{3/2}(1+p_r)^{7/2}}\exp\left[-\nu k^2 \frac{1+2p_r}{p_r(1+p_r)}\left(t-t_0+\frac{p_r}{1+2p_r}\Delta t\right]\right[\; \frac{3p_r k^4}{2\nu^2(t-t_0+\Delta t)^{5/2}}$$

$$+\frac{p_r(7p_r-6)k^6}{3\nu(1+p_r)(t-t_0)^{3/2}}-\frac{4(3p_r^2-2p_r+3)k^8}{3(1+p_r)^{5/2}(t-t_0+\Delta t)^{1/2}}+\frac{8\nu^{1/2}(3p_r^2-2p_r+3)k^9}{3(1+p_r)^{5/2}p_r^{1/2}}F(\eta)$$

(2.4.13)

where $F(\eta)=e^{-\eta^2}\int_0^{\eta} e^{x^2}dx$, $\quad \eta = k\sqrt{\dfrac{\nu(t-t_0)}{p_r(1+p_r)}}$ or $\eta = k\sqrt{\dfrac{\nu(t-t_0+\Delta t)}{p_r(1+p_r)}}$.

By setting $\hat{r}=0$ in equation (2.2.9) and use is made of the definition of E, the result is

$$\frac{\langle TT'\rangle}{2}=\frac{\langle T^2\rangle}{2}=\int_0^{\infty} E\,dk \;. \qquad (2.4.14)$$

Substituting equation (2.4.13) into the equation (2.4.14) and integrating with respect to k, gives

$$\frac{\langle T^2\rangle}{2}=\frac{N_0 p_r^{3/2}\left(\Gamma+\frac{\Delta\Gamma}{2}\right)^{-3/2}}{8\nu^{3/2}\sqrt{2\pi}}+\frac{\delta_0\pi p_r^{6}}{4\nu^{6}(1+p_r)(1+2p_r)^{5/2}}\Bigg[\;\frac{9}{16\Gamma^{5/2}\left(\Gamma+\frac{1+p_r}{1+2p_r}\Delta\Gamma\right)^{5/2}}$$

$$+\frac{9}{16(\Gamma+\Delta\Gamma)^{5/2}\left(\Gamma+\frac{p_r}{1+2p_r}\Delta\Gamma\right)^{5/2}}+\frac{5p_r(7p_r-6)}{16(1+2p_r)\Gamma^{3/2}\left(\Gamma+\frac{1+p_r}{1+2p_r}\Delta\Gamma\right)^{7/2}}$$

$$+\frac{5p_r(7p_r-6)}{16(1+2p_r)(\Gamma+\Delta\Gamma)^{3/2}(\Gamma+\frac{p_r}{1+2p_r}\Delta\Gamma)^{7/2}}+\frac{35p_r(3p_r^2-2p_r+3)}{8(1+2p_r)\Gamma^{1/2}(\Gamma+\frac{1+p_r}{1+2p_r}\Delta\Gamma)^{9/2}}$$

$$+\frac{35p_r(3p_r^2-2p_r+3)}{8(1+2p_r)(\Gamma+\Delta\Gamma)^{1/2}(\Gamma+\frac{p_r}{1+2p_r}\Delta\Gamma)^{9/2}}+\cdots\cdots\cdots\cdots \qquad (2.4.15)$$

where $\Gamma = t - t_0$.

Equation (2.4.15) is the decay law of temperature energy fluctuation before the final period for the case of multi-point and multi-time.

If we put $\Delta\Gamma = 0$ in equation (2.4.15), we obtain the decay law for the multi-point and single-time as

$$\langle\frac{T^2}{2}\rangle = [\ A(t-t_0)^{-\frac{3}{2}} + B(t-t_0)^{-5}\ ] \qquad (2.4.16)$$

where

$$A = \frac{N_0 p_r^{3/2}}{8\nu^{3/2}\sqrt{2\pi}} \quad \text{and}$$

$$B = \frac{\delta_0 \pi p_r^{6}}{2\nu^6(1+p_r)(1+2p_r)^{5/2}}\left[\ \frac{9}{16}+\frac{5}{16}\frac{p_r(7p_r-6)}{1+2p_r}-\frac{35}{8}\frac{p_r(3p_r^2-2p_r+3)}{(1+2p_r)^2}+\cdots\cdots\ \right]$$

which is obtained earlier by Loeffler and Deissler [57].

The first term of the right side of equation (2.4.16) corresponds to the temperature energy for two-point correlation and the second term represents temperature energy for the three-point correlation. For large times, the second term in the equation becomes negligible, leaving the –3/2 power decay law for the final period previously found by corrsin [18].

2.5. CONCLUDING RERARKS

The results of the present study, obtained by neglecting the quadruple correlations in the three-point, three-time correlation equations, appear to represent the decay law of temperature fluctuation for times before the final period.

Corrsin [18] has previously pointed out that for the final period, as well as for self-preserving and inertial spectrums at very large Reynolds and Peclet numbers, temperature fluctuations die out more slowly than velocity fluctuations. This analysis indicates that the same is true for times before the final period for the case of multi-point and multi-time.

CHAPTER - III

DECAY OF MHD TURBULENCE BEFORE THE FINAL PERIOD FOR THE CASE OF MULTI-POINT AND MULTI-TIME.

3.1. INTRODUCTION

In [21,22], Dissler developed a theory for homogeneous turbulence for time before the final period. Using Deissler's theory, Loeffer and Deissler [57] studied temperature fluctuation in homogeneous turbulence. In their study, they considered the two and three point correlation equations and solved these equations after neglecting the fourth-order correlation terms in comparison to the second and third order correlation terms. By considering the Deissler's same theory, Kumar and Patel [52] studied the first order reactant in homogeneous turbulence before the final period for the case of multi-point and single time consideration. The problem [52] is extended to the case of multi-point and multi-time concentration correlation by Kumar and Patel [53], and numerical result of [53] carried-out by Patel [78].

Following Deissler's approach, Sarker & Kishore [91] also studied the decay of MHD turbulence before the final period for the case of multi-point and single time.

In this problem, the decay of MHD turbulence before the final period for the case of multi-point and multi-time has been studied. Finally we obtained the decay law of magnetic energy fluctuation before the final period for the case of multi-point and multi-time.

3.2. FUNDAMENTAL EQUATIONS

The equations of motion for viscous, incompressible MHD turbulent flow are given by

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial W}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k}, \quad (3.2.1)$$

$$\frac{\partial h_i}{\partial t} + \frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \lambda \frac{\partial^2 h_i}{\partial x_k x_k} \quad (3.2.2)$$

with

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial h_i}{\partial x_i} \quad (3.2.3)$$

where $u_i(x,t)$, ith-component of turbulent velocity; $h_i(x,t)$, ith-component of magnetic field fluctuation; $W(x,t) = \frac{p}{\rho} + \frac{1}{2}\langle h \rangle^2$, total MHD pressure; $p(x,t)$, hydrodynamic pressure; ρ, fluid density; ν, kinematic viscosity ; $\lambda = \frac{\nu}{P_M}$, magnetic diffusivity; P_M, magnetic prandtle number; x_k, space coordinate; the subscripts are taken on the values 1, 2 or 3 and the repeated subscripts in a term indicate a summation.

Equations, (3.2.1) - (3.2.3) are derived by S. Chandrasekhar [13], the basis of Batchelor's discussion by coupling Maxwell's equation for the electromagnetic field and the Navier-Stokes equations for the velocity field. The Maxwell equations are modified to include the induced electric field due to the fluid motion, and the Navier-Stokes equations are modified to include to the Lorentz force on fluid elements due to the magnetic field.

3.3. TWO-POINT, TWO-TIME CORRELATION AND SPECTRAL EQUATIONS.

Induction equation of a magnetic field at the points p and p' separated by the vector $\hat{r}$ may be written as

$$\frac{\partial h_i}{\partial t} + u_k \frac{\partial h_i}{\partial x_k} - h_k \frac{\partial u_i}{\partial x_k} = \lambda \frac{\partial^2 h_i}{\partial x_k \partial x_k} \tag{3.3.1}$$

and

$$\frac{\partial h'_j}{\partial t'} + u'_k \frac{\partial h'_j}{\partial x'_k} - h'_k \frac{\partial u'_j}{\partial x'_k} = \lambda \frac{\partial^2 h'_j}{\partial x'_k \partial x'_k}. \tag{3.3.2}$$

Multiplying equation (3.3.1) by h'_i and equation (3.3.2) by h_i and taking ensemble average, we get

$$\frac{\partial \langle h_i h'_j \rangle}{\partial t} + \frac{\partial}{\partial x_k}[\langle u_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle] = \lambda \frac{\partial^2 \langle h_i h_j \rangle}{\partial x_k \partial x_k} \tag{3.3.3}$$

and

$$\frac{\partial \langle h_i h'_j \rangle}{\partial t'} + \frac{\partial}{\partial x'_k}[\langle u'_k h_i h'_j \rangle - \langle u'_j h_i h'_j \rangle] = \lambda \frac{\partial^2 \langle h_i h'_j \rangle}{\partial x'_k \partial x'_k}. \tag{3.3.4}$$

Angular bracket $\langle .... \rangle$ which is used to denote an ensemble average.

Using the transformations

$$\frac{\partial}{\partial x_k} = -\frac{\partial}{\partial r_k}, \frac{\partial}{\partial x'_k} = \frac{\partial}{\partial r_k}, \qquad (\frac{\partial}{\partial t})_{t'} = (\frac{\partial}{\partial t})_{\Delta t} - \frac{\partial}{\partial \Delta t}, \qquad \frac{\partial}{\partial t'} = \frac{\partial}{\partial \Delta t} \tag{3.3.5}$$

equations, (3.3.3) and (3.3.4) can be written as

$$\frac{\partial \langle h_i h'_j \rangle}{\partial t} + \frac{\partial}{\partial r_k}[\langle u'_k h_i h'_j \rangle - \langle u'_j h_i h'_k \rangle](\hat{r}, \Delta t, t) - \frac{\partial}{\partial r_k}[\langle u_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle](\hat{r}, \Delta t, t)$$

$$= 2\lambda \frac{\partial^2 \langle h_i h'_j \rangle}{\partial r_k \partial r_k} \tag{3.3.6}$$

and

$$\frac{\partial \langle h_i h'_j \rangle}{\partial \Delta t} + \frac{\partial}{\partial r_k}[\langle u'_k h_i h'_j \rangle - \langle u'_j h_i h'_k \rangle](r, \Delta t, t) = \lambda \frac{\partial^2 \langle h_i h'_j \rangle}{\partial r_k \partial r_k} . \tag{3.3.7}$$

Using the relations (cf. Chandrasekhar [13])

$$\langle u_k h_i h'_j \rangle = -\langle u'_k h_i h'_j \rangle \quad , \qquad \langle u'_i h_i h'_k \rangle = -\langle u_i h_k h'_i \rangle$$

equations,(3.3.6) and (3.3.7) becomes

$$\frac{\partial \langle h_i h'_j \rangle}{\partial t} + 2 \frac{\partial}{\partial r_k} [\langle u'_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle] = 2\lambda \frac{\partial^2 \langle h_i h'_j \rangle}{\partial r_k \partial r_k} \tag{3.3.8}$$

and

$$\frac{\partial \langle h_i h'_j \rangle}{\partial \Delta t} + \frac{\partial}{\partial r_k}[\langle u'_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle] = \lambda \frac{\partial^2 \langle h_i h'_j \rangle}{\partial r_k \partial r_k} . \tag{3.3.9}$$

In order to convert equations, (3.3.8) and (3.3.9) to spectral form, we define the following three-dimensional Fourier transforms

$$\langle h_i h'_j \rangle(\hat{r}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \psi_i \psi'_j \rangle(\hat{K}, \Delta t, t) \exp[\hat{i}(\hat{K}.\hat{r})] d\hat{K} \tag{3.3.10}$$

$$\langle u_i h_k h'_j \rangle(\hat{r}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \alpha_i \psi_k \psi'_j \rangle(\hat{K}, \Delta t, t) \exp[\hat{i}(\hat{K}.\hat{r})] d\hat{K} . \tag{3.3.11}$$

Interchanging the subscripts i and j and then interchanging the points p and p' gives

$$\langle u'_k h_i h'_j \rangle(\hat{r},\Delta t,t) = \langle u_k h_i h'_j \rangle(-\hat{r},-\Delta t,t+\Delta t)$$

$$= \int_{-\infty}^{\infty} \langle \alpha_i \psi_i \psi'_j \rangle(-\hat{K},-\Delta t,t+\Delta t)\exp[\hat{i}(\hat{K}.\hat{r})]d\hat{K} \qquad (3.3.12)$$

where $\hat{K}$ is known as a wave-number vector and $d\hat{K} = dK_1 dK_2 dK_3$. The magnitude of $\hat{K}$ has the dimension 1/length and can be considered to be the reciprocal of an eddy size. Substitution of equations. (3.3.10) - (3.3.12) into equations, (3.3.8) and (3.3.9) leads to the spectral equations

$$\frac{d\langle \psi_i \psi'_j \rangle}{dt} + 2\lambda k^2 \langle \psi_i \psi'_j \rangle = 2ik_k[\langle \alpha_i \psi_k \psi'_j \rangle(\hat{K},\Delta t,t) - \langle \alpha_k \psi_i \psi'_j \rangle(-\hat{K},-\Delta t,t+\Delta t)], \qquad (3.3.13)$$

$$\frac{d\langle \psi_i \psi'_j \rangle}{d\Delta t} + \lambda k^2 \langle \psi_i \psi'_j \rangle = ik_k[\langle \alpha_i \psi_k \psi'_j \rangle(\hat{K},\Delta t,t) - \langle \alpha_k \psi_i \psi'_j \rangle(-\hat{K},-\Delta t,t+\Delta t)]. \qquad (3.3.14)$$

The tensors equations. (3.3.13) and (3.3.14) becomes a scalar equation by contraction of the indices i and j

$$\frac{d\langle \psi_i \psi'_i \rangle}{dt} + 2\lambda k^2 \langle \psi_i \psi'_i \rangle = 2ik_k[\langle \alpha_i \psi_k \psi'_i \rangle(\hat{K},\Delta t,t) - \langle \alpha_k \psi_i \psi'_i \rangle(\hat{K},-\Delta t,t+\Delta t)], \qquad (3.3.15)$$

$$\frac{d\langle \psi_i \psi'_i \rangle}{d\Delta t} + \lambda k^2 \langle \psi_i \psi'_i \rangle = ik_k[\langle \alpha_i \psi_k \psi'_i \rangle(\hat{K},\Delta t,t) - \langle \alpha_k \psi_i \psi' \rangle_i(-\hat{K},-\Delta t,t+\Delta t)]. \qquad (3.3.16)$$

The terms on the right side of equations. (3.3.15) and (3.3.16) are known as the magnetic energy transfer term. They account for the transfer of energy from one wave number to another or from one eddy size to another.

3.4. THREE-POINT, THREE-TIME CORRELATION EQUATIONS AND SOLUTION FOR TIMES BEFORE THE FINAL PERIOD.

In the present investigation, under the same assumption as before it is proposed to obtain an expression for the transfer term applicable at time before the final period from the three-point correlation or spectral equations. To obtain the three-point three-time correlation equations, we take the momentum equation of MHD turbulence at the point p and induction equations of magnetic field fluctuation at p' and p'' separated by the vector $\hat{r}$ and $\hat{r}'$ as

$$\frac{\partial u_l}{\partial t} + u_k \frac{\partial u_l}{\partial x_k} - h_k \frac{\partial h_l}{\partial x_k} = -\frac{\partial W}{\partial x_l} + \nu \frac{\partial^2 u_l}{\partial x_k \partial x_k}, \tag{3.4.1}$$

$$\frac{\partial h'_i}{\partial t'} + u'_k \frac{\partial h'_i}{\partial x'_k} - h'_k \frac{\partial u'_i}{\partial x'_k} = \lambda \frac{\partial^2 h'_i}{\partial x'_k \partial x'_k}. \tag{3.4.2}$$

$$\frac{\partial h''_j}{\partial t''} + u''_k \frac{\partial h''_j}{\partial x''_k} - h''_k \frac{\partial u''_j}{\partial x''_k} = \lambda \frac{\partial^2 h''_j}{\partial x''_k \partial x''_k}. \tag{3.4.3}$$

Multiplying equations. (3.4.1)- (3.4.3) by $h'_i h''_j$, $u_l h''_j$ and $u_l h'_i$ respectively and taking ensemble average, we obtain

$$\frac{\partial \langle u_l h'_i h''_j \rangle}{\partial t} + \frac{\partial}{\partial x_k}[\langle u_k u_l h'_i h''_j \rangle - \langle h_k h_l h'_i h''_j \rangle] = -\frac{\partial \langle W h'_i h''_j \rangle}{\partial x_l} + \nu \frac{\partial^2 \langle u_l h'_i h''_j \rangle}{\partial x_k \partial x_k}, \tag{3.4.4}$$

$$\frac{\partial \langle u_l h'_i h''_j \rangle}{\partial t'} + \frac{\partial}{\partial x'_k}[\langle u_l u'_k h'_i h''_j \rangle - \langle u_l u'_i h'_k h''_j \rangle] = \lambda \frac{\partial^2 \langle u_l h'_i h''_j \rangle}{\partial x_k \partial x_k} \tag{3.4.5}$$

and

$$\frac{\partial \langle u_l h'_i h''_j \rangle}{\partial t''} + \frac{\partial}{\partial x''_k}[\langle u_l u''_k h'_i h''_j \rangle - \langle u_l u''_j h'_i h''_k \rangle] = \lambda \frac{\partial^2 \langle u_l h'_i h''_j \rangle}{\partial x''_k \partial x''_k}. \tag{3.4.6}$$

If we use the transformations

$$\frac{\partial}{\partial x_k} = -(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r_k'}), \quad \frac{\partial}{\partial x_k'} = \frac{\partial}{\partial r_k}, \quad \frac{\partial}{\partial x_k''} = \frac{\partial}{\partial r_k'} (\frac{\partial}{\partial t})_{r,r'} = (\frac{\partial}{\partial t})_{\Delta t, \Delta t'} - \frac{\partial}{\partial \Delta t} - \frac{\partial}{\partial \Delta t'}$$

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial \Delta t}, \quad \frac{\partial}{\partial t''} = \frac{\partial}{\partial \Delta t'}$$

into equations, (3.4.4) - (3.4.6), we have

$$\frac{\partial \langle u_l h_i' h_j'' \rangle}{\partial t} - (\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r_k'})[\langle u_k u_l h_i' h_j'' \rangle - \langle h_k h_l h_i' h_j'' \rangle] + \frac{\partial}{\partial r_k}[\langle u_l u_k' h_i' h_j'' \rangle$$

$$- \langle u_l u_i' h_k' h_j'' \rangle] + \frac{\partial}{\partial r_k'}[\langle u_l u_k'' h_i' h_j'' \rangle - \langle u_l u_j'' h_i' h_k'' \rangle = -(\frac{\partial}{\partial r_l} + \frac{\partial}{\partial r_l'})\langle W h_i' h_j'' \rangle$$

$$+ \nu(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r_k'})^2 \langle u_l h_i' h_j'' \rangle + \lambda[\frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial r_k \partial r_k} + \frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial r_k' \partial r_k'}], \tag{3.4.7}$$

$$\frac{\partial \langle u_l h_i' h_j'' \rangle}{\partial \Delta t} + \frac{\partial}{\partial r_k}[\langle u_l u_k' h_i' h_j'' \rangle - \langle u_l u_i' h_k' h_j'' \rangle] = \lambda[\frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial r_k \partial r_k}, \tag{3.4.8}$$

$$\frac{\partial \langle u_l h_i' h_j'' \rangle}{\partial \Delta t'} + \frac{\partial}{\partial r_k'}[\langle u_l u_k'' h_i' h_j'' \rangle - \langle u_l u_j'' h_i' h_k'' \rangle] = \lambda[\frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial r_k' \partial r_k'}. \tag{3.4.9}$$

Using the six dimensional Fourier transforms of the type

$$\langle u_l h_i' h_j'' \rangle(\hat{r}, \hat{r}', \Delta t, \Delta t', t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_l \beta_i' \beta_j'' \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) \exp[\hat{i}(\hat{K}.\hat{r} + \hat{K}'.\hat{r}')] d\hat{K} d\hat{K}', \tag{3.4.10}$$

$$\langle u_l u_k' h_i' h_j'' \rangle(\hat{r}, \hat{r}', \Delta t, \Delta t', t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_l \phi_k' \beta_i' \beta_j'' \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t)$$

$$\times \exp[\hat{i}(\hat{K}.\hat{r} + \hat{K}'.\hat{r}')] d\hat{K} d\hat{K}', \tag{3.4.11}$$

$$\langle Wh_i'h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\gamma\beta_i'\beta_j''\rangle(\hat{K},\hat{K},\Delta t,\Delta t',t)\exp[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}d\hat{K}' \qquad (3.4.12)$$

into equations,(3.4.7) - (3.4.9) and after neglecting the quadruple correlation terms (as they decay faster than the lower-order correlation terms), one obtains

$$\frac{d}{dt}\langle\phi_l\beta_i'\beta_j''\rangle(\hat{K},\hat{K},\Delta t,\Delta t',t)+\lambda[(1+P_M)(k^2+k'^2)+2P_M kk']$$
$$\times\langle\phi_l\beta_i'\beta_j''\rangle(\hat{K},\hat{K},\Delta t,\Delta t',t)=0\,, \qquad (3.4.13)$$

$$\frac{d}{d\Delta t}\langle\phi_l\beta_i'\beta_j''\rangle(\hat{K},\hat{K},\Delta t,\Delta t',t)+\lambda k^2\langle\phi_l\beta_i'\beta_j''\rangle(\hat{K},\hat{K},\Delta t,\Delta t',t)=0\,, \qquad (3.4.14)$$

$$\frac{d}{d\Delta t'}\langle\phi_l\beta_i'\beta_j''\rangle(\hat{K},\hat{K},\Delta t,\Delta t',t)+\lambda k'^2\langle\phi_l\beta_i'\beta_j''\rangle(\hat{K},\hat{K},\Delta t,\Delta t',t)=0\,. \qquad (3.4.15)$$

The term $\langle\gamma\beta_i'\beta_j''\rangle$ associated with the pressure correlation term are also neglected because it is related to the quadruple correlation's (equation (3.16) of Sarker & Kishore [91]).

The tensor equations, (3.4.13)-(3.4.15) can be converted to scalar equations by contraction of the indices i and j and inner multiplication by k_l

$$\frac{d}{dt}k_l\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)+\lambda[(1+P_M)(k^2+k'^2)+2P_M kk']$$
$$\times\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=0\,, \qquad (3.4.13a)$$

$$\frac{d}{d\Delta t}k_l\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K},\Delta t,\Delta t',t)+\lambda k^2\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K},\Delta t,\Delta t',t)=0\,, \qquad (3.4.14a)$$

$$\frac{d}{d\Delta t'}k_l\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)+\lambda k'^2\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=0\,. \qquad (3.4.15a)$$

Integrating equations, (3.4.13a) - (3.4.15a) between t_0 to t, we obtain

$$k_l\langle\phi_l\beta_i'\beta_i''\rangle=f_l\exp\{-\lambda[(1+P_M)(k^2+k'^2)+2P_M kk'\cos\theta](t-t_0)\}\,,$$

$$k_l \langle \phi_l \beta'_i \beta''_i \rangle = g_l \exp[-\lambda k^2 \Delta t],$$

$$k_l \langle \phi_l \beta'_i \beta''_i \rangle = q_l \exp[-\lambda k'^2 \Delta t'].$$

For these relations to be consistent, we have

$$k_l \langle \phi_l \beta'_i \beta''_i \rangle = k_l \langle \phi_l \beta'_i \beta''_i \rangle_0 \exp\{-\lambda[(1 + P_M)(k^2 + k'^2)(t - t_0)$$

$$+ k^2 \Delta t + k'^2 \Delta t' + 2P_M kk' \cos\xi (t - t_0)\} \qquad (3.4.16)$$

where the subscript 0 refers to the value of $\langle \phi \beta'_i \beta''_i \rangle$ at $t = t_0$, $\Delta t = \Delta t' = 0$ and ξ is the angle between $\hat{K}$ and $\hat{K}'$.

By letting $\hat{r}' = 0$, $\Delta t' = 0$ in the equation (3.4.10) and comparing with equation (3.3.11) and (3.3.12), we obtain the relations

$$\langle \alpha_i \psi_k \psi'_i \rangle(\hat{K}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \phi_l \beta'_i \beta''_i \rangle(\hat{K}, \hat{K}', \Delta t, 0, t) d\hat{K}', \qquad (3.4.17)$$

$$\langle \alpha_k \psi_i \psi'_i \rangle(-\hat{K}, -\Delta t, t + \Delta t) = \int_{-\infty}^{\infty} \langle \phi_l \beta'_i \beta''_i \rangle(-\hat{K}, \hat{K}', -\Delta t, 0, t) d\hat{K}'. \qquad (3.4.18)$$

Substituting equations. (3.4.16) - (3.4.18) into equation (3.3.15), one obtains

$$\frac{d}{dt} \langle \psi_i \psi'_i \rangle(\hat{K}, \Delta t, t) + 2\lambda k^2 \langle \psi_i \psi'_i \rangle(\hat{K}, \Delta t, t) =$$

$$= \int_{-\infty}^{\infty} 2ik_l [\langle \phi_l \beta'_i \beta''_i \rangle(\hat{K}, \hat{K}') - \langle \phi_l \beta'_i \beta''_i \rangle(-\hat{K}, -\hat{K}')]_0 \times$$

$$\times \exp[-\lambda\{(1 + P_M)(k^2 + k'^2)(t - t_0) + k^2 \Delta t + 2P_M (t - t_0) kk' \cos\xi d(\cos\xi)\}] dk', \qquad (3.4.19)$$

or,

$$\frac{d}{dt}\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)+2\lambda k^2\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)=2\int_0^\infty 2\pi i k_l[\langle\phi_l\beta_i'\beta_i''\rangle-\langle\phi_l\beta_i'\beta_i''\rangle]_0 k'^2$$

$$\times\exp[-\lambda\{(1+P_M)(k^2+k'^2)(t-t_0)+k^2\Delta t+2P_M(t-t_0)kk'\cos\xi d(\cos\xi)\}]dk' \tag{3.4.20}$$

where $d\hat{K}'=dK_1'dK_2'dK_3'$ is written in terms of k' and ξ as $-2\pi k'^2 d(\cos\xi)dk'$ (cf. Deissler [22]) and the quantity $[\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}')-\langle\phi_l\beta_i'\beta_i''\rangle(-\hat{K},-\hat{K}')]_0$ depends on the initial condition of the turbulence.

In order to make further calculation it is necessary to assume a relation, which gives $ik_l[\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}')-\langle\phi_l\beta_i'\beta_i''\rangle(-\hat{K},-\hat{K}')]_0$ as a function of k and k'. The relation assumed here is

$$(2\pi)^2 ik_l[\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}')-\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}')]_0=-\delta_0[k^2k'^4-k^4k'^2] \tag{3.4.21}$$

where δ_0 is a constant determined by the initial conditions.

Substituting equation (3.4.21) into equation (3.4.20), and multiplying both sides by k^2 and writing $\langle\psi_i\psi_i'\rangle$ in terms of the magnetic energy spectrum function as $E_M=2\pi k^2\langle\psi_i\psi_i'\rangle$, we get

$$\frac{dE_M}{dt}+2\lambda k^2 E_M=M_T \tag{3.4.22}$$

where M_T is the magnetic energy transfer term and is given by

$$M_T=-2\delta_0\int_0^\infty (k^2k'^4-k^4k'^4)k^2k'^2\times[\int_1^1 \exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_0)$$

$$+k^2\Delta t + 2P_M(t-t_0)kk'\cos\xi]d(\cos\xi)\}]dk'. \qquad (3.4.23)$$

Integrating equation (3.4.23) with respect to $\cos\xi$ and k', we have

$$M_T = -\frac{\delta_0 P^{\frac{5}{2}}{}_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_0)^{\frac{3}{2}}(1+P_M)^{\frac{5}{2}}}\exp[-k^2\lambda(\frac{1+2P_M}{1+P_M})(t-t_0+\frac{1+P_M}{1+2P_M}\Delta t)]$$

$$\times[\frac{15k^4}{4P_M{}^2(t-t_0)^2\lambda^2}(\frac{P_M}{1+P_M})+\{5(\frac{P_M}{1+P_M})^2-\frac{3}{2}\}\frac{k^6}{P_M\lambda(t-t_0)}$$

$$+\{(\frac{P_M}{1+P_M})^3-\frac{P_M}{1+P_M}\}k^8-\frac{\delta_0 P_M\sqrt{\pi}}{4\lambda^{\frac{3}{2}}(t-t_0+\Delta t)^{\frac{3}{2}}(1+P_M)^{\frac{5}{2}}}$$

$$\times\exp[-k^2\lambda(\frac{1+2P_M}{1+P_M})(t-t_0+\frac{P_M}{1+P_M}\Delta t)][\frac{15k^4}{4\lambda^2 P_M{}^2(t-t_0+\Delta t)^2}(\frac{P_M}{1+P_M})$$

$$+\{5(\frac{P_M}{1+P_M})^2-\frac{3}{2}\}\frac{k^6}{P_M\lambda(t-t_0+\Delta t)}+\{(\frac{P_M}{1+P_M})^3-\frac{P_M}{1+P_M}\}k^8]. \qquad (3.4.24)$$

The series of equation (3.4.24) contains only even power of k and the equation represents the transfer function arising owing to consideration of magnetic field at three-point and three-times.

If we integrate equation (3.4.24) for $\Delta t = 0$ over all wave numbers, we find that

$$\int_0^\infty M_T . dk = 0, \qquad (3.4.25)$$

which indicating that the expression for M_T satisfying the conditions of continuity and homogeneity. Physically it was to be expected as M_T is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero.

For obtaining the magnetic energy spectrum function E_M, equation (3.4.22) can be written in the integral form as

$$E_M = \exp[-2\lambda k^2(t+t_0+\Delta t/2)]\int M_T \exp[2\lambda k^2(t-t_0+\Delta t/2)]dt$$

$$+J(k)\exp[-2\lambda k^2(t-t_0+\Delta t/2)] \qquad (3.4.26)$$

where $J(k)=\frac{N_0k^2}{\pi}$ is a constant of integration and can be obtained as by Corrsin [18].

Substituting the value of M_T as given by equation (3.4.24) into equation (3.4.26), gives the equation

$$E_M = \frac{N_0k^2}{\pi}\exp[-2\lambda k^2(t-t_0+\Delta t/2)]+\frac{\delta_0 P_M\sqrt{\pi}}{4\lambda^{3/2}(1+P_M)^{7/2}}$$

$$\times\exp[-k^2\lambda(\frac{1+2P_M}{1+P_M})(t-t_0+\frac{1+P_M}{1+2P_M}\Delta t)][\frac{3k^4}{2P_M\ (t-t_0)^{5/2}\lambda^2}+$$

$$+\frac{(7P_M-6)k^6}{3\lambda(1+P_M)(t-t_0)^{3/2}}-\frac{4(3P_M{}^2-2P_M+3)}{3(1+P_M)^2(t-t_0)^{1/2}}+\frac{8\lambda^{1/2}(3P_M{}^2-2P_M+3)}{3(1+P_M)^{5/2}}k^9F(\omega)$$

$$+\frac{\delta_0\sqrt{\pi}P_M}{4\lambda^{3/2}(1+P_M)}\exp[-\lambda k^2(\frac{1+2P_M}{1+P_M})(t-t_0+\frac{P_M}{1+P_M}\Delta t)][\frac{3k^4}{2\lambda^2P_M(t-t_0+\Delta t)^{5/2}}$$

$$+\frac{(7P_M-6)k^6}{3\lambda(1+P_M)(t-t_0+\Delta t)^{3/2}}+\frac{4(3P_M{}^2-2P_M+3)}{3(1+P_M)^2(t-t_0+\Delta t)^{\frac{1}{2}}}+$$

$$+\frac{8\lambda^{1/2}(3P_M{}^2-2P_M+3)}{(1+P_M)^{5/2}}k^9F(\omega) \qquad (3.4.27)$$

where, $F(\omega)=e^{-\omega^2}\int_0^\omega e^{x^2}dx$, $\quad \omega=k\sqrt{\frac{\lambda(t-t_0)}{1+P_M}}$ or $k\sqrt{\frac{\lambda(t-t_0+\Delta t)}{1+P_M}}$.

The expression for the magnetic energy decay is obtained from equation (3.3.10) by setting $\hat{r}=0, j=i$, $d\hat{K}=-2\pi k^2 d(\cos\xi)d\hat{k}$ and $E_M=2\pi k^2\langle\psi_i\psi'_j\rangle$ as

$$\frac{\langle h_i h'_i\rangle}{2}=\int_0^\infty E_M dk. \tag{3.4.28}$$

Substituting equation (3.4.27) into equation (3.4.28) and after integration, one can obtains

$$\frac{\langle h_i h'_i\rangle}{2}=\frac{N_0}{8\lambda^{3/2}\sqrt{2\pi}(T+\Delta T/2)^{3/2}}+\frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}}$$

$$\times[\frac{9}{16T^{5/2}(T+\frac{1+P_M}{1+2P_M}\Delta T)^{5/2}}+\frac{9}{16(T+\Delta T)^{5/2}(T+\frac{P_M}{1+P_M}\Delta T)^{5/2}}$$

$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)T^{3/2}(T+\frac{1+P_M}{1+2P_M}\Delta T)^{7/2}}+\frac{5P_M(7P_M-6)}{16(1+2P_M)(T+\Delta T)(T+\frac{P_M}{1+2P_M}\Delta T)^{7/2}}$$

$$+\frac{35P_M(3P_M{}^2-2P_M+3)}{8(1+2P_M)T^{1/2}(T+\frac{1+P_M}{1+2P_M}\Delta T)^{9/2}}+\frac{35P_M(3P_M{}^2-2P_M+3)}{8(1+2P_M)(T+\Delta T)^{1/2}(T+\frac{P_M}{1+2P_M}\Delta T)^{9/2}}$$

$$+\frac{8P_M(3P_M{}^2-2P_M+3)(1+2P_M)^{5/2}}{3.2^{23/2}(1+P_M)^{11/2}}\sum_{n=0}^{\infty}\frac{1.3.5.-----(2n+9)}{n!(2n+1)2^{2n}(1+P_M)^n}$$

$$\times\{\frac{T^{(2n+1)/2}}{(T+\Delta T)^{(2n+11)/2}}+\frac{(T+\Delta T)^{(2n+1)/2}}{(T+\Delta T/2)^{(2n+11)/2}}\}] \tag{3.4.29}$$

where $T=t-t_0$.

Or,

$$\frac{\langle h_i h_i' \rangle}{2} = \frac{N_0}{8\lambda^{3/2}\sqrt{2\pi}T_m^{3/2}} + \frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}}$$

$$\times\left[\frac{9}{16(T_m - \Delta T/2)^{5/2}(T_m + \frac{\Delta T}{1+2P_M})^{5/2}} + \frac{9}{16(T_m + \Delta T/2)^{5/2}(T_m - \frac{\Delta T}{2(1+P_M)})^{5/2}}\right.$$

$$+\frac{5P_M(7P_M - 6)}{16(1+2P_M)(T_m - \Delta T/2)^{3/2}(T_m + \frac{\Delta T}{2(1+2P_M)})^{7/2}}$$

$$\left.+\frac{5P_M(7P_M - 6)}{16(1+2P_M)(T_m + \Delta T/2)^{3/2}(T_m - \frac{\Delta T}{2(1+2P_M)})^{7/2}} + \text{----------}\right] \tag{3.4.30}$$

where $T_m = T + \Delta T/2$.

This is the decay law of magnetic energy fluctuation before the final period for the case of multi-point and multi-time.

If we put $\Delta T = 0$, we can easily find-out that

$$\frac{\langle h^2 \rangle}{2} = \frac{N_0}{8\lambda^{3/2}\sqrt{2\pi}} T^{-3/2} + \frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}} T^{-5}\left\{\frac{9}{16} + \frac{5}{16}\frac{P_M(7P_M - 6)}{1+2P_M} + \text{---}\right\}$$

$$= \frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}} T^{-3/2} + \frac{\delta_0 S}{2\lambda^6} T^{-5} \tag{3.4.31}$$

where

$$S = \frac{\pi}{(1+P_M)(1+2P_M)^{5/2}}\left\{\frac{9}{16} + \frac{5}{16}\frac{P_M(7P_M - 6)}{1+2P_M} + \text{---}\right.$$

which is same as obtained earlier by Kishore and Sarker [91].

3.5. CONCLUDING REMARKS.

This study shows that the terms associated with the higher-order correlation's die out faster than those associated with the lower order ones. Therefore, from this assumption we conclude that the higher-order correlation terms may be neglected in comparison with lower-order correlation terms. By neglecting the quadruple correlation terms in the three-point, three-time correlation equation the result (3.4.30) applicable to the MHD turbulence before the final period of decay. If higher order correlation equations are considered in the analysis, it appears that more terms of higher power of time would be added to the equation (3.4.30).

Another result is that the decay of magnetic field fluctuations are more slowly than the velocity fluctuations.

CHAPTER – IV

FIRST ORDER REACTANT IN MHD TURBULENCE BEFORE THE FINAL PERIOD OF DECAY

4.1. INTRODUCTION

Loeffer and Dissler [57] used the theory, developed by Deissler [21,22] to study the temperature fluctuations in homogeneous turbulence before the final period. In their approach it is considered the two- and three-point correlation equations and solutions were obtained of these equations after neglecting the fourth and higher order correlation terms. Using Deissler's theory, Kumar and Patel [52] studied the first order reactant in homogeneous turbulence before the final period for the case of multi-point and single-time consideration. Kumar and Patel [53] extended their problem [52] for the case of multi-point and multi-time concentration correlation. Patel [78] also studied in detail the same problem to carryout the numerical results. In [91], Sarker and Kishore studied the decay of MHD turbulence at time before the final period using Chandrasekher's relation [13].

In our present work, the same approach of Deissler [21] is applied to the study of magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in MHD turbulence before the final period. Here, we have considered the two-and three-point correlation equations and solved these equations after neglecting the fourth-order correlation terms. Finally, we have obtained the decay law for magnetic energy fluctuation of concentration of dilute contaminant undergoing a first order chemical reaction in the form

$$\langle h^2 \rangle = \left[X(t-t_0)^{-3/2} + Y(t-t_0)^{-5} \right] \exp[-R(t-t_0)]$$

where $\langle h^2 \rangle$ denotes the total 'energy' (mean square of the magnetic field fluctuations of concentration), t is the time, and X, Y and t_0 are constants determined by the initial conditions.

4.2. FUNDAMENTAL EQUATIONS

The equations of motion for viscous, incompressible MHD turbulent flow are given by Chandrasekhar [13] as

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial W}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k} \tag{4.2.1}$$

$$\frac{\partial h_i}{\partial t} + \frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \lambda \frac{\partial^2 h_i}{\partial x_k \partial x_k} \tag{4.2.2}$$

with

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial h_i}{\partial x_i} = 0 \tag{4.2.3}$$

where $u_i(\hat{x},t)$, ith-component of turbulent velocity at a point $P(\hat{x},t)$; $h_i(x,t)$, ith-component of magnetic field fluctuation of concentration at a point $P(\hat{x},t)$; $W(x,t) = \frac{p}{\rho} + \frac{1}{2}\langle h \rangle^2$, total MHD pressure; $p(\hat{x},t)$, hydrodynamic pressure; ρ, fluid density; $\lambda = \frac{\nu}{P_m}$, magnetic diffusivity; ν, kinematic viscosity ; P_M, magnetic prandtle number; x_k, space coordinate; the subscripts can take on the values 1, 2 or 3 and the repeated subscripts in a term indicates a summation.

4.3.TWO-POINT CORRELATION AND SPECTRAL EQUATIONS

If the turbulence and the concentration magnetic field are homogeneous, chemical reaction and the local mass transfer have no effect on the velocity field, the reaction rate and the magnetic diffusivity are constant, then the induction equation of a magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction at the points P and P' separated by the vector $\hat{r}$ could be written as

$$\frac{\partial h_i}{\partial t} + u_k \frac{\partial h_i}{\partial x_k} - h_k \frac{\partial u_i}{\partial x_k} = \lambda \frac{\partial^2 h_i}{\partial x_k \partial x_k} - Rh_i \tag{4.3.1}$$

and

$$\frac{\partial h'_j}{\partial t'} + u'_k \frac{\partial h'_j}{\partial x'_k} - h'_k \frac{\partial u'_j}{\partial x'_k} = \lambda \frac{\partial^2 h'_j}{\partial x'_k \partial x'_k} - Rh'_j \tag{4.3.2}$$

where R is the constant reaction rate.

Multiplying equation (4.3.1) by h'_j and equation (4.3.2) by h_i, adding and taking ensemble average, we get the two-point correlation equation for the fluctuating concentration as

$$\frac{\partial \langle h_i h'_j \rangle}{\partial t} + \frac{\partial}{\partial x_k}[\langle u_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle] + \frac{\partial}{\partial x'_k}[\langle u'_k h_i h'_j \rangle - \langle u'_j h_i h'_k \rangle]$$

$$= \lambda [\frac{\partial^2 \langle h_i h'_j \rangle}{\partial x_k \partial x_k} + \frac{\partial^2 \langle h_i h'_j \rangle}{\partial x'_k \partial x'_k}] - 2R\langle h_i h'_j \rangle \tag{4.3.3}$$

where the bracket $\langle ....\rangle$ is used to denote an ensemble average.

Using the transformations

$$\frac{\partial}{\partial r_k} = -\frac{\partial}{\partial x_k} = \frac{\partial}{\partial x'_k}$$

and the Chandrasekhar relations [13]

$$\langle u_k h_i h'_j\rangle = -\langle u'_k h_i h'_j\rangle \quad , \qquad \langle u'_l h_i h'_k\rangle = -\langle u_l h_k h'_i\rangle .$$

equation (4.3.3) becomes

$$\frac{\partial\langle h_i h'_j\rangle}{\partial t} + 2\frac{\partial}{\partial r_k}[\langle u'_k h_i h'_j\rangle - \langle u_i h_k h'_j\rangle] = 2\lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k \partial r_k} - 2R\langle h_i h'_j\rangle . \qquad (4.3.4)$$

It is desirable to write equation (4.3.4) in spectral form in order to reduce it to an ordinary differential equation and because of the physical significance of spectral quantities. For this purpose it is usual to introduce three-dimensional Fourier-transforms

$$\langle h_i h'_j(\hat{r})\rangle = \int_{-\infty}^{\infty}\langle \psi_i \psi'_j(\hat{K})\rangle \exp[\hat{i}(\hat{K}.\hat{r})]d\hat{K} , \qquad (4.3.5)$$

$$\langle u_i h_k h'_j(\hat{r})\rangle = \int_{-\infty}^{\infty}\langle \alpha_i \psi_k \psi'_j(\hat{K})\rangle \exp[\hat{i}(\hat{K}.\hat{r})]d\hat{K} , \qquad (4.3.6)$$

$$\langle u'_k h_i h'_j(\hat{r})\rangle = \langle u_k h_i h'_j(-\hat{r})\rangle = \int_{-\infty}^{\infty}\langle \alpha_i \psi_i \psi'_j(-\hat{k})\rangle \exp[\hat{i}(\hat{K}.\hat{r})]d\hat{K} \qquad (4.3.7)$$

into equation (4.3.4), gives

$$\frac{d\langle \psi_i \psi'_j(\hat{K})\rangle}{dt} + 2[\lambda k^2 + R]\langle \psi_i \psi'_j(\hat{K})\rangle = 2ik_k[\langle \alpha_i \psi_k \psi'_j(\hat{K})\rangle - \langle \alpha_k \psi_i \psi'_j(-\hat{K})\rangle] \quad (4.3.8)$$

The tensor equation (4.3.8) becomes a scalar equation by contraction of the indices i and j

$$\frac{d\langle\psi_i\psi_i'(\hat{K})\rangle}{dt}+2\lambda[k^2+R/\lambda]\langle\psi_i\psi_i'(\hat{K})\rangle=2ik_k[\langle\alpha_i\psi_k\psi_i'(\hat{K})\rangle-\langle\alpha_k\psi_i\psi_i'(-\hat{K})\rangle]. \quad (4.3.9)$$

The term on the right hand side of equation (4.3.9) is called energy transfer term while the 2nd term on the left- hand side is the dissipation term.

Solution for the final period of decay the third-order correlation terms can be neglected in comparison to the second-order correlation terms. With this truncation approximation, the solution of equation (4.3.9) may be written as

$$E_m=J(k)\exp[-2\lambda(k^2+R/\lambda)(t-t_0)]=\frac{N_0k^2}{\pi}\exp[-2\lambda(k^2+R/\lambda)(t-t_0)] \quad (4.3.10)$$

where $E_m=2\pi k^2\langle\psi_i\psi_i'\rangle$ is the magnetic energy spectrum and $J(k)=\frac{N_0k^2}{\pi}$ is the constant of integration and can be obtain as by corrsin [18].

By integration equation (4.3.10) with respect to k, we obtain the magnetic energy decay law for the final period

$$\frac{\langle h_ih_i'\rangle}{2}=\frac{N_0\lambda^{3/2}}{8\sqrt{2\pi}}(t-t_0)^{-3/2}\exp[-2R(t-t_0)]. \quad (4.3.11)$$

4.4. THREE-POINT CORRELATION AND SPECTRAL EQUATIONS.

The same procedure can be used to find the three-point correlation equation i,e by taking the momentum equation of MHD turbulence at the point p and induction equations of magnetic field fluctuation, governing the concentration of a dilute contaminant undergoing a first order chemical reaction at p' and p'' separated by the vector $\hat{r}$ and $\hat{r}'$ as

$$\frac{\partial u_l}{\partial t}+u_k\frac{\partial u_l}{\partial x_k}-h_k\frac{\partial h_l}{\partial x_k}=-\frac{\partial W}{\partial x_l}+\nu\frac{\partial^2 u_l}{\partial x_k\partial x_k}, \tag{4.4.1}$$

$$\frac{\partial h_i'}{\partial t'}+u_k'\frac{\partial h_i'}{\partial x_k'}-h_k'\frac{\partial u_i'}{\partial x_k'}=\lambda\frac{\partial^2 h_i'}{\partial x_k'\partial x_k'}-Rh_i, \tag{4.4.2}$$

$$\frac{\partial h_j''}{\partial t''}+u_k''\frac{\partial h_j''}{\partial x_k''}-h_k''\frac{\partial u_j''}{\partial x_k''}=\lambda\frac{\partial^2 h_j''}{\partial x_k''\partial x_k''}-Rh_j''. \tag{4.4.3}$$

Multiplying equations, (4.4.1) - (4.4.3) by $h_i'h_j''$, u_lh_j' and u_lh_i' respectively, adding and taking ensemble average, one obtains

$$\frac{\partial\langle u_lh_i'h_j''\rangle}{\partial t}+\frac{\partial}{\partial x_k}[\ \langle u_ku_lh_i'h_j''\rangle-\langle h_kh_lh_i'h_j''\rangle\ ]+\frac{\partial}{\partial x_k'}[\ \langle u_lu_k'h_i'h_j''\rangle-\langle u_lu_i'h_k'h_j''\rangle\ ]$$

$$+\frac{\partial}{\partial x_k''}[\ \langle u_lu_k''h_i'h_j''\rangle-\langle u_lu_j''h_i'h_k''\rangle\ ]=-\frac{\partial\langle Wh_i'h_j''\rangle}{\partial x_l}+\nu\frac{\partial^2\langle u_lh_i'h_j''\rangle}{\partial x_k\partial x_k}$$

$$+\lambda\left[\frac{\partial^2\langle u_lh_i'h_j''\rangle}{\partial x_k'x_k'}+\frac{\partial^2\langle u_lh_i'h_j''\rangle}{\partial x_k''\partial x_k''}\right]-2R\langle u_lh_i'h_j''\rangle\cdot \tag{4.4.4}$$

Using the transformations

$$\frac{\partial}{\partial x_k}=-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right),\quad \frac{\partial}{\partial x_k'}=\frac{\partial}{\partial r_k},\quad \frac{\partial}{\partial x_k''}=\frac{\partial}{\partial r_k'}$$

into equation (4.4.4), we get

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t} - \lambda\left[(1+P_M)\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k \partial r_k} + (1+P_M)\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k' \partial r_k'} + 2P_M\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k \partial r_k'}\right]$$

$$= \frac{\partial}{\partial r_k}\langle u_l u_k' h_i' h_j''\rangle + \frac{\partial}{\partial r_k'}\langle u_l u_k' h_i' h_j''\rangle - \frac{\partial}{\partial r_k'}\langle h_l h_k h_i' h_j''\rangle - \frac{\partial}{\partial r_k'}\langle h_l h_k h_i' h_k''\rangle$$

$$- \frac{\partial}{\partial r_k}\langle u_l u_k' h_i' h_j''\rangle + \frac{\partial}{\partial r_k}\langle u_l u_i' h_k' h_j''\rangle - \frac{\partial\langle u_l u_k'' h_i' h_j''\rangle}{\partial r_k'} + \frac{\partial\langle u_l u_j'' h_i' h_k''\rangle}{\partial r_k'}$$

$$+ \frac{\partial\langle W h_i' h_j''\rangle}{\partial r_l} + \frac{\partial\langle W h_i' h_j''\rangle}{\partial r_l'} - 2R\langle u_l h_i' h_j''\rangle. \tag{4.4.5}$$

Using the six dimensional Fourier transforms of the type

$$\langle u_l h_i'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \phi_l \beta_i'(\hat{K})\beta_j''(\hat{K}')\rangle \exp[\hat{i}(\hat{K}.\hat{r} + \hat{k}'.\hat{r}')]d\hat{K}d\hat{K}', \tag{4.4.6}$$

$$\langle u_l u_k'(\hat{r}) h_i'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \phi_l \phi_k'(\hat{K})\beta_i'(\hat{K})\beta_j''(\hat{K}')\rangle \exp[\hat{i}(\hat{K}.\hat{r} + \hat{k}'.\hat{r}')]d\hat{K}d\hat{K}', \tag{4.4.7}$$

$$\langle u_l u_i'(\hat{r}) h_k'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \phi_l \phi_i'(\hat{K})\beta_k'(\hat{K})\beta_j''(\hat{K}')\rangle \exp[\hat{i}(\hat{K}.\hat{r} + \hat{k}'.\hat{r}')]d\hat{K}d\hat{K}', \tag{4.4.8}$$

$$\langle u_l u_k h_i'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \phi_l \phi_k \beta_i''(\hat{K})\beta_j''(\hat{K}')\rangle \exp[\hat{i}(\hat{K}.\hat{r} + \hat{k}'.\hat{r}')]d\hat{K}d\hat{K}', \tag{4.4.9}$$

$$\langle h_l h_k h_i'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \beta_l \beta_k \beta_i'(\hat{K})\beta_j''(\hat{K}')\rangle \exp[\hat{i}(\hat{K}.\hat{r} + \hat{k}'.\hat{r}')]d\hat{K}d\hat{K}', \tag{4.4.10}$$

$$\langle W h_i'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \gamma \beta_i'(\hat{K})\beta_j''(\hat{K}')\rangle \exp[\hat{i}(\hat{K}.\hat{r} + \hat{k}'.\hat{r}')]d\hat{K}d\hat{K}', \tag{4.4.11}$$

with the facts that

$$\langle u_l u''_k h''_j h'_i \rangle = \langle u_l u'_k h'_i h''_j \rangle, \qquad \langle u_l u''_j h'_i h''_k \rangle = \langle u_l u'_i h'_k h''_j \rangle$$

we can write equation (4.4.5) in the form

$$\frac{d}{dt}\langle \phi_l \beta'_i \beta''_j \rangle + \lambda\Big[(1+P_M)(k^2+k'^2)+2P_M k_k k'_k + 2\frac{R}{\lambda} \Big]\langle \phi_l \beta'_i \beta''_j \rangle =$$

$$i(k_k + k'_k)\langle \phi_l \phi_k \beta'_i \beta''_j \rangle - i(k_k + k'_k)\langle \beta_l \beta_k \beta'_i \beta''_j \rangle - i(k_k + k'_k)\langle \phi_l \phi'_k \beta'_i \beta''_j \rangle$$

$$+ i(k_k + k'_k)\langle \phi_l \phi'_i \beta'_k \beta''_j \rangle + i(k_l + k'_l)\langle \gamma \beta'_i \beta'_j \rangle . \tag{4.4.12}$$

In order to relate the terms on the right side of equation (4.4.12) derived from the quadruple correlation terms and from the pressure force terms in equation (4.4.5), we take the derivative with respect to x_l of the momentum equation (4.4.1) at p and combine with the continuity equation to give

$$-\frac{\partial^2 W}{\partial x_l \partial x_l} = \frac{\partial^2}{\partial x_l \partial x_k}(u_l u_k - h_l h_k) . \tag{4.4.13}$$

Multiplying equation (4.4.13) by $h'_i h'_j$, taking time averages and writing the equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$-\left[\frac{\partial^2}{\partial r_l \partial r_l} + \frac{\partial^2}{\partial r'_l \partial r'_l} + 2\frac{\partial}{\partial r_l \partial r'_l}\right]\langle W h'_i h''_j \rangle$$

$$= \left[\frac{\partial^2}{\partial r_l \partial r_k} + \frac{\partial^2}{\partial r'_l \partial r_k} + \frac{\partial^2}{\partial r_l \partial r'_k} + \frac{\partial^2}{\partial r'_l \partial r'_k}\right]\Big(\langle u_l u_k h'_i h''_j \rangle - \langle h_l h_k h'_i h''_j \rangle\Big) . \tag{4.4.14}$$

Which in Fourier-space can be written as

$$-\langle \gamma \beta_i' \beta_j'' \rangle = \frac{\left(k_l k_k' + k_l' k_k + k_l k_k' + k_l' k_k'\right)\left(\langle \phi_l \phi_k \beta_i' \beta_j'' \rangle - \langle \beta_l \beta_k \beta_i' \beta_j'' \rangle\right)}{k^2 + k'^2 + 2k_l k_l'}. \tag{4.4.15}$$

Thus, the equations, (4.4.14) and (4.4.15) are the spectral equation corresponding to the three-point correlation equations. Equation (4.4.15) can be used to eliminate $\langle \gamma \beta_i' \beta_j'' \rangle$ from the equation (4.4.12).

4.5. SOLUTION FOR TIMES BEFORE THE FINAL PERIOD

It is known that the equation for final period of decay is obtained by considering the two-point correlation equation after neglecting the third order correlation terms. To study the decay for times before the final period, the three-point correlation equations are considered and the quadruple correlation terms are neglected. But, to get a better picture of the MHD homogeneous turbulence decay from its initial period to its final period, three-point correlation equations are to be considered. Here, we neglect the quadruple correlation terms since they decay faster than the lower-order correlation terms.

Putting the value of $\langle \gamma \beta_i' \beta_j'' \rangle$ from equation (4.4.15) into equation (4.4.12) and neglecting all the quadruple correlation terms, we have

$$\frac{d}{dt}\langle \phi_l \beta_i' \beta_j'' \rangle + \lambda\left[(1+P_M)(k^2 + k'^2) + 2P_M k_k k_k' + 2\frac{R}{\lambda} \right]\langle \phi_l \beta_i' \beta_j'' \rangle = 0 . \tag{4.5.1}$$

The tensor equation (4.5.1) can be converted to a scalar equation by contraction of the indices i and j, and inner multiplication by k_l

$$\frac{d}{dt}(k_l \langle \phi_l \beta_i' \beta_i'' \rangle) + \lambda\left[(1+P_M)(k^2 + k'^2) + 2P_M k_k k_k' + 2\frac{R}{\lambda} \right](k_l \langle \phi_l \beta_i' \beta_i'' \rangle) = 0 . \tag{4.5.2}$$

Integrating the equation (4.5.2) between t_0 and t, and gives

$$k_l\langle\phi_l\beta_i'\beta_i''\rangle = k_l[\langle\phi_l\beta_i'\beta_i''\rangle]\exp\{-\lambda[(1+P_M)(k^2+k'^2)+2P_M k_k k_k'\cos\theta-2\frac{R}{\lambda}](t-t_0)\}$$
(4.5.3)

where $\langle\phi_l\beta_i'\beta_i''\rangle_0$ is the value of $\langle\phi\beta_i'\beta_i''\rangle$ at $t=t_0$, and θ is the angle between $\hat{K}$ and $\hat{K}'$. Now, by letting $r'=0$ in the equation (4.4.6) and comparing with equation (4.3.6) and (4.3.7), we obtain the relation

$$\langle\alpha_i\psi_k\psi_i'(\hat{K})\rangle = \int_{-\infty}^{\infty}\langle\phi_l\beta_i'(\hat{K})\beta_i''(\hat{K}')\rangle d\hat{K}' \tag{4.5.4}$$

and

$$\langle\alpha_k\psi_i\psi_i'(-\hat{K})\rangle = \int_{-\infty}^{\infty}\langle\phi_l\beta_i'(-\hat{K})\beta_i''(-\hat{K}')\rangle d\hat{K}'. \tag{4.5.5}$$

Substituting equations, (4.5.3), (4.5.4), and (4.5.5) in equation (4.3.9), one obtains

$$\frac{d}{dt}\langle\psi_i\psi_i'(\hat{K})\rangle + 2\lambda[k^2+\frac{R}{\lambda}]\langle\psi_i\psi_i'(\hat{K})\rangle = \int_{-\infty}^{\infty}2ik_l\Big[\langle\phi_l\beta_i'(\hat{K})\beta_i''(\hat{K}')\rangle - \langle\phi_l\beta_i'(-\hat{K})\beta_i''(-\hat{K}'$$

$$\times\exp[\ -\lambda\{(1+P_M)(k^2+k'^2)+2P_M kk'\cos\theta-\frac{2R}{\lambda}\}(t-t_0\ \Big]d\hat{K}. \tag{4.5.7}$$

Now, $d\hat{K}'$ can be expressed in terms of $\hat{k}'$ and θ as $-2\pi k'^2 d(\cos\theta)d\hat{k}'$ (cf. Deissler [22])

With the above relation, equation (5.7) to give

$$\frac{d}{dt}\langle\psi_i\psi_i'(\hat{K})\rangle + 2\lambda[k^2+\frac{R}{\lambda}]\langle\psi_i\psi_i'(\hat{K})\rangle = 2\int_0^{\infty}2\pi ik_l\Big[\langle\phi_l\beta_i'(\hat{K})\beta_i''(\hat{K}')\rangle - \langle\phi_l\beta_i'(-\hat{K})\beta_i''(-\hat{K}')\rangle\Big]_0$$

$$\times k'^2\left[\int_{-1}^{1}\exp\left\{-\lambda(t-t_0)\left[(1+P_M)(k^2+k'^2)+2P_M kk'\cos\theta-\frac{R}{\lambda}\right]\right\}d(\cos\theta)\right]dk'. \tag{4.5.8}$$

In order to make further calculation it is necessary to assume a relation which gives $[\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}')-\langle\phi_l\beta_i'\beta_i''\rangle(-\hat{K},-\hat{K}')]_0$ as a function of k and k'.

Following Loeffler and Deissler [57], we assume that

$$ik_l[\langle\phi_l\beta_i'(\hat{K})\beta_i''(\hat{K}')\rangle-\langle\phi_l\beta_i'(-\hat{K})\beta_i''(-\hat{K}')\rangle]_0=-\frac{\delta_0}{(2\pi)^2}[k^2k'^4-k^4k'^2] \tag{4.5.9}$$

where δ_0 is a constant depending on the initial conditions. The negative sign is placed in front of δ_0 in order to make the transfer of energy from small to large wave numbers for positive value of δ_0.

Combining equations, (4.5.8) and (4.5.9), and completing the integration with respect to $\cos\theta$, one obtains

$$\frac{d}{dt}(2\pi\langle\psi_i\psi_i'(\hat{K})\rangle)+2\lambda[k^2+\frac{R}{\lambda}](2\pi\langle\psi_i\psi_i'(\hat{K})\rangle)=-\frac{\delta_0}{\nu(t-t_0)}\int_0^\infty\left(k^3k'^5-k^5k'^3\right)\times$$

$$\times\Big[\exp\Big\{-\lambda(t-t_0)[(1+P_M)(k^2+k'^2)-2P_Mkk'+2\frac{R}{\lambda}]\Big\}$$

$$-\exp\Big\{-\lambda(t-t_0)[(1+P_M)(k^2+k'^2)+2P_Mkk'+2\frac{R}{\lambda}]\Big\}\Big]dk'. \tag{4.5.10}$$

Multiplying both sides of equation (4.5.10) by k^2, we get

$$\frac{dE_m}{dt}+2\lambda[\;k^2+\frac{R}{\lambda}\;]E_m=G \tag{4.5.11}$$

where, $E_m=2\pi k^2\langle\psi_i\psi_i'(\hat{K})\rangle$ is the magnetic energy spectrum function and G is the magnetic energy transfer term and is given by

$$G = -\frac{\delta_0}{\nu(t-t_0)}\int_0^{\infty}\left(k^3k'^5 - k^5k'^3\right)\left[\exp\left\{-\lambda(t-t_0)[(1+P_M)(k^2+k'^2) - 2P_M kk' + 2\frac{R}{\lambda}]\right\}\right.$$

$$\left. - \exp\left\{-\lambda(t-t_0)[(1+P_M)(k^2+k'^2) + 2P_M kk' + 2\frac{R}{\lambda}]\right\}\right]dk'. \tag{4.5.12}$$

Integrating Eq. (4.5.12) with respect to k', we have

$$G = -\frac{\delta_0 P_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_0)^{3/2}(1+P_M)^{5/2}}\exp[-2R(t-t_0)]\exp\left[-\lambda(t-t_0)(\frac{1+2P_M}{1+P_M})k^2\right]$$

$$\times\left[\frac{15P_M k^4}{4\nu^2(t-t_0)^2(1+P_M)} + \frac{1}{(t-t_0)}\left\{\frac{5P_M{}^2}{\nu(1+P_M)^2} - \frac{3}{2\nu}\right\}k^6 + \frac{P_M}{(1+P_M)}\left\{\frac{P_M{}^2}{(1+P_M)^2} - 1\right\}k^8\right].$$

(4.5.13)

The series of equation (4.5.13) contains only even power of k and the equation represents the transfer function arising owing to consideration of magnetic field at three-point at a time.

It is interesting to note that if we integrate equation (5.13) over all wave numbers, we find that

$$\int_0^{\infty} G\,dk = 0 \tag{4.5.14}$$

which indicating that the expression for G satisfies the condition of continuity and homogeneity.

The linear equation (4.5.11) can be solved to give

$$E_m = \exp[\ -2\lambda(k^2 + \frac{R}{\lambda})(t-t_0)\ ]\int G\exp[\ 2\lambda(k^2+\frac{R}{\lambda})(t-t_0)\ ]dt$$

$$+J(k)\exp[\ -2\lambda(k^2+\frac{R}{\lambda})(t-t_0)\ ] \qquad (4.5.15)$$

where $J(k) = \dfrac{N_0 k^2}{\pi}$ is a constant of integration and can be obtained as by Corrsin [18]

Substituting the value of G as given by equation (4.5.13) into equation (4.5.15), and integrating with respect to t, we get

$$E_m = \frac{N_0 k^2}{\pi}\exp[\ -2\lambda(k^2+\frac{R}{\lambda})(t-t_0)\ ]$$

$$+\frac{\delta_0 P_M\sqrt{\pi}}{4\lambda^{3/2}(1+P_M)^{7/2}}\exp[-2R(t-t_0)]\exp[\ -k^2\lambda(\frac{1+2P_M}{1+P_M})(t-t_0)\ ][\ \frac{3k^4}{2P_M\ (t-t_0)^{5/2}\lambda^2}$$

$$+\frac{(7P_M-6)k^6}{3\lambda(1+P_M)(t-t_0)^{3/2}}-\frac{4(3P_M{}^2-2P_M+3)}{3(1+P_M)^2(t-t_0)^{1/2}}+\frac{8\lambda^{1/2}(3P_M{}^2-2P_M+3)}{3(1+P_M)^{5/2}}k^9 N(\omega)\ ]$$

$$(4.5.16)$$

where, $N(\omega) = e^{-\omega^2}\int_0^{\omega} e^{x^2}dx$, $\qquad \omega = k\sqrt{\dfrac{\lambda(t-t_0)}{1+P_M}}$.

By setting $r=0, j=i, d\hat{K} = -2\pi k^2 d(\cos\theta)d\hat{k}$ and $E_m = 2\pi k^2\langle\psi_i\psi_j'\rangle$ in Eq.(4.3.5), we get the expression for magnetic energy decay with the fluctuating concentration as

$$\frac{\langle h_i h_i'\rangle}{2} = \int_0^{\infty} E_m d\hat{k} \qquad (4.5.17)$$

The substitution of equation (4.5.16) and subsequent integration with respect to k leads to the result

$$\frac{\langle h_i h_i' \rangle}{2} = \exp[-2R(t-t_0)][\frac{N_0(t-t_0)^{-3/2}}{8\lambda^{3/2}\sqrt{2\pi}} + \frac{\pi\delta_0(t-t_0)^{-5/2}}{4\lambda^6(1+P_M)(1+2P_M)}\{\frac{9}{16}$$

$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)} - \frac{35P_M(3P_M^2-2P_M+3)}{8(1+2P_M)^2}$$

$$+\frac{8P_M(3P_M^2-2P_M+3)}{3.2^6(1+2P_M)^3}\sum\frac{1.3.5.\cdots\cdots(2n+9)}{n!(2n+1)2^{2n}(1+P_M)^n}\}]$$

or,

$$\frac{\langle h_i h_i' \rangle}{2} = \exp[-2R(t-t_0)][\frac{N_0(t-t_0)^{-3/2}}{8\lambda^{3/2}\sqrt{2\pi}} + \delta_0 Q(t-t_0)^{-5}] \tag{4.5.18}$$

where

$$Q = \frac{\pi}{(1+P_M)(1+2P_M)^{5/2}}[\frac{9}{16} + \frac{5}{16}\frac{P_M(7P_M-6)}{1+2P_M} - \frac{35}{8}\frac{P_M(3P_M^2-2P_M+3)}{(1+2P_M)^2} + \cdots\cdots].$$

Thus, the decay law for magnetic energy fluctuation governing the concentration of a dilute contaminant undergoing a first order chemical reaction before the final period may be written as

$$\langle h^2 \rangle = \exp[-2R(t-t_0)]\left[X(t-t_0)^{-3/2} + Y(t-t_0)^{-5}\right]. \tag{4.5.19}$$

The first term of right side of equation (4.5.19) corresponds to the energy of magnetic field fluctuation of concentration for the two-point correlation and the second term represents that energy for the three-point correlation.

4.6. CONCLUDING RERARKS

This study shows that the magnetic field fluctuation of concentration decays slowly than the velocity fluctuation and if the chemical reaction of the first order is selected in the concentration, then the effect is that the magnetic field fluctuation of concentration is much more rapid and the faster rate is governed by $\exp[-2R(t-t_0)]$.

In equation (4.5.19), the term associated with the three-point correlation die out faster than the two-point correlation. For large times, the last term of equation (4.5.19) becomes negligible and the decay law for the final period becomes

$$\langle h^2 \rangle = X \exp[-2R(t-t_0)](t-t_0)^{-3/2}$$

In absence of chemical reaction, i.e., if we put $R=0$, the result shows completely accords with the result obtained earlier by sarker and kishor [91].

CHAPTER – V

PART – A

DECAY OF DUSTY FLUID TURBULENCE BEFORE THE FINAL PERIOD IN A ROTATING SYSTEM.

5.1. INTRODUCTION

In geophysical flows, the system is usually rotating with a constant angular velocity. Such large-scale flows are generally turbulent. When the motion is referred to axes, which rotate steadily with the bulk of the fluid, the coriolis force and centrifugal force must be supposed to act on the fluid. The coriolis force due to rotation plays an important role in a rotating system of turbulent flow, while the centrifugal force with the potential is incorporated to the pressure. Kishore and Dixit [43], Kishore and Singh [41], Dixit and Upadhyay [25] and Kishore and Golsefied [45] discussed the effect of coriolis force on acceleration covariance in ordinary and MHD turbulent flows. Shimomura and Yoshizawa [97], Shimomura [98,99] discussed the statistical analysis of turbulent viscosity, turbulent scalar flux and turbulent shear flows respectively in a rotating system by two-scale Direct-interaction approach.

Saffman [89] derived an equation that described the motion of a fluid containing small dust particles, which is applicable to laminar flow as well as turbulent flow. Using the equations given by Saffman, Micheal and Miller [64] discussed the motion of dusty gas occupying the semi-infinite above a rigid plane boundary. Sinha [100] and Sarker [92] considered dust particles on their own works.

Batchelor and Townsend [2] studied the decay of turbulence in the final period. The decay of turbulence in the final period occurs when the effects of the inertia forces are negligible. Diessler [21,22] developed a theory for the decay of homogeneous turbulence at times before the final period. Loeffer and Diessler [57] discussed the decay of temperature fluctuation in homogeneous turbulence. In their approach they considered the two- and three-point correlation equations and solved these equations after neglecting the fourth and higher order correlation terms. Using Deisser's theory Kumar and Patel [52,53] studied the first order reactant in homogeneous turbulence before the final period, Sarker and Kishore [91] studied the decay of MHD turbulence at the time before the final period.

Kishore and Upathdyay [49] studied the decay of MHD turbulence in rotating system. In the next, Sarker and Islam [96] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time.

By considering the above theories we have studied the decay of dusty fluid turbulence before the final period in a rotating system. In this problem we have considered the two- and three-point correlation equations and solved these equations after neglecting the quadruple correlation terms. Finally the energy decay law of fluctuating velocity of dusty fluid turbulence in a rotating system is obtained.

5.2 BASIC EQUATIONS

The equation of motion and continuity for turbulent flow of dusty incompressible fluid in a rotating system are given by

$$\frac{\partial u_i}{\partial t}+\frac{\partial(u_i u_k)}{\partial x_k}=-\frac{\partial P_m}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k \partial x_k}-2\in_{mki}\Omega_m u_i+f(u_i-v_i)\,, \tag{5.2.1}$$

$$\frac{\partial v_i}{\partial t}+v_k\frac{\partial v_i}{\partial x_k}=-\frac{k}{m_s}(v_i-u_i)\,, \tag{5.2.2}$$

and

$$\frac{\partial u_i}{\partial x_i}=\frac{\partial v_i}{\partial x_i}=0\ . \tag{5.2.3}$$

Here u_i, turbulent velocity components; v_i, dust particle velocity components; ρ, fluid density; ν, kinematic viscosity; Ω_m ,constant angular velocity components; $\in_{mki}$, alternating tensor; P_m, modified pressure (sum of hydrodynamics pressure divided by fluid density and potential of a centrifugal force); $m_s=\frac{4}{3}\pi R_s^3\rho_s$, mass of a single spherical dust particle of radius R_s ; ρ_s, constant density of the material in dust particles; k, stock's drug resistance; $f=\frac{kN}{\rho}$, dimensions of frequency; N, constant number density of dust particle;

5.3. CORRELATION AND SPECTRAL EQUATIONS

The equation of motion of dusty fluid turbulence in rotating system for the point P and P' separated by the vector $\hat{r}$

$$\frac{\partial u_i}{\partial t}+\frac{\partial(u_i u_k)}{\partial x_k}=-\frac{\partial P_m}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k \partial x_k}-2\in_{mki}\Omega_m u_i+f(u_i-v_i) \tag{5.3.1}$$

and

$$\frac{\partial u'_j}{\partial t}+\frac{\partial(u'_j u'_k)}{\partial x'_k}=-\frac{\partial P'_m}{\partial x'_j}+\nu\frac{\partial^2 u'_j}{\partial x'_k \partial x'_k}-2\in_{mki}\Omega_m u'_j+f(u'_j-v'_j)\,, \tag{5.3.2}$$

Multiplying equation (5.3.1) by u'_j and equation (5.3.2) by u_i and taking the ensemble average, we have

$$\frac{\partial\langle u_i u'_j\rangle}{\partial t}+\frac{\partial\langle u_i u'_j u_k\rangle}{\partial x_k}+\frac{\partial\langle u_i u'_j u'_k\rangle}{\partial x'_k}=-\left(\frac{\partial\langle P_m u'_j\rangle}{\partial x_i}+\frac{\partial\langle P'_m u_i\rangle}{\partial x'_j}\right)+\nu\left(\frac{\partial^2\langle u_i u'_j\rangle}{\partial x_k \partial x_k}+\frac{\partial\langle u_i u'_j\rangle}{\partial x'_k \partial x'_k}\right)$$

$$-2(\in_{mki}\Omega_m\langle u_i u'_j\rangle+\in_{nkj}\Omega_m\langle u_i u'_j\rangle)+f(2\langle u_i u'_j-v_i u'_j-u_i v'_j)\,. \tag{5.3.3}$$

By use of the transformation

$$\frac{\partial}{\partial r_i}=-\frac{\partial}{\partial x_i}=\frac{\partial}{\partial x'_i}\,,$$

equation (5.3.3) can be written as

$$\frac{\partial\langle u_i u'_j\rangle}{\partial t}+\frac{\partial\langle u_i u'_j u_k\rangle}{\partial r_k}-\frac{\partial\langle u_i u'_j u'_k\rangle}{\partial r_k}=-\left(-\frac{\partial\langle P_m u'_j\rangle}{\partial r_i}+\frac{\partial\langle P'_m u_i\rangle}{\partial r_j}\right)+2\nu\frac{\partial^2\langle u_i u'_j\rangle}{\partial r_k \partial r_k}$$

$$-2(\in_{mki}\Omega_m\langle u_i u'_j\rangle+\in_{nkj}\Omega_m\langle u_i u'_j\rangle)+f(2\langle u_i u'_j\rangle-\langle v_i u'_j\rangle-\langle u_i v'_j\rangle)\,. \tag{5.3.4}$$

Now we write equation (5.3.4) in spectral form in order to reduce it to an ordinary differential equation because of the physical significance of the spectral quantities. For this, we use three-dimensional Fourier transforms defined as follows:

$$\langle u_i u'_j(\hat{r})\rangle=\int_{-\infty}^{\infty}\langle\psi_i\psi'_j(\hat{K})\rangle\exp(i\hat{K}.\hat{r})d\hat{K}\,. \tag{5.3.5}$$

$$\langle u_i u_k u'_j(\hat{r})\rangle = \int_{-\infty}^{\infty} \langle \psi_i \psi_k \psi'_j(\hat{K})\rangle \exp(i\hat{K}.\hat{r})d\hat{K}\,, \tag{5.3.6}$$

$$\langle P_m u'_j(\hat{r})\rangle = \int_{-\infty}^{\infty} \langle \lambda \psi'_j(\hat{K})\rangle \exp(i\hat{K}.\hat{r})d\hat{K}\,, \tag{5.3.7}$$

and $$\langle v_i u'_j\rangle = \int_{-\infty}^{\infty} \langle \mu_i \psi'_j(\hat{K})\rangle \exp(i\hat{K}.\hat{r})d\hat{K} \tag{5.3.8}$$

where $\hat{K}$ is known as a wave number vector and $d\hat{K} = dK_1 dK_2 dK_3$.

From equation (5.3.6), we have

$$\langle u_i u_k u'_j(-\hat{r})\rangle = \int_{-\infty}^{\infty} \langle \psi_i \psi_k \psi'_j(\hat{K})\rangle \exp(-i\hat{K}.\hat{r})d\hat{K} = \int_{-\infty}^{\infty} \psi_i \psi_k \psi'_j(-\hat{K}) \exp(i\hat{K}.\hat{r})d\hat{K}\,.$$

Interchanging the subscripts i and j and then interchanging the point P and P', gives

$$\langle u_i u'_j u'_k(\hat{r})\rangle = \langle u_i u_k u'_i(-r)\rangle = \int_{-\infty}^{\infty} \langle \psi_i \psi_k \psi'_i(-\hat{K})\rangle \exp(i\hat{K}.\hat{r})d\hat{K}\,, \tag{5.3.6a}$$

$$\langle u_i P'_m(\hat{r})\rangle = \langle P_m u'_j(-\hat{r})\rangle = \int_{-\infty}^{\infty} \langle \lambda \psi'_j(-\hat{K})\rangle \exp(i\hat{K}.\hat{r})d\hat{K}\,, \tag{5.3.7a}$$

$$\langle u_i v'_j(\hat{r})\rangle = \langle v_i u'_j(-\hat{r})\rangle = \int_{-\infty}^{\infty} \langle \mu_i \psi'_j(-\hat{K})\rangle \exp(i\hat{K}.\hat{r})d\hat{K}\,. \tag{5.3.8a}$$

Substituting equations, (5.3.5), (5.3.6), (5.3.6a), (5.3.7), (5.3.7a), (5.3.8) and (5.3.8a) into equation (5.3.4) and making it in scalar form by contraction of the indices i and j, we get

$$\frac{d}{dt}\langle\psi_i\psi'_j\rangle+(2\nu k^2+2\in_{mki}\Omega_m+2\in_{nki}\Omega_n-Rf)\langle\psi_i\psi'_i\rangle=$$

$$=ik_k[\psi_i\psi_k\psi'_i(\hat{K})-\psi_i\psi_k\psi'_i(-K')]. \quad (5.3.10)$$

The pressure terms drop out of equation (5.3.10) because of the continuity relation

$$\frac{\partial u_i}{\partial x_i}=\frac{\partial v_i}{\partial x_i}=0 .$$

In order to obtain the three-point equation, we consider the equation of motion of dusty fluid turbulence in rotating system at the points p, p' and p'' as

$$\frac{\partial u_i}{\partial t}+\frac{\partial(u_iu_l)}{\partial x_l}=-\frac{\partial P}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_l\partial x_l}-2\in_{mli}\Omega_m u_i+f(u_i-v_i) \quad , \quad (5.3.11)$$

$$\frac{\partial u'_j}{\partial t}+\frac{\partial(u'_ju'_l)}{\partial x'_l}=-\frac{\partial P'}{\partial x'_j}+\nu\frac{\partial^2 u'_j}{\partial x'_l\partial x'_l}-2\in_{nli}\Omega_n u'_j+f(u'_j-v'_j) \quad (5.3.12)$$

and

$$\frac{\partial u''_k}{\partial t}+\frac{\partial(u''_ku''_l)}{\partial x''_l}=-\frac{\partial'' P}{\partial x''_k}+\nu\frac{\partial^2 u''_k}{\partial x'_l\partial x'_l}-2\in_{mli}\Omega_q u''_k+f(u''_k-v''_k), \quad (5.3.13)$$

Multiplying equation (5.3.11) by $u'_ku''_k$, equation (5.3.12) by $u_iu''_k$ and equation (5.3.13) by $u_iu'_j$, adding and taking ensemble average and using the transformations

$$\frac{\partial}{\partial x'_l}=\frac{\partial}{\partial r_l},\quad \frac{\partial}{\partial x''_l}=\frac{\partial}{\partial r'_l},\quad \text{and}\ \frac{\partial}{\partial x_l}=-\frac{\partial}{\partial r_l}-\frac{\partial}{\partial r'_l}$$, one obtains as

$$\frac{\partial}{\partial t}\langle u_iu'_ju''_k\rangle-\frac{\partial}{\partial r_j}\langle u_iu'_ju''_ku_l\rangle-\frac{\partial}{\partial r'_l}\langle u_iu'_ju''_ku_l\rangle+\frac{\partial}{\partial r_l}\langle u_iu'_ju''_ku'_l\rangle+\frac{\partial}{\partial r'_l}\langle u_iu'_ju''_ku''_l\rangle$$

$$=-(-\frac{\partial}{\partial r_i}\langle Pu'_ju''_k\rangle-\frac{\partial}{\partial r'_j}\langle Pu'_ju''_k\rangle+\frac{\partial}{\partial r_i}\langle P'u_iu''_k+\frac{\partial}{\partial r'_k}\langle P''u_iu'_j\rangle)+$$

$$+2\nu\left(\frac{\partial^2\langle u_i u'_j u''_k\rangle}{\partial r_l \partial r_l}+\frac{\partial\langle u_i u'_j u''_k\rangle}{\partial r_l \partial r'_l}+\frac{\partial^2\langle u_i u'_j u''_k\rangle}{\partial r'_l \partial r'_l}\right)-2(\in_{mli}\Omega_m\langle u_l u'_j u''_k\rangle+\in_{nlj}\Omega_n\langle u_i u'_l u''_k\rangle$$

$$+\in_{qlk}\Omega_q\langle u_i u'_j u''_l\rangle)+f(3\langle u_i u'_j u''_k\rangle-\langle v_i u'_j u''_k\rangle-\langle u_i v'_j u''_k\rangle-\langle u_i u'_j v''_k\rangle). \qquad (5.3.14)$$

In order to convert equation (5.3.14) to spectral form, we can define following six dimensional Fourier transforms:

$$\langle u_i u'_j(\hat{r}) u''_k(\hat{r}')\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\beta_i\beta'_j(\hat{K})\beta''_k(\hat{K}')\exp[i(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}d\hat{K}', \qquad (5.3.15)$$

$$\langle u_i u_l u'_j(\hat{r}) u''_k(\hat{r}')\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\beta_i\beta_l\beta'_j(\hat{K})\beta''_k(\hat{K}')\exp[i(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}d\hat{K}', \qquad (5.3.16)$$

$$\langle Pu'_j(\hat{r}) u''_k(\hat{r}')\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\alpha\beta'_j(\hat{K})\beta''_k(\hat{K}')\exp[i(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}d\hat{K}' \qquad (5.3.17)$$

and

$$\langle v_i u'_j(\hat{r}) u''_k(\hat{r}')\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\gamma_i\beta'_j(\hat{K})\beta''_k(\hat{K}')\exp[i(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}d\hat{K}'. \qquad (5.3.18)$$

By using the method used in obtaining equation (5.3.6a), the following relations hold

$$\langle u_i u'_l(\hat{r}) u'_j(\hat{r}) u''_k(\hat{r}')\rangle=\langle u_j u_l u'_i(-\hat{r}) u''_k(\hat{r}'-\hat{r})\rangle=$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\beta_j\beta_l\beta'_i(-\hat{K}-\hat{K}')\beta''_k(\hat{K}')\exp[i(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}d\hat{K}' \qquad (5.3.16a)$$

$$\langle u_i u'_j(\hat{r}) u''_k(\hat{r}) u''_l(\hat{r}')\rangle=\langle u_k u_l u'_i(-\hat{r}) u''_j(\hat{r}-\hat{r}')\rangle=$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\beta_k\beta_l\beta'_i(-\hat{K}-\hat{K}')\beta''_j(\hat{K}')\exp[i(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}d\hat{K}' \qquad (5.3.16b)$$

where the points p and p' are interchanged to obtain equation (5.3.16a). For equation (5.3.16b), p is replaced by p', p' is replaced by p'', and p'' is replaced by p.

Similarly,

$$\langle u_i P'(\hat{r}) u''_k(r') \rangle = \langle P u'_i(-\hat{r}) u''_k(\hat{r}' - \hat{r}) =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \alpha \beta'_i(-\hat{K} - \hat{K}') \beta''_k(\hat{K}') \exp[i(\hat{K}.\hat{r} + \hat{K}'.\hat{r}')] d\hat{K} d\hat{K}' \qquad (5.3.17a)$$

$$\langle u_i u'_j(\hat{r}) P''(\hat{r}') \rangle = \langle P u'_i(-\hat{r}) u''_j(\hat{r} - \hat{r}') =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \alpha \beta'_i(-\hat{K} - \hat{K}') \beta''_k(\hat{K}') \exp[i(\hat{K}.\hat{r} + \hat{K}'.\hat{r}')] d\hat{K} d\hat{K}' \qquad (5.3.17b)$$

$$\langle u_i v'_j u''_k(\hat{r}') \rangle = \langle v_j u'_i(-\hat{r}) u''_k(\hat{r}' - \hat{r}) =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \gamma_i \beta'_j(\hat{K}) \beta''_k(\hat{K}') \exp[i(\hat{K}.\hat{r} + \hat{K}'.\hat{r}')] d\hat{K} d\hat{K}' \qquad (5.3.18a)$$

$$\langle u_i u'_j(\hat{r}) v''_k(\hat{r}') \rangle = \langle v_k u'_i(-\hat{r}) u''_j(\hat{r} - \hat{r}') =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \gamma_k \beta'_i(-\hat{K} - \hat{K}') \beta''_j(\hat{K}) \exp[i(\hat{K}.\hat{r} + \hat{K}'.\hat{r}')] d\hat{K} d\hat{K}' \qquad (5.3.18b)$$

Substituting the preceding relation into equation (5.3.14), we get

$$\frac{d}{dt}\langle \beta_i \beta'_j \beta''_k \rangle = 2\nu(k^2 + k_l k'_l + k'^2)\langle \beta_i \beta'_j \beta''_k \rangle = [i(k_l + k'_l)\langle \beta_i \beta_l \beta'_j \beta''_k \rangle$$

$$- ik_l \langle \beta_j \beta_l \beta'_i(\hat{K} - \hat{K}') \beta''_k(\hat{K}') \rangle - ik'_l \langle \beta_k \beta_l \beta'_i(-\hat{K} - \hat{K}') \beta''_j(\hat{K}) \rangle]$$

$$- \frac{1}{\rho}[-i(k_i + k'_i)\langle \alpha \beta'_j \beta''_k \rangle + ik_j \langle \alpha \beta'_i(-\hat{K} - \hat{K}') \beta''_k(\hat{K}) \rangle + ik'_k \langle \alpha \beta'_i(-\hat{K} - \hat{K}') \beta''_j(\hat{K}) \rangle$$

$$-2[\in_{mli}\ \Omega_m+\in_{nlj}\ \Omega_n+\in_{qlk}\ \Omega_q]\langle\beta_i\beta'_j\beta''_k\rangle+f[3\langle\beta_i\beta'_j(\hat{K})\beta''_k(\hat{K}')\rangle$$

$$-\langle\gamma_i\beta'_j(\hat{K})\beta''_k(\hat{K}')\rangle-\langle\gamma_j\beta'_i(-\hat{K}-\hat{K}')\beta''_k(\hat{K}')\rangle-\langle\gamma_k\beta'_i(-\hat{K}-\hat{K}')\beta''_j(\hat{K}).\quad (5.3.19)$$

The tensor equation (5.3.19) can be converted to a scalar form by contraction of the indexes i and j and inner multiplication by k_k;

$$\frac{d}{dt}\langle k_k\beta_i\beta'_i\beta''_k\rangle+2\nu(k^2+k_lk'_l+k'^2)\langle k_k\beta_i\beta'_i\beta''_k\rangle=[ik_k(k_l+k'_l)\langle\beta_i\beta_l\beta'_i\beta''_k\rangle-$$

$$-ik_kk_l\langle\beta_i\beta_l\beta'_i(\hat{K}-\hat{K}')\beta''_k(\hat{K}')\rangle-ik_kk'_l\langle\beta_k\beta_l\beta'_i(-\hat{K}-\hat{K}')\beta''_i(\hat{K})\rangle]$$

$$-\frac{1}{\rho}k_k[-i(k_i+k'_i)\langle\alpha\beta'_j\beta''_k\rangle+ik_j\langle\alpha\beta'_i(-\hat{K}-\hat{K}')\beta''_k(\hat{K})\rangle+ik'_k\langle\alpha\beta'_i(-\hat{K}-\hat{K}')\beta''_i(\hat{K})\rangle]$$

$$-2k_k[\in_{mli}\ \Omega_m+\in_{nlj}\ \Omega_n+\in_{qlk}\ \Omega_q]\langle\beta_i\beta'_j\beta''_k\rangle+fk_k[3\langle\beta_i\beta'_j(\hat{K})\beta''_k(\hat{K}')\rangle$$

$$-\langle\gamma_i\beta'_j(\hat{K})\beta''_k(\hat{K}')\rangle-\langle\gamma_j\beta'_i(-\hat{K}-\hat{K}')\beta''_k(\hat{K}')\rangle-\langle\gamma_k\beta'_i(-\hat{K}-\hat{K}')\beta''_j(\hat{K}).\quad (5.3.20)$$

To obtain a relation between the terms on the right hand side of equation (5.3.20) derived from the quadruple correlation terms, pressure terms, rotational terms and the dust particle terms in equation (5.3.14), take the divergence of the equation of motion and combine with the continuity equation to give

$$\frac{1}{\rho}\frac{\partial^2 P}{\partial x_i\partial x_i}=-\frac{\partial^2(u_iu_l)}{\partial x_i\partial x_l}\ .\qquad (5.3.21)$$

Multiplying the equation (5.3.21) by $u'_iu''_k$, taking ensemble average and writing the resulting equation in terms of the independent variables r and r', gives

$$\frac{1}{\rho}\left(\frac{\partial^2\langle Pu'_iu''_k\rangle}{\partial r_i\partial r_i}+2\frac{\partial^2\langle Pu'_iu''_k\rangle}{\partial r_i\partial r'_i}+\frac{\partial^2\langle Pu'_iu''_k\rangle}{\partial r'_i\partial r'_i}\right)=-\frac{\partial^2\langle u_iu_lu'_iu''_k\rangle}{\partial r_i\partial r_l}-\frac{\partial^2\langle u_iu_lu'_iu''_k\rangle}{\partial r'_i\partial r_l}-$$

$$-\frac{\partial^2 \langle u_i u_l u'_i u''_k \rangle}{\partial r_i \partial r'_l} - \frac{\partial^2 \langle u_i u_l u'_i u''_k \rangle}{\partial r'_l \partial r'_i}. \qquad (5.3.22)$$

The Fourier transform of equation (5.3.22) is

$$-\frac{1}{\rho}\langle \alpha \beta_i \beta''_k \rangle = \frac{(k_i k_l + k'_i k_l + k_i k'_l + k'_i k'_l)\langle \beta_i \beta_l \beta'_i \beta''_k \rangle}{k^2 + kk + k'^2}. \qquad (5.3.23)$$

Equation (5.3.23) can be used to eliminate the quantities $\langle \alpha \beta'_i \beta''_k \rangle$, $\langle \alpha \beta'_i(-\hat{K} - \hat{K}')\beta''_k \rangle$, etc. from equation (5.3.20).

5.4. SOLUTION FOR TIMES BEFORE THE FINAL PERIOD

In order to obtain the equation for final period of decay the third order correlation terms are neglected compared to the second order correlation terms. Analogously, it would be possible to obtain a solution for times before the final period of decay by neglecting the fourth order correlation terms. If this assumption is made, all the fourth order correlation terms in the right side of equation (5.3.23) should be neglected. Thus from equation (5.3.20) and equation (5.3.23), we obtain

$$\frac{d}{dt}\langle k_k \beta_i \beta'_i \beta''_k \rangle + 2[\nu(k^2 + k_l k'_l + k'^2) + (\in_{mli} \Omega_m + \in_{nli} \Omega_n + \in_{qlk} \Omega_q) - Sf] \times$$

$$\times \langle k_k \beta_i \beta'_i \beta''_k \rangle = 0 \qquad (5.4.1)$$

where

$$3\langle \beta_i \beta'_i \beta''_k \rangle - \langle \gamma_i \beta'_i(\hat{K})\beta''_k(\hat{K}') \rangle - \langle \gamma_i \beta'_i(-\hat{K} - \hat{K}')\beta''_k(\hat{K}') \rangle - \langle \gamma_k \beta'_i(-\hat{K} - \hat{K}')\beta''_j(\hat{K})$$

$$= S\beta_i \beta'_i \beta''_k \quad \text{(say)}$$

and S is an arbitrary constant.

Integrating equation (5.4.1) between the limits t_0 and t, we get

$$\langle k_k \beta_i \beta_i' \beta_k'' \rangle = k_k (\beta_i \beta_i' \beta_{k\,i}'')_0 \exp[\{-2\nu(k^2 + kk'\cos\theta + k'^2) + 2(\in_{mli} \Omega_m +$$

$$+ \in_{nli} \Omega_n + \in_{qlk} \Omega_q) - Sf\}(t - t_0)] \tag{5.4.2}$$

where $\langle \beta_i \beta_i' \beta_k'' \rangle_0$ is the value of $\langle \beta_i \beta_i' \beta_k'' \rangle$ at $t = t_0$ and θ is the angle between k and k'.

Letting $r' = 0$ in equation (5.3.15) and comparing the result with equation (5.3.6) shows that

$$\psi_i \psi_k \psi_i'(\hat{K}) = \int_{-\infty}^{\infty} \beta_i \beta_i' \beta_k''(\hat{K}) d\hat{K}'. \tag{5.4.3}$$

Substitution of equation (5.4.2) and (5.4.3) in equation (5.3.10) result in

$$\frac{d}{dt}\langle \psi_i \psi_i' \rangle + (2\nu k^2 + 2 \in_{mki} \Omega_m + 2 \in_{nki} \Omega_n - Rf)\langle \psi_i \psi_i' \rangle$$

$$= \int_0^{\infty} 2\pi i k_k [\beta_i \beta_i' \beta_k'' - \beta_i \beta_i'(-\hat{K}) \beta_k''(-\hat{K}')]_0 k'^2 [\int_{-1}^{1} \exp[-\{2\nu(k^2 + kk'\cos\theta + k')$$

$$+ 2(\in_{mli} \Omega_m + \in_{nli} \Omega_m + \in_{qlk} \Omega_q) - Sf\}(t - t_0)] d(\cos\theta)] dk' \tag{5.4.4}$$

where $d\hat{K}' = dK_1' dK_2' dK_3'$ written in terms of k' and θ as $-2\pi k'^2 d(\cos\theta) dk'$.

In order to find the solution completely and following Deissler [22], we assume that

$$ik_k [\beta_i \beta_i' \beta_k'' - \beta_i \beta_i'(-\hat{K}) \beta_k''(-\hat{K}')]_0 = -\beta_0 (k^4 k'^6 - k^6 k'^4) \tag{5.4.5}$$

where β_0 is a constant determined by the initial conditions. The negative sign is placed in front of β_0 in order to make the transfer of energy from small to large wave numbers or positive value of β_0.

Substituting equation (5.4.5) in equation (5.4.4), writing $\langle\psi_i\psi_i'\rangle$ in terms of the energy spectrum function as

$$E = 2\pi k^2 \langle\psi_i\psi_i'\rangle \tag{5.4.6}$$

and carrying out the integration with respect to $\cos\theta$, result in

$$\frac{dE}{dt} + (2\nu k^2 + 2 \in_{mki} \Omega_m + 2 \in_{nki} \Omega_n - Rf)E = W \tag{5.4.7}$$

where W is the energy transfer term and is given by

$$W = -\frac{\beta_0}{2\nu(t-t_0)} \int_0^\infty (k^5 k'^7 - k^7 k'^5)\{\exp[-\{2\nu(k^2 - kk' + k'^2) + 2(\in_{mli} \Omega_m + \in_{nli} \Omega_m +$$

$$+ \in_{qlk} \Omega_q) - Sf\}(t-t_0)] - \exp[\{-2\nu(k^2 + kk' + k'^2) + 2(\in_{mli} \Omega_m + \in_{nli} \Omega_m +$$

$$+ \in_{qlk} \Omega_q) - Sf\}(t-t_0)]\}dk', \tag{5.4.8}$$

Integrating equation (5.4.8) with respect to k', we get

$$W = -\frac{\beta_0\sqrt{\frac{\pi}{2}}}{2\nu(t-t_0)} \exp[-\frac{3}{2\nu k^2(t-t_0)} - \{2(\in_{mli} \Omega_m + \in_{nli} \Omega_n + \in_{qlk} \Omega_q) - Sf\}(t-t_0)] \times$$

$$\times [105\frac{k^6}{(t-t_0)^{\frac{9}{2}}} + 45\frac{k^8}{(t-t_0)^{\frac{7}{2}}} - 19\frac{k^{10}}{(t-t_0)^{\frac{5}{2}}} - 3\frac{k^{12}}{(t-t_0)^{\frac{3}{2}}}] \, . \tag{5.4.9}$$

The series of equation (5.4.9) contains only even power of k , it is interesting to note that

$$\int_0^\infty W dk = 0 \, . \tag{5.4.10}$$

For obtaining the energy spectrum function E, equation (5.4.7) can be written in integral form as

$$E = \exp[-(2\nu k^2 + 2\in_{mki}\Omega_m + 2\in_{nki}\Omega_n - Rf\}(t-t_0)]\times\int\exp[(2\nu k^2 + 2\in_{mki}\Omega_m + $$

$$+ 2\in_{nki}\Omega_n - Rf\}(t-t_0)]Wdt + C(k)\exp[-(2\nu k^2 + 2\in_{mki}\Omega_m + 2\in_{nki}\Omega_n - Rf)(t-t_0)] \qquad (5.4.11)$$

where $C(k) = \frac{J_0 k^4}{3\pi}$ is a constant of integration and can be obtain following deissler [22]. The constant J_0 is known, as Loitziansky's invariant when the turbulence is isotropic.

Substituting the values of W from equation (5.4.9) in equation (5.4.11) and integrating with respect to t, we get

$$E = \frac{J_0 k^4}{3\pi}\exp[-(2\nu k^2 + 2\in_{mki}\Omega_m + 2\in_{nki}\Omega_n - Rf\}(t-t_0)] -$$

$$-\frac{\beta_0\sqrt{\pi}}{256\nu}\exp[-\frac{3}{2\nu k^2(t-t_0)} - \{2(\in_{mli}\Omega_m + \in_{nli}\Omega_n + \in_{qlk}\Omega_q) - Sf\}(t-t_0)]\times$$

$$\times[-\frac{\frac{15\sqrt{2}}{7}}{\nu^2}\frac{k^6}{\frac{7}{(t-t_0)^2}} - \frac{\frac{12\sqrt{2}}{5}}{\nu^2}\frac{k^8}{\frac{5}{(t-t_0)^2}} + \frac{\frac{7\sqrt{2}}{3}}{3\nu^2}\frac{k^{10}}{\frac{5}{(t-t_0)^2}} + \frac{\frac{16\sqrt{2}}{1}}{3\nu^2}\frac{k^{12}}{\frac{3}{(t-t_0)^2}} - \frac{32}{3}k^{13}F(\omega)] \qquad (5.4.12)$$

where $F(\omega) = \exp(-\omega^2)\int_0^\infty \exp(x^2)dx, \qquad \omega = k[\frac{\nu(t-t_0)}{2}]^{\frac{1}{2}}$.

The expression for the energy decay is obtained from equation (5.3.5) by setting $\hat{r} = 0, \quad j = i$, $d\hat{K} = -2\pi k^2 d(\cos\theta)dk$ and $E = 2\pi k^2\langle\psi_i\psi_i'\rangle$.

Thus,

$$\frac{\langle u_i u_i \rangle}{2} = \int_0^\infty E dk = \frac{J_0}{32(2\pi)^{1/2}} \nu^{-\frac{5}{2}} \exp[-(2 \in_{mki} \Omega_m + \in_{nki} \Omega_n - Rf)(t-t_0)](t-t_0)^{\frac{-5}{2}} +$$

$$+ 0.2296\beta_0 \nu^{-8} \exp[-2\{(\in_{mli} \Omega_m + \in_{nli} \Omega_n + \in_{qlk} \Omega_q) - Sf\}(t-t_0)](t-t_0)^{-7}. \qquad (5.4.13)$$

Thus, the energy decay law of velocity fluctuations of dusty fluid turbulence in a rotating system may be written as

$$\langle u^2 \rangle = A \exp[-(2 \in_{mki} \Omega_m + 2 \in_{nki} \Omega_n - Rf)(t-t_0)](t-t_0)^{\frac{-5}{2}} +$$

$$+ B \exp[-2\{(\in_{mli} \Omega_m + \in_{nli} \Omega_n + \in_{qlk} \Omega_q) - Sf\}(t-t_0)](t-t_0)^{-7} \qquad (5.4.14)$$

5.4. CONCLUDING REMARKS

In equation (5.4.14) we obtained the decay law of dusty fluid turbulence in a rotating system before the final period considering three-point correlation equation after neglecting quadruple correlation terms. If the system is non-rotating and the fluid is clean (Ω's $= 0$, $f = 0$), the equation (5.4.14) becomes

$$\langle u^2 \rangle = A\,(t-t_0)^{\frac{-5}{2}} + B\,(t-t_0)^{-7}$$

which is obtained earlier by Deissler [22].

This study shows that the effect of rotation in presence of dust particles in the flow field, the turbulent energy decays more rapidly than the energy for non-rotating clean fluid.

For large times, the effect of the higher order inertia terms is very negligible and gives the $-5/2$ power decay law for the final period.

If the higher order correlations were considered in the analysis, it appears that more terms in higher power of $(t-t_0)$ would be added to equation (5.4.14).

CHAPTER – V

PART - B

DECAY OF DUSTY FLUID MHD TURBULENCE BEFORE THE FINAL PERIOD IN ROTATING SYSTEM.

5.1. INTRODUCTION

Magnetohydrodynamics (MHD) is an important branch of Fluid dynamics. MHD is the science, which deals with the motion of highly conducting fluids in the presence of a magnetic field. The motion of the conducting fluid across the magnetic field generates electric currents which change the magnetic field, and the action of the magnetic field on these currents gives rise to mechanical force which modifies the flow of the field.

Funada, Tutiya and Ohji [29] considered the effect of coriolis force on turbulent motion in the presence of strong magnetic field with the assumption that the coriolis force term is balanced by the geostropic wind approximation

The problem considered here is an extension of the part-A of this chapter. In part-A, we have considered the ordinary turbulence but in this part, we have considered the MHD turbulence.

Following all the references, which are given in part-A of this chapter and Funada, Tutiya and Ohji [29], we have obtained the decay law of magnetic energy fluctuation of dusty fluid turbulence in rotating system.

5.2. BASIC EQUATIONS

The equations of motion for viscous, incompressible MHD dusty fluid turbulent flow in a rotating system are given by

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial W}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2\in_{mki}\Omega_m u_i + f(u_i - v_i) \qquad (5.2.1)$$

$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \lambda\frac{\partial^2 h_i}{\partial x_k \partial x_k} \qquad (5.2.2)$$

$$\frac{\partial v_i}{\partial t}+v_k\frac{\partial v_i}{\partial x_k} = -\frac{k}{m_s}(v_i - u_i) \qquad (5.2.3)$$

with

$$\frac{\partial u_i}{\partial x_i}=\frac{\partial v_i}{\partial x_i}=\frac{\partial h_i}{\partial x_i}=0 \qquad (5.2.4)$$

where $u_i(\hat{x},t)$, ith-component of turbulent velocity about the mean at a point $P(\hat{x},t)$; $h_i(\hat{x},t)$, ith-component of magnetic field fluctuation about the mean at a point $P(\hat{x},t)$; $W(x,t)=\frac{p}{\rho}+\frac{1}{2}\langle h^2\rangle+\frac{1}{2}\left|\hat{\Omega}\times\hat{x}\right|^2$, total MHD pressure inclusive of potential and centrifugal force; $p(\hat{x},t)$, hydrodynamic pressure; Ω_m, constant angular velocity components : $\in_{mki}$, alternating tensor; ρ, fluid density; $\lambda=\frac{\nu}{P_M}$, magnetic diffusivity;

ν, kinematic viscosity ; P_M, magnetic prandtle number; $m_s = \frac{4}{3}\pi R_s^3 \rho_s$, mass of single spherical dust particle of radius R_s ; ρ_s, constant density of the material in dust particles; $f = \frac{kN}{\rho}$, dimension of frequency; N, constant number density of dust particle; x_k, space coordinate; the subscripts can take on the values 1, 2 or 3 and the repeated subscripts in a term indicates a summation.

5.3. TWO-POINT CORRELATION AND SPECTRAL EQUATIONS

The induction equation of a magnetic field at the point P is

$$\frac{\partial h_i}{\partial t} + u_k \frac{\partial h_i}{\partial x_k} - h_k \frac{\partial u_i}{\partial x_k} = \lambda \frac{\partial^2 h_i}{\partial x_k \partial x_k} \tag{5.3.1}$$

and the point P' is

$$\frac{\partial h_i'}{\partial t'} + u_k' \frac{\partial h_j'}{\partial x_k'} - h_k' \frac{\partial u_j'}{\partial x_k'} = \lambda \frac{\partial^2 h_j'}{\partial x_k' \partial x_k'} \ . \tag{5.3.2}$$

Multiplying equation (5.3.1) by h_j' and equation (5.3.2) by h_i, adding and taking ensemble average, we get the two-point correlation equation for the fluctuating magnetic field as

$$\frac{\partial \langle h_i h_j' \rangle}{\partial t} + \frac{\partial}{\partial x_k}[\langle u_k h_i h_j' \rangle - \langle u_i h_k h_j' \rangle] + \frac{\partial}{\partial x_k'}[\langle u_k' h_i h_j' \rangle - \langle u_j' h_i h_k' \rangle]$$

$$= \lambda [\frac{\partial^2 \langle h_i h_j' \rangle}{\partial x_k \partial x_k} + \frac{\partial^2 \langle h_i h_j' \rangle}{\partial x_k' \partial x_k'}] . \tag{5.3.3}$$

Angular bracket $\langle .... \rangle$ which is used to denote an ensemble average.

Using the transformation

$$\frac{\partial}{\partial r_k} = -\frac{\partial}{\partial x_k} = \frac{\partial}{\partial x'_k}$$

and the Chandrasekhar relations [13]

$$\langle u_k h_i h'_j \rangle = -\langle u'_k h_i h'_j \rangle \quad , \qquad \langle u'_i h_i h'_k \rangle = -\langle u_i h_k h'_i \rangle$$

equation (5.3.3) becomes

$$\frac{\partial \langle h_i h'_j \rangle}{\partial t} + 2\frac{\partial}{\partial r_k}[\langle u'_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle] = 2\lambda \frac{\partial^2 \langle h_i h'_j \rangle}{\partial r_k \partial r_k} - 2R\langle h_i h'_j \rangle . \qquad (5.3.4)$$

Now we write equation (5.3.4) in spectral form in order to reduce it to an ordinary differential equation by use of the following three-dimensional Fourier-transforms

$$\langle h_i h'_j(\hat{r}) \rangle = \int_{-\infty}^{\infty} \langle \psi_i \psi'_j(\hat{K}) \rangle \exp[\hat{i}(\hat{K}.\hat{r})] d\hat{K} , \qquad (5.3.5)$$

$$\langle u_i h_k h'_j(\hat{r}) \rangle = \int_{-\infty}^{\infty} \langle \alpha_i \psi_k \psi'_j(\hat{K}) \rangle \exp[\hat{i}(\hat{K}.\hat{r})] d\hat{K} , \qquad (5.3.6)$$

$$\langle u'_k h_i h'_j(\hat{r}) \rangle = \langle u_k h_i h'_j(-\hat{r}) \rangle = \int_{-\infty}^{\infty} \langle \alpha_i \psi_i \psi'_j(-K') \rangle \exp[\hat{i}(\hat{K}.\hat{r})] d\hat{K} \qquad (5.3.7)$$

(equation (5.3.7) is obtained by interchanging the subscripts i and j and then the points p and p') and hence

$$\frac{d\langle \psi_i \psi'_j(\hat{K}) \rangle}{dt} + 2\lambda k^2 \langle \psi_i \psi'_j(\hat{K}) \rangle = 2ik_k [\langle \alpha_i \psi_k \psi'_j(\hat{K}) \rangle - \langle \alpha_k \psi_i \psi'_j(-\hat{K}) \rangle] . \qquad (5.3.8)$$

The tensor equation (5.3.8) becomes a scalar form by contraction of the indices i and j

$$\frac{d\langle\psi_i\psi'_i(\hat{K})\rangle}{dt}+2\lambda k^2\langle\psi_i\psi'_i(\hat{K})\rangle=2ik_k[\langle\alpha_i\psi_k\psi'_i(\hat{K})\rangle-\langle\alpha_k\psi_i\psi'_i(-\hat{K})\rangle]. \qquad (5.3.9)$$

The term on the right hand side of equation (5.3.9) is called energy transfer term while the 2[nd] term on the left- hand side is the dissipation term.

5.4. THREE-POINT CORRELATION AND SPECTRAL EQUATIONS

Similar procedure can be used to find the three-point correlation equation. For this purpose we take the momentum equation of MHD dusty fluid turbulence in a rotating system at the point P and induction equations of magnetic field fluctuation at the points P' and P'' separated by the vector $\hat{r}$ and $\hat{r}'$ as

$$\frac{\partial u_l}{\partial t}+u_k\frac{\partial u_l}{\partial x_k}-h_k\frac{\partial h_l}{\partial x_k}=-\frac{\partial W}{\partial x_l}+\nu\frac{\partial^2 u_l}{\partial x_k\partial x_k}-2\in_{mkl}\Omega_m u_l+f(u_l-v_l), \qquad (5.4.1)$$

$$\frac{\partial h'_i}{\partial t'}+u'_k\frac{\partial h'_i}{\partial x'_k}-h'_k\frac{\partial u'_i}{\partial x'_k}=\lambda\frac{\partial^2 h'_i}{\partial x'_k\partial x'_k}, \qquad (5.4.2)$$

$$\frac{\partial h''_j}{\partial t''}+u''_k\frac{\partial h''_j}{\partial x''_k}-h''_k\frac{\partial u''_j}{\partial x''_k}=\lambda\frac{\partial^2 h''_j}{\partial x''_k\partial x''_k}. \qquad (5.4.3)$$

Multiplying equations, (5.4.1) - (5.4.3) by $h'_i h''_j$, $u_l h'_j$ and $u_l h'_i$ respectively, adding and taking ensemble average, one obtains

$$\frac{\partial\langle u_l h'_i h''_j\rangle}{\partial t}+\frac{\partial}{\partial x_k}[\langle u_k u_l h'_i h''_j\rangle-\langle h_k h_l h'_i h''_j\rangle]+\frac{\partial}{\partial x'_k}[\langle u_l u'_k h'_i h''_j\rangle-\langle u_l u'_i h'_k h''_j\rangle]$$

$$+\frac{\partial}{\partial x''_k}[\langle u_l u''_k h'_i h''_j\rangle-\langle u_l u''_j h'_i h''_k\rangle]=-\frac{\partial\langle W h'_i h''_j\rangle}{\partial x_l}+\nu\frac{\partial^2\langle u_l h'_i h''_j\rangle}{\partial x_k\partial x_k}+$$

$$+\lambda\left[\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k' x_k'}+\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k''\partial x_k''}\right]-2\in_{mkl}\Omega_m\langle u_l h_i' h_j''\rangle+f(\langle u_l h_i' h_j''\rangle-\langle v_l h_i' h_j''\rangle). \tag{5.4.4}$$

Using the transformations

$$\frac{\partial}{\partial x_k}=-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right),\quad \frac{\partial}{\partial x_k'}=\frac{\partial}{\partial r_k},\quad \frac{\partial}{\partial x_k''}=\frac{\partial}{\partial r_k'}$$

into equations (5.4.4), one obtains

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t}-\lambda\left[(1+P_M)\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k\partial r_k}+(1+P_M)\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k'\partial r_k'}+2P_M\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k\partial r_k'}\right]$$

$$=\frac{\partial}{\partial r_k}\langle u_l u_k' h_i' h_j''\rangle+\frac{\partial}{\partial r_k'}\langle u_l u_k' h_i' h_j''\rangle-\frac{\partial}{\partial r_k'}\langle h_l h_k h_i' h_j''\rangle-\frac{\partial}{\partial r_k'}\langle h_l h_k h_i' h_k''\rangle$$

$$-\frac{\partial}{\partial r_k}\langle u_l u_k' h_i' h_j''\rangle+\frac{\partial}{\partial r_k}\langle u_l u_i' h_k' h_j''\rangle-\frac{\partial\langle u_l u_k'' h_i' h_j''\rangle}{\partial r_k'}+\frac{\partial\langle u_l u_j'' h_i' h_k''\rangle}{\partial r_k'}$$

$$+\frac{\partial\langle W h_i' h_j''\rangle}{\partial r_l}+\frac{\partial\langle W h_i' h_j''\rangle}{\partial r_l'}-2\in_{mkl}\Omega_m\langle u_l h_i' h_j''\rangle+f(\langle u_l h_i' h_j''\rangle-\langle v_i h_i' h_j''\rangle). \tag{5.4.5}$$

Using the six dimensional Fourier transforms of the type

$$\langle u_l h_i'(\hat{r})h_j''(\hat{r}')\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\phi_l\beta_i'(\hat{K})\beta_j''(\hat{K}')\rangle\exp[\hat{i}(\hat{K}.\hat{r}+\hat{k}'.\hat{r}')]d\hat{K}d\hat{K}', \tag{5.4.6}$$

$$\langle u_l u_k'(\hat{r})h_i'(\hat{r})h_j''(\hat{r}')\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\phi_l\phi_k'(\hat{K})\beta_i'(\hat{K})\beta_j''(\hat{K}')\rangle\exp[\hat{i}(\hat{K}.\hat{r}+\hat{k}'.\hat{r}')]d\hat{K}d\hat{K}', \tag{5.4.7}$$

$$\langle u_l u_i'(\hat{r})h_k'(\hat{r})h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \phi_l \phi_i'(\hat{K})\beta_k'(\hat{K})\beta_j''(\hat{K}')\rangle \exp[\hat{i}(\hat{K}.\hat{r} + \hat{k}'.\hat{r}')]d\hat{K}d\hat{K}', \quad (5.4.8)$$

$$\langle u_l u_k h_i'(\hat{r})h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \phi_l \phi_k \beta_i'(\hat{K})\beta_j''(\hat{K}')\rangle \exp[\hat{i}(\hat{K}.\hat{r} + \hat{k}'.\hat{r}')]d\hat{K}d\hat{K}', \quad (5.4.9)$$

$$\langle h_l h_k h_i'(\hat{r})h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \beta_l \beta_k \beta_i'(\hat{K})\beta_j''(\hat{K}')\rangle \exp[\hat{i}(\hat{K}.\hat{r} + \hat{k}'.\hat{r}')]d\hat{K}d\hat{K}', \quad (5.4.10)$$

$$\langle W h_i'(\hat{r})h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \gamma \beta_i'(\hat{K})\beta_j''(\hat{K}')\rangle \exp[\hat{i}(\hat{K}.\hat{r} + \hat{k}'.\hat{r}')]d\hat{K}d\hat{K}', \quad (5.4.11)$$

and

$$\langle v_l h_i'(\hat{r})h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \mu_l \beta_i(\hat{K})\beta_j''(\hat{K}')\rangle \exp[\hat{i}(\hat{K}.\hat{r} + \hat{k}'.\hat{r}')]d\hat{K}d\hat{K}' \quad (5.4.12)$$

with the facts that

$$\langle u_l u_k'' h_j'' h_i'\rangle = \langle u_l u_k' h_i' h_j''\rangle, \quad \langle u_l u_j'' h_i' h_k''\rangle = \langle u_l u_i' h_k' h_j''\rangle,$$

we can write equation (5.4.5) in the form

$$\frac{d}{dt}\langle \phi_l \beta_i' \beta_j''\rangle + \lambda\left[(1+P_M)(k^2 + k'^2) + 2P_M k_k k_k' + 2\frac{\in_{mkl}\Omega_m}{\lambda} - \frac{f}{\lambda}\right]\langle \phi_l \beta_i' \beta_j''\rangle$$

$$= i(k_k + k_k')\langle \phi_l \phi_k \beta_i' \beta_j''\rangle - i(k_k + k_k')\langle \beta_l \beta_k \beta_i' \beta_j''\rangle - i(k_k + k_k')\langle \phi_l \phi_k' \beta_i' \beta_j''\rangle$$

$$+ i(k_k + k_k')\langle \phi_l \phi_i' \beta_k' \beta_j''\rangle + i(k_l + k_l')\langle \gamma \beta_i' \beta_j'\rangle - f\langle \mu_i \beta_i' \beta_j''\rangle. \quad (5.4.13)$$

In order to relate the terms on the right side of equation (5.4.12) derived from the quadruple correlation terms and from the pressure force terms in equation (5.4.5), we take the derivative with respect to x_l of the momentum equation (5.4.1) at p and combine with the continuity equation to give

$$-\frac{\partial^2 W}{\partial x_l \partial x_l} = \frac{\partial^2}{\partial x_l \partial x_k}(u_l u_k - h_l h_k). \tag{5.4.13}$$

Multiplying equation (4.4.13) by $h_i' h_j'$, taking time averages and writing the equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$-\left[\frac{\partial^2}{\partial r_l \partial r_l} + \frac{\partial^2}{\partial r_l' \partial r_l'} + 2\frac{\partial}{\partial r_l \partial r_l'}\right]\langle W h_i' h_j'' \rangle$$
$$= \left[\frac{\partial^2}{\partial r_l \partial r_k} + \frac{\partial^2}{\partial r_l' \partial r_k} + \frac{\partial^2}{\partial r_l \partial r_k'} + \frac{\partial^2}{\partial r_l' \partial r_k'}\right]\left(\langle u_l u_k h_i' h_j'' \rangle - \langle h_l h_k h_i' h_j'' \rangle\right). \tag{5.4.14}$$

Which in Fourier-space can be written as

$$-\langle \gamma \beta_i' \beta_j'' \rangle = \frac{(k_l k_k' + k_l' k_k + k_l k_k' + k_l' k_k')(\langle \phi_l \phi_k \beta_i' \beta_j'' \rangle - \langle \beta_l \beta_k \beta_i' \beta_j'' \rangle)}{k^2 + k'^2 + 2k_l k_l'} . \tag{5.4.15}$$

Thus, the equations (5.4.14) and (5.4.15) are the spectral equation corresponding to the three-point correlation equations. Equation (5.4.15) can be used to eliminate $\langle \gamma \beta_i' \beta_j'' \rangle$ from the equation (5.4.13).

5.5. SOLUTION FOR TIMES BEFORE THE FINAL PERIOD.

It is known that the equation for final period decay is obtained by considering the two-point correlation equations after neglecting the third order correlation terms. To study the decay for times before the final period, the three-point correlation equations are considered and the quadruple correlation terms are neglected because the quadruple correlation terms decays faster than the lower-order correlation terms.

From equation (5.4.15) and (5.4.13) after neglecting all the quadruple correlation terms, we have

$$\frac{d}{dt}\langle \phi_l \beta'_i \beta''_j \rangle + \lambda\Big[\,(1+P_M)(k^2+k'^2)+2P_M k_k k'_k + 2\frac{\in_{mkl}\Omega_m}{\lambda} - \frac{f}{\lambda}\,\Big]\langle \phi_l \beta'_i \beta''_j \rangle$$

$$+ f\langle \mu_l \beta'_i \beta'' \rangle_j = 0 \,. \tag{5.5.1}$$

The tensor equation (5.5.1) can be converted to a scalar equation by contraction of the indices i and j, and inner multiplication by k_l

$$\frac{d}{dt}(k_l\langle \phi_l \beta'_i \beta''_i \rangle) + \lambda\Big[\,(1+P_M)(k^2+k'^2)+2P_M k_k k'_k$$

$$+2\frac{\in_{mkl}\,\Omega_m}{\lambda} - \frac{fS}{\lambda}\,\Big](k_l\langle \phi_l \beta'_i \beta''_i \rangle) = 0 \tag{5.5.2}$$

where $\langle \mu_l \beta'_i \beta''_i \rangle = R\langle \phi_l \beta'_i \beta''_i \rangle$ and $1-R=S$, here R and S are arbitrary constant.

Integrating the equation (5.5.2) between t_0 and t, and gives

$$k_l\langle \phi_l \beta'_i \beta''_i \rangle = k_l[\langle \phi_l \beta'_i \beta''_i \rangle]\exp\{-\lambda[(1+P_M)(k^2+k'^2)+$$

$$+2P_M k_k k'_k \cos\theta + 2\frac{\in_{mkl}\,\Omega_m}{\lambda} - \frac{fS}{\lambda}](t-t_0)\} \tag{5.5.3}$$

where $\langle \phi_l \beta_i' \beta_i'' \rangle_0$ is the value of $\langle \phi \beta_i' \beta_i'' \rangle$ at $t = t_0$, and θ is the angle between $\hat{K}$ and $\hat{K}'$. Now, by letting $r' = 0$ in the equation (5.4.6) and comparing with equations, (5.3.6) and (5.3.7), we obtain the relation

$$\langle \alpha_i \psi_k \psi_i'(\hat{K}) \rangle = \int_{-\infty}^{\infty} \langle \phi_l \beta_i'(\hat{K}) \beta_i''(\hat{K}') \rangle d\hat{K}' \tag{5.5.4}$$

and

$$\langle \alpha_k \psi_i \psi_i'(-\hat{K}) \rangle = \int_{-\infty}^{\infty} \langle \phi_l \beta_i'(-\hat{K}) \beta_i''(-\hat{K}') \rangle d\hat{K}' . \tag{5.5.5}$$

Substituting equations, (5.5.3)-(5.5.5) in equation (5.3.9), we obtain

$$\frac{d}{dt}\langle \psi_i \psi_i'(\hat{K}) \rangle + 2\lambda k^2 \langle \psi_i \psi_i'(\hat{K}) \rangle = \int_{-\infty}^{\infty} 2ik_l \left[\langle \phi_l \beta_i'(\hat{K}) \beta_i''(\hat{K}') \rangle - \langle \phi_l \beta_i'(-\hat{K}) \beta_i''(-\hat{K}') \rangle \right]_0$$

$$\times \exp\left[-\lambda\{(1 + P_M)(k^2 + k'^2) + 2P_M kk' \cos\theta + 2\frac{\in_{mkl} \Omega_m}{\lambda} - \frac{fS}{\lambda}\}(t - t_0) \right] d\hat{K} . \tag{5.5.7}$$

Now, $d\hat{K}'$ can be expressed in terms of $\hat{k}'$ and θ as $-2\pi k'^2 d(\cos\theta) d\hat{k}'$.(cf. Deissler [22])

.

With the above relation, equation (5.5.7) to give

$$\frac{\partial}{\partial t}\langle \psi_i \psi_i'(\hat{K}) \rangle + 2\lambda k^2 \langle \psi_i \psi_i'(\hat{K}) \rangle = 2\int_0^{\infty} 2\pi i k_l \left[\langle \phi_l \beta_i'(\hat{K}) \beta_i''(\hat{K}') \rangle - \langle \phi_l \beta_i'(-\hat{K}) \beta_i''(-\hat{K}') \rangle \right]_0 k'^2$$

$$\times \left[\int_{-1}^{1} \exp\left\{ -\lambda(t - t_0)\left[(1 + P_M)(k^2 + k'^2) + 2P_M kk' \cos\theta + 2\frac{\in_{mkl} \Omega_m}{\lambda} - \frac{fS}{\lambda} \right] \right\} d(\cos\theta) \right] dk'. \tag{5.5.8}$$

In order to make further calculation it is necessary to assume a relation, which gives $[\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K}, \hat{K}') - \langle \phi_l \beta_i' \beta_i'' \rangle(-\hat{K}, -\hat{K}')]_0$ as a function of k and k'.

Following Loeffler and Deissler [57], we assume that

$$ik_l[\langle\phi_l\beta_i'(\hat{K})\beta_i''(\hat{K}')\rangle - \langle\phi_l\beta_i'(-\hat{K})\beta_i''(-\hat{K}')\rangle]_0 = -\frac{\delta_0}{(2\pi)^2}[k^2k'^4 - k^4k'^2] \tag{5.5.9}$$

where δ_0 is a constant depending on the initial conditions. The negative sign is placed in front of δ_0 in order to make the transfer of energy from small to large wave numbers for positive value of δ_0.

Combining equations (5.5.8) and (5.5.9), and completing the integration with respect to $\cos\theta$, one obtains

$$\frac{d}{dt}(2\pi\langle\psi_i\psi_i'(\hat{K})\rangle) + 2\lambda k^2(2\pi\langle\psi_i\psi_i'(\hat{K})\rangle) = -\frac{\delta_0}{\nu(t-t_0)}\int_0^\infty\left(k^3k'^5 - k^5k'^3\right)\times$$

$$\times\left[\exp\left\{-\lambda(t-t_0)[(1+P_M)(k^2+k'^2) - 2P_M kk' + 2\frac{\in_{mkl}\Omega_m}{\lambda} - \frac{fS}{\lambda}]\right\}\right.$$

$$\left.-\exp\left\{-\lambda(t-t_0)[(1+P_M)(k^2+k'^2) + 2P_M kk' + 2\frac{\in_{mkl}\Omega_m}{\lambda} - \frac{fS}{\lambda}]\right\}\right]dk'. \tag{5.5.10}$$

Multiplying both sides of equation (5.5.10) by k^2, we have the magnetic energy spectrum function $E_m = 2\pi k^2\langle\psi_i\psi'_j\rangle$ and then we obtain

$$\frac{\partial E_m}{\partial t} + 2\lambda k^2 E_m = G \tag{5.5.11}$$

where, G is the magnetic energy transfer term and is given by

$$G = -\frac{\delta_0}{\nu(t-t_0)}\int_0^\infty\left(k^3k'^5 - k^5k'^3\right)\left[\exp\left\{-\lambda(t-t_0)[(1+P_M)(k^2+k'^2) - 2P_M kk' + 2\frac{\in_{mkl}\Omega_m}{\lambda}\right.\right.$$

$$-\frac{fS}{\lambda}]\} \quad -\exp\left\{-\lambda(t-t_0)[(1+P_M)(k^2+k'^2)+2P_M kk'+2\frac{\in_{mkl}\Omega_m}{\lambda}-\frac{fS}{\lambda}]\}\right] dk'. \quad (5.5.12)$$

Integrating equation (5.5.12) with respect to k', we have

$$G=-\frac{\delta_0 P_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_0)^{3/2}(1+P_M)^{5/2}}\exp\left[-\{2\frac{\in_{mkl}\Omega_m}{\lambda}-\frac{fS}{\lambda}(t-t_0)\right]$$

$$\times\exp\left[-\lambda(t-t_0)(\frac{1+2P_M}{1+P_M})k^2\right]\left[\frac{15P_M k^4}{4\nu^2(t-t_0)^2(1+P_M)}\right.$$

$$\left.+\frac{1}{(t-t_0)}\left\{\frac{5P_M{}^2}{\nu(1+P_M)^2}-\frac{3}{2\nu}\right\}k^6+\frac{P_M}{(1+P_M)}\left\{\frac{P_M{}^2}{(1+P_M)^2}-1\right\}k^8\right]. \quad (5.5.13)$$

The series of equation (5.5.13) contains only even power of k and the equation represents the transfer function arising owing to consideration of magnetic field at three-point at a time.

It is interesting to note that if we integrate equation (5.5.13) over all wave numbers, we find that

$$\int_0^\infty G dk = 0 \quad (5.5.14)$$

which indicating that the expression for G satisfies the condition of continuity and homogeneity.

The linear equation (5.5.11) can be solved to give

$$E_m = \exp[\ -2\lambda k^2(t-t_0)\ ]\int G\exp[\ 2\lambda k^2(t-t_0)\ ]dt$$

$$+J(k)\exp[\ -2\lambda k^2(t-t_0)\ ] \qquad (5.5.15)$$

where $J(k) = \dfrac{N_0 k^2}{\pi}$ is a constant of integration and can be obtained as by Corrsin [17]

Substituting the value of G as given by equation (5.5.13) into equation (5.5.15), and integrating with respect to t, we get

$$E_m = \frac{N_0 k^2}{\pi}\exp[\ -2\lambda k^2(t-t_0)\ ] + \frac{\delta_0 P_M\sqrt{\pi}}{4\lambda^{3/2}(1+P_M)^{7/2}}$$

$$\times\exp[-\{2\in_{mkl}\Omega_m - fS\}(t-t_0)]\exp[\ -k^2\lambda(\frac{1+2P_M}{1+P_M})(t-t_0)\ ][\ \frac{3k^4}{2P_M\ (t-t_0)^{5/2}\lambda^2}$$

$$+\frac{(7P_M-6)k^6}{3\lambda(1+P_M)(t-t_0)^{3/2}} - \frac{4(3P_M{}^2-2P_M+3)}{3(1+P_M)^2(t-t_0)^{1/2}} + \frac{8\lambda^{1/2}(3P_M{}^2-2P_M+3)}{3(1+P_M)^{5/2}}k^9 N(\omega)\ ] \qquad (5.5.16)$$

where, $N(\omega) = e^{-\omega^2}\int_0^{\omega} e^{x^2}dx$, $\qquad \omega = k\sqrt{\dfrac{\lambda(t-t_{0)}}{1+P_M}}$.

By setting $\hat{r}=0, j=i, d\hat{K} = -2\pi k^2 d(\cos\theta)d\hat{k}$ and $E_m = 2\pi k^2\langle\psi_i\psi_j'\rangle$ in equation (5.3.5), we get the expression for magnetic energy decay law as

$$\frac{\langle h_i h_i'\rangle}{2} = \int_0^{\infty} E_m d\hat{k}\ . \qquad (5.5.17)$$

On substitution of equation (5.5.16) and subsequent integration with respect to k leads to the result

$$\frac{\langle h_i h_i' \rangle}{2} = \frac{N_0 (t-t_0)^{-3/2}}{8\lambda^{3/2}\sqrt{2\pi}} + \exp[-\{2\in_{mkl}\Omega_m - fS\}] \frac{\pi\delta_0 (t-t_0)^{-5/2}}{4\lambda^6 (1+P_M)(1+2P_M)}$$

$$\times\left\{ \frac{9}{16} + \frac{5P_M(7P_M-6)}{16(1+2P_M)} - \frac{35P_M(3P_M^2-2P_M+3)}{8(1+2P_M)^2} + \right.$$

$$\left. + \frac{8P_M(3P_M^2-2P_M+3)}{3.2^6(1+2P_M)^3} \sum \frac{1.3.5.\cdots\cdots(2n+9)}{n!(2n+1)2^{2n}(1+P_M)^n} \right\}]$$

or,

$$\frac{\langle h_i h_i' \rangle}{2} = \frac{N_0 (t-t_0)^{-3/2}}{8\lambda^{3/2}\sqrt{2\pi}} + \exp[-\{2\in_{mkl}\Omega_m - fS\}]\delta_0 Q(t-t_0)^{-5} \tag{4.5.18}$$

where

$$Q = \frac{\pi}{(1+P_M)(1+2P_M)^{5/2}} [\frac{9}{16} + \frac{5}{16}\frac{P_M(7P_M-6)}{1+2P_M} - \frac{35}{8}\frac{P_M(3P_M^2-2P_M+3)}{(1+2P_M)^2} + \cdots\cdots].$$

Thus, the decay law for magnetic energy fluctuation of dusty fluid MHD turbulence in a rotating system before the final period may be written as

$$\langle h^2 \rangle = X(t-t_0)^{-\frac{3}{2}} + \exp[-\{2\in_{mkl}\Omega_m - fs\}]\, Y(t-t_0)^{-5}. \tag{4.5.19}$$

5.6. CONCLUDING RERARKS

The first term in the right side of equation (5.5.19) corresponds to the magnetic energy for two-point correlation and the second term represent magnetic energy for the three-point correlation. The term associated with the three-point correlation die out faster than the two-point correlation term. For large times, the last term in the equation becomes negligible, leaving the $-3/2$ power decay law for the final period.

This study shows that for a given magnetic field flùctuation of dusty fluid turbulence in a rotating system, the energy decays more slowly than the energy of velocity fluctuation of dusty fluid turbulence in a rotating system which are obtained in part A of this chapter.

If we consider non-rotating clean fluid, equation (5.5.19) will be reduced to Sarker and Kishore [91]. If the effect of dust particle is not taken in to account, the result will be completely same with the result obtained by Kishore and Upathdyay [49].

CHAPTER -VI

PART- A

DISTRIBUTION FUNCTION IN THE STATISTICAL THEORY OF TURBULENCE FOR VELOCITY AND CONCENTRATION UNDERGOING A FIRST ORDER REACTION.

6.1. INTRODUCTION

The starting points for modern studies of kinetic theory are the Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) equations. These are a coupled infinite hierarchy of equations for multi-particle distribution functions, which are obtained by integrating the Liouville equation over some of the variables. Two major and distinct areas of investigation in non-equilibrium statistical mechanics are the kinetic theory of gases and the statistical theory of fluid turbulence. Various analytical theories in the statistical theory of turbulence have been discussed in the past by Hopf [35], Kraichanan [51], Edward [26] and Herring [33]. Lundgren [58] derived hierarchy of coupled equations for multi-point turbulent velocity distributions in the statistical theory of turbulence that resemble with BBGKY hierarchy of equations in the kinetic theory of gases. Pope [81] considered the probability density function for the instantaneous composition of reacting mixture of gases. Kishore and Singh [40,42] derived transport equation for the bivariate joint distribution function of velocity, temperature and velocity, temperature and concentration in convective turbulent flow respectively.

In the following, we have defined the distribution functions for the simultaneous velocity and concentrations of dilute contaminant undergoing a first order chemical reaction and derived the equations for evolution of distribution functions. These equations are similar to the BBGKY equation in structure.

6.2. FUNDAMENTAL EQUATIONS

The equation of motion for viscous, incompressible turbulent flow [58] and field equation of concentration undergoing a first order chemical reaction [52] are given by

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=-\frac{\partial}{\partial x}\frac{1}{4\pi}\int\frac{\frac{\partial}{\partial x'}\{u(x',t)\frac{\partial u(x',t)}{\partial x'}\}}{|x-x'|}dx'+v\frac{\partial}{\partial x}\frac{\partial}{\partial x}u \quad , \tag{6.2.1}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}=D\frac{\partial}{\partial x}\frac{\partial}{\partial x}C-RC \tag{6.2.2}$$

with

$$\frac{\partial u}{\partial x}=0 \tag{6.2.3}$$

where u is the fluctuating velocity component, v is the kinematic viscosity, C is the fluctuation of concentration, D is the diffusive coefficient of contaminant, R is the constant reaction rate. u and x are the vectors in the whole process of this part.

6.3. DISTRIBUTION FUNCTIONS AND SOME OF THEIR PROPERTIES

We define now the joint distribution function of velocity and concentration in terms of Dirac-Delta functions. The one point distribution, $f^{(1)}(v^{(1)},\phi^{(1)})$ is defined such that

$f_1^{(1)}(v^{(1)},\phi^{(1)})dv^{(1)}d\phi^{(1)}$ is the probability that the fluid velocity and concentration at a time t are in the element $dv^{(1)}$ about $v^{(1)}$ and $d\phi^{(1)}$ about $\phi^{(1)}$ and is given by

$$f_1^{(1)}(x^{(1)},v^{(1)},\phi^{(1)},t) = \langle \delta\,(u^{(1)} - v^{(1)})\,\delta\,(C^{(1)},\phi^{(1)})\rangle . \tag{6.3.1}$$

Similarly, the two-point distribution function is given by

$$f_2^{(1,2)}(x^{(1)},v^{(1)},\phi^{(1)},x^{(2)},v^{(2)},\phi^{(2)},t) =$$

$$\langle \delta\,(u^{(1)} - v^{(1)})\,\delta\,(C^{(1)},\phi^{(1)})\delta(u^{(2)}-\phi^{(2)})\delta(C^{(2)} - \phi^{(2)})\rangle . \tag{6.3.2}$$

And so on, an infinite number of multi-point distribution functions $f_3^{(1,2,3)}$, $f_4^{(1,2,3,4)}$ etc. can be defined.

The following properties of the constructed distribution functions can be deduced from the above definitions.

6.3.1. REDUCTION PROPERTY

Integration with respect to pair of variables at one point lowers the order of distribution by one

$$\iint f_1^{(1)}dv^{(1)}d\phi^{(1)} = 1$$

$$\iint f_2^{(1,2)}dv^{(2)}d\phi^{(2)} = f_1^{(1)}$$

$$\iint f_3^{(1,2,3)}dv^{(3)}d\phi^{(3)} = f_2^{(1,2)}$$

and so on. Also integration with respect to any one of the variables reduces the number of Delta-functions in the distribution by one

$$\int f_1^{(1)} dv^{(1)} = \langle \delta(C^{(1)} - \phi^{(1)}) \rangle,$$

$$\int f_1^{(1)} d\phi^{(1)} = \langle \delta(u^{(1)} - v^{(1)}) \rangle$$

and

$$\int f_2^{(1,2)} d\phi^{(2)} = \langle \delta(u^{(1)} - v^{(1)}) \delta(C^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \rangle,$$

$$\int f_2^{(1,2)} dv^{(2)} = \langle \delta(u^{(1)} - v^{(1)}) \delta(C^{(1)} - \phi^{(1)}) \delta(C^{(2)} - \phi^{(2)}) \rangle.$$

6.3.2. SEPARATION PROPERTY

If two points are far apart from each other than the pair of variables at these points should be statistically independent of each other i.e.,

$$\lim_{\left|x^{(2)} - x^{(1)}\right| \to \infty} f_2^{(1,2)} \doteq f_1^{(1)} f_1^{(2)}$$

and similarly

$$\lim_{\substack{\left|x^{(2)} - x^{(1)}\right| \\ \left|x^{(3)} - x^{(2)}\right|} \to \infty} f_3^{(1,2,3)} = f_2^{(1,2)} f_1^{(3)}, \text{ and so on.}$$

6.3.3. COINCIDENCE PROPERTY

When two points coincide in a flow field, the components at these points should be obviously the same, that is $f_2^{(1,2)}$ must be zero unless $v^{(2)} = v^{(1)}$ and $\phi^{(2)} = \phi^{(1)}$. But $f_2^{(1,2)}$ must also have the property

$$\iint f_2^{(1,2)} dv^{(2)} d\phi^{(2)} = f_1^{(1)}$$

and, hence, it follows that

$$\lim_{x^{(2)} \to x^{(1)}} f_2^{(1,2)} = f_1^{(1)} \delta(v^{(2)} - v^{(1)}) \delta(\phi^{(2)} - \phi^{(1)})$$

and similarly,

$$\lim_{x^{(3)} \to x^{(1)}} f_3^{(1,2,3)} = f_2^{(1,2)} \delta(v^{(3)} - v^{(1)}) \delta(\phi^{(3)} - \phi^{(1)}) \,, \quad \text{etc.}$$

6.4. CONTONUITY EQUATIONS EXPRESSED IN TERMS OF DISTRIBUTION FUNCTION

An infinite number of continuity equations can be derived which will be satisfied for the initial values of distribution functions. These can be derived directly from $div\, u = 0$. Taking the ensemble average of this equation gives

$$0 = \langle \frac{\partial}{\partial x^{(1)}} u^{(1)} \rangle = \langle \frac{\partial}{x^{(1)}} u^{(1)} \iint f_1^{(1)} dv^{(1)} d\phi^{(1)} \rangle = \frac{\partial}{\partial x^{(1)}} \iint v^{(1)} f_1^{(1)} dv^{(1)} d\phi^{(1)} \,. \qquad (6.4.1)$$

Similarly, multiplying the continuity equation by $\delta(u^{(2)} - v^{(1)}) \delta(C^{(2)} - \phi^{(1)})$ and taking ensemble average, one obtains

$$0 = \langle \delta(u^{(2)} - v^{(2)}) \delta(C^{(2)} - \phi^{2)}) \frac{\partial}{\partial x^{(1)}} u^{(1)} \rangle$$

$$= \frac{\partial}{\partial x^{(1)}} \langle \delta(u^{(2)} - v^{(2)}) \delta(C^{(2)} - \phi^{2)}) u^{(1)} \rangle$$

$$= \frac{\partial}{\partial x^{(1)}} \iint \langle u^{(1)} \delta(u^{(1)} - v^{(1)}) \delta(C^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(C^{(2)} - \phi^{2)}) \rangle$$

$$= \frac{\partial}{\partial x^{(1)}} \iint v^{(1)} f_2^{(1,2)} dv^{(1)} d\phi^{1)} \quad . \tag{6.4.2}$$

The N-th order continuity equation is given in similar way

$$\frac{\partial}{\partial x^{(\alpha)}} \iint v^{(\alpha)} f_N^{(1,2,......N)} dv^{(\alpha)} d\phi^{(\alpha)} = 0 \quad . \tag{6.4.3}$$

The continuity equations are symmetric in their arguments, i.e.,

$$\frac{\partial}{\partial x^{(\alpha)}} \iint v^{(\alpha)} f_N^{(1,2,......N)} dv^{(\alpha)} d\phi^{(\alpha)}$$

$$= \frac{\partial}{\partial x^{(\beta)}} \iint v^{(\beta)} f_N^{(1,2,......N)} dv^{(\beta)} d\phi^{(\beta)} \quad . \tag{6.4.4}$$

Since the divergence property is an important property and it is easily verified by the use of the property of distribution function as

$$\frac{\partial}{\partial x^{(1)}} \iint v^{(1)} f_2^{(1,2)} dv^{(1)} d\phi^{(1)} = \frac{\partial}{\partial x^{(1)}} \langle u^{(1)} \rangle = \langle \frac{\partial}{\partial x^{(1)}} u^{(1)} \rangle = 0 \tag{6.4.5}$$

and all the properties of the distribution function obtained in sec. (3) can also be easily verified.

6.5. EQUATION FOR EVOLUTION OF BIVARIATE DISTRIBUTION FUNCTION

The equation for bivariate distribution function is obtained from the definition of the constructed distribution function and equations, (6.2.1), (6.2.2) and (6.2.3). If we differentiate equation (6.2.1) partially with respect to time, we get

$$\frac{\partial f_1^{(2)}}{\partial t} = \frac{\partial}{\partial t} \langle \delta(u^{(1)} - v^{(1)}) \delta((^{(1)} - \phi^{(1)})) \rangle$$

$$= \langle \delta(C^{(1)} - \phi^{(1)}) \frac{\partial}{\partial t} \{\delta(u^{(1)} - v^{(1)})\}\rangle + \langle \delta(u^{(1)} - v^{(1)}) \frac{\partial}{\partial t} \{\delta(C^{(1)} - \phi^{(1)})\}\rangle$$

$$= \langle -\delta(C^{(1)} - \phi^{(1)}) \frac{\partial u^{(1)}}{\partial t} \frac{\partial}{\partial v^{(1)}} \delta(u^{(1)} - v^{(1)})\rangle$$

$$+ \langle -\delta(u^{(1)} - v^{(1)}) \frac{\partial C^{(1)}}{\partial t} \frac{\partial}{\partial \phi^{(1)}} \delta(C^{(1)} - \phi^{(1)})\rangle \; . \qquad (6.5.1)$$

Now substituting the values of $\frac{\partial u^{(1)}}{\partial t}$ and $\frac{\partial C^{(1)}}{\partial t}$ from equation (6.2.1) and (6.2.2) in equation (6.4̸5.1), we get

$$\frac{\partial f_1^{(1)}}{\partial t} + \langle -\delta(C^{(1)} - \phi^{(1)}) u^{(1)} \frac{\partial u^{(1)}}{\partial x^{(1)}} \frac{\partial}{\partial v^{(1)}} \delta(u^{(1)} - v^{(1)})\rangle$$

$$+ \langle -\delta(u^{(1)} - v^{(1)}) u^{(1)} \frac{\partial C^{(1)}}{\partial x^{(1)}} \frac{\partial}{\partial \phi^{(1)}} \delta(C^{(1)} - \phi^{(1)})\rangle$$

$$+ \langle \delta(C^{(1)} - \phi^{(1)}) [-\frac{\partial}{\partial x^{(1)}} \{\frac{1}{4\pi} \int \frac{\frac{\partial}{\partial x^{(2)}} \frac{\partial}{\partial x^{(2)}} u^{(2)} u^{(2)}}{\left| x^{(1)} - x^{(2)} \right|} dx^{(2)}\} \frac{\partial}{\partial v^{(1)}} \delta(u^{(1)} - v^{(1)})]\rangle$$

$$+ \langle \delta(C^{(1)} - \phi^{(1)}) \nu (\frac{\partial}{\partial x^{(1)}} \frac{\partial}{\partial x^{(1)}} u^{(1)}) \frac{\partial}{\partial v^{(1)}} \delta(u^{(1)} - v^{(1)})\rangle$$

$$+ \langle \delta(u^{(1)} - v^{(1)}) D (\frac{\partial}{\partial x^{(1)}} \frac{\partial}{\partial x^{(1)}} C^{(1)}) \frac{\partial}{\partial \phi^{(1)}} \delta(C^{(1)} - \phi^{(1)})\rangle$$

$$+ \langle -\delta(u^{(1)} - v^{(1)}) R \, C^{(1)} \frac{\partial}{\partial \phi^{(1)}} \delta(C^{(1)} - \phi^{(1)})\rangle = 0 \; . \qquad (6.5.2)$$

Various terms in equation (6.5.2) can be reduced one at a time. for example, the 2nd term on the left hand side of the equation is simplified as

$$\langle -\delta(C^{(1)}-\phi^{(1)})u^{(1)}\frac{\partial u^{(1)}}{\partial x^{(1)}}\frac{\partial}{\partial v^{(1)}}\delta(u^{(1)}-v^{(1)})\rangle$$

$$=\langle -\delta(C^{(1)}-\phi^{(1)})u^{(1)}\frac{\partial}{\partial x^{(1)}}\delta(u^{(1)}-v^{(1)})\rangle \ . \qquad (6.5.3)$$

Similarly 3rd term can be simplified as

$$\langle -\delta(u^{(1)}-v^{(1)})u^{(1)}\frac{\partial C^{(1)}}{\partial x^{(1)}}\frac{\partial}{\partial \phi^{(1)}}\delta(C^{(1)}-\phi^{(1)})\rangle$$

$$=\langle \delta(u^{(1)}-v^{(1)})u^{(1)}\frac{\partial}{\partial x^{(1)}}\delta(C^{(1)}-\phi^{(1)})\rangle \qquad (6.5.4)$$

Now adding equation (6.4.3) and (6.4.4), we get

$$\langle -\delta(C^{(1)}-\phi^{(1)})u^{(1)}\frac{\partial}{\partial x^{(1)}}\delta(u^{(1)}-v^{(1)})\rangle+\langle \delta(u^{(1)}-v^{(1)})u^{(1)}\frac{\partial}{\partial x^{(1)}}\delta(C^{(1)}-\phi^{(1)})\rangle$$

$$=\langle u^{(1)}\rangle\langle \frac{\partial}{\partial x^{(1)}}\delta(u^{(1)}-v^{(1)})\delta(C^{(1)}-\phi^{(1)})\rangle \ . \qquad (6.5.5)$$

The 4th term in equation (6.5.2) can be simplified in such a way

$$\langle \delta(C^{(1)}-\phi^{(1)})[-\frac{\partial}{\partial x^{(1)}}\{\frac{1}{4\pi}\frac{\int \frac{\partial}{\partial x^{(2)}}\frac{\partial}{\partial x^{(2)}}u^{(2)}u^{(2)}}{\left|x^{(1)}-x^{(2)}\right|}dx^{(2)}\}\frac{\partial}{\partial v^{(1)}}\delta(u^{(1)}-v^{(1)})]\rangle$$

$$=\frac{\partial}{\partial v^{(1)}}[-\frac{1}{4\pi}\iiint\frac{\partial}{\partial x^{(1)}}\frac{1}{\left|x^{(1)}-x^{(2)}\right|}(v^{(2)}\frac{\partial}{\partial x^{(2)}})^2\delta(u^{(1)}-v^{(1)})$$

$$\times\delta(C^{(1)}-\phi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(C^{(2)}-\phi^{2)})\rangle dx^{(2)}dv^{(2)}d\phi^{(2)}]$$

$$=\frac{\partial}{\partial v^{(1)}}[-\frac{1}{4\pi}\iiint\frac{\partial}{\partial x^{(1)}}\frac{1}{\left|x^{(1)}-x^{(2)}\right|}(v^{(2)}\frac{\partial}{\partial x^{(2)}})^2 f_2^{(1,2)}dx^{(2)}dv^{(2)}d\phi^{(2)}], \qquad (6.5.6)$$

The viscous term can be simplified as

$$\langle \delta(C^{(1)} - \phi^{(1)}) \nu (\frac{\partial}{\partial x^{(1)}} \frac{\partial}{\partial x^{(1)}} u^{(1)}) \frac{\partial}{\partial v^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle$$

$$= \frac{\partial}{\partial v^{(1)}} \lim_{x^{(2)} \to x^{(1)}} \nu \frac{\partial}{\partial x^{(2)}} \frac{\partial}{\partial x^{(2)}} \langle u^{(2)} \delta(u^{(1)} - v^{(1)}) \delta(C^{(1)} - \phi^{(1)}) \rangle$$

$$= \frac{\partial}{\partial v^{(1)}} \lim_{x^{(2)} \to x^{(1)}} \nu \frac{\partial}{\partial x^{(2)}} \frac{\partial}{\partial x^{(2)}} \iint v^{(2)} f_2^{(1,2)} dv^{(2)} d\phi^{(2)} . \qquad (6.5.7)$$

The diffusive term can be simplified similarly, i.e.,

$$\langle \delta(u^{(1)} - v^{(1)}) D (\frac{\partial}{\partial x^{(1)}} \frac{\partial}{\partial x^{(1)}} C^{(1)}) \frac{\partial}{\partial \phi^{(1)}} \delta(C^{(1)} - \phi^{(1)}) \rangle$$

$$= \frac{\partial}{\partial \phi^{(1)}} \lim_{x^{(2)} - x^{(1)}} D \frac{\partial}{\partial x^{(2)}} \frac{\partial}{\partial x^{(2)}} \iint \phi^{(2)} f_2^{(1,2)} dv^{(2)} d\phi^{(2)} . \qquad (6.5.8)$$

And the reaction term can be simplified as

$$\langle -\delta(u^{(1)} - v^{(1)}) R\, C^{(1)} \frac{\partial}{\partial \phi^{(1)}} \delta(C^{(1)} - \phi^{(1)}) \rangle$$

$$= R \frac{\partial}{\partial \phi^{(1)}} \langle -\delta(u^{(1)} - v^{(1)}) C^{(1)} \delta(C^{(1)} - \phi^{(1)}) \rangle = -R\phi^{(1)} \frac{\partial}{\partial \phi^{(1)}} f_1^{(1)} \qquad (6.5.9)$$

Now summing up the whole process, the equation for the one point distribution function $f_1^{(1)}$ is obtained as

$$\frac{\partial f_1^{(1)}}{\partial t} + v^{(1)} \frac{\partial f_1^{(1)}}{\partial x^{(1)}} + \frac{\partial}{\partial v^{(1)}} [-\frac{1}{4\pi} \iiint \frac{\partial}{\partial x^{(1)}} \frac{1}{|x^{(1)} - x^{(2)}|} (v^{(2)} \frac{\partial}{\partial x^{(2)}})^2$$

$$\times f_2^{(1,2)} dx^{(2)} dv^{(2)} d\phi^{(2)} + \lim_{x^{(2)} \to x^{(1)}} \nu \frac{\partial}{\partial x^{(2)}} \frac{\partial}{\partial x^{(2)}} \iint v^{(2)} f_2^{(1,2)} dv^{(2)} d\phi^{(2)}]$$

$$+ \frac{\partial}{\partial \phi^{(1)}} \lim_{x^{(2)} - x^{(1)}} D \frac{\partial}{\partial x^{(2)}} \frac{\partial}{\partial x^{(2)}} \iint C^{(2)} f_2^{(1,2)} dv^{(2)} d\phi^{(2)}$$

$$- R\phi^{(1)} \frac{\partial}{\partial \phi^{(1)}} f_1^{(1)} \quad . \tag{6.5.10}$$

Similarly, an equation for two points bivariate distribution function $f_2^{(1,2)}$ can be derived by differentiating equation (6.3.2), and by use of equations, (6.2.1) and (6.2.2) and simplifying in the same manner written as

$$\frac{\partial f_2^{(1,2)}}{\partial t} + (v^{(1)} \frac{\partial}{\partial x^{(1)}} + v^{(2)} \frac{\partial}{\partial x^{(2)}}) f_2^{(1,2)} + \frac{\partial}{\partial v^{(1)}} \{ - \frac{1}{4\pi} \iiint \frac{\partial}{\partial x^{(1)}} \frac{1}{\left| x^{(1)} - x^{(2)} \right|}$$

$$\times (v^{(3)} \frac{\partial}{\partial x^{(3)}})^2 f_3^{(1,2,3)} dx^{(3)} dv^{(3)} d\phi^{(3)} + \lim_{x^{(3)} \to x^{(1)}} \nu \frac{\partial}{\partial x^{(3)}} \frac{\partial}{\partial x^{(3)}}$$

$$\times \iint v^{(3)} f_3^{(1,2,3)} dv^{(3)} d\phi^{(3)} \} + \frac{\partial}{\partial \phi^{(1)}} \lim_{x^{(3)} \to x^{(1)}} D \frac{\partial}{\partial x^{(3)}} \frac{\partial}{\partial x^{(3)}} \iint \phi^{(3)} f_3^{(1,2,3)} dv^{(3)} d\phi^{(3)}$$

$$+ \frac{\partial}{\partial v^{(2)}} [- \frac{1}{4\pi} \iiint \frac{\partial}{\partial x^{(2)}} \frac{1}{\left| x^{(2)} - x^{(3)} \right|} (v^{(3)} \frac{\partial}{\partial x^{(3)}})^2 f_3^{(1,2,3)} dx^{(3)} dv^{(3)} d\phi^{3)}]$$

$$+ \lim_{x^{(3)} \to x^{(2)}} \nu \frac{\partial}{\partial x^{(3)}} \frac{\partial}{\partial x^{(3)}} \iint v^{(3)} f_3^{(1,2,3)} dv^{(3)} d\phi^{(3)}] + \frac{\partial}{\partial \phi^{(2)}} \lim_{x^{(3)} \to x^{(2)}}$$

$$\times D \frac{\partial}{\partial x^{(3)}} \frac{\partial}{\partial x^{(3)}} \iint \phi^{(3)} f_3^{(1,2,3)} dv^{(3)} d\phi^{3)} - R(\phi^{(1)} \frac{\partial}{\partial \phi^{(1)}} + \phi^{(2)} \frac{\partial}{\partial \phi^{(2)}}) f_2^{(1,2)} = 0 \quad .$$

(6.5.11)

This process can be continued to obtain equations for $f_3^{(1,2,3)}$, $f_4^{(1,2,3,4)}$ and so on. Logically it is possible to have an equation for every f_N (N is an integral value), but the system of equation so obtain are not closed. It seems that certain approximation will be required for the closure of the system of equations thus obtained.

6.6. DISCUSSION AND CONCLUTION

Firstly, we can show an analogy the equations derived above with the equation BBGKY hierarchy in the kinetic theory of gases. The first equation of BBGKY hierarchy is given as [109]

$$\frac{\partial}{\partial t} f_1^{(1)} + \frac{1}{m} v_\alpha^{(1)} \frac{\partial f_1^{(1)}}{\partial x_\alpha^{(1)}} = n \iint \frac{\partial \psi_{1,2}}{\partial x_\alpha^{(1)}} \frac{\partial f_2^{(1,2)}}{\partial v_\alpha^{(1)}} dx^{(2)} dv^{(2)} \tag{6.6.1}$$

where $\psi_{1,2} = \psi(\left|v_\alpha^{(2)} - v_\alpha^{(1)}\right|)$ is intermolecular potential energy. If we drop the viscous, diffusive and constant reaction rate terms from our one point hierarchy, it strongly resembles with the above BBGKY hierarchy.

CHAPTER–VI

PART-B

DISTRIBUTION FUNCTIONS IN THE STATISTICAL THEORY OF MHD TURBULENCE FOR VELOCITY AND CONCENTRATION UNDERGOING A FIRST ORDER REACTION

6.1 INTRODUCTION

Kishore [39] studied the distribution functions in the statistical theory of MHD turbulence. He has made an attempt for defining a hierarchy of distribution functions for the simultaneous velocity and magnetic fields. Dixit and Upadhyay [25] studied the same problem of Kishore [39] in the presence of coriolis force. In the next, Sarker and Kishore [83] studied the distribution functions in the statistical theory of convective MHD turbulence. Beside these, there are also other theories already discussed in part-A of this chapter.

In the following, an attempt is made for defining the distribution functions for the simultaneous velocity, magnetic and concentration fields in MHD turbulence and derives the equations for evolution of distribution functions. These equations are similar to the BBGKY equations in structure.

6.2. FUNDAMENTAL EQUATIONS

The equation of motion for viscous, incompressible MHD turbulent flow [13] and field equation of concentration undergoing a first order chemical reaction [52] are given by

$$\frac{\partial u_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(u_\alpha u_\beta - h_\alpha h_\beta\right)=-\frac{\partial W}{\partial x_\alpha}+\nu\nabla^2 u_\alpha \tag{6.2.1}$$

$$\frac{\partial h_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(h_\alpha u_\beta - u_\alpha h_\beta\right)=\lambda\nabla^2 h_\alpha \tag{6.2.2}$$

$$\frac{\partial C}{\partial t}+u_\beta\frac{\partial C}{\partial x_\beta}=D\nabla^2 C - RC \tag{6.2.3}$$

and

$$\frac{\partial u_\alpha}{\partial x_\alpha}=\frac{\partial h_\alpha}{\partial x_\alpha}=0 \tag{6.2.4}$$

where u_α, α- component of turbulent velocity; h_α, α- component of magnetic field; C, concentration field; $W = p_t + p_h$, stands for the generalized pressure; p_t, hydrodynamic pressure divided by fluid density ρ; $p_h = 1/2\langle h^2\rangle$, MHD pressure; ν, kinematic viscosity; λ, magnetic diffusivty $=(4\pi\mu\sigma)^{-1}$; σ, electrical conductivity; μ, magnetic permeability; D, diffusive coefficient of contaminant; R constant reaction rate.

The total pressure W which occurs in equation (6.2.1) may be eliminated with the help of the equation obtained by taking the divergence of equation (6.2.1) –viz.,

$$\nabla^2 W = -\frac{\partial^2}{\partial x_\alpha \partial x_\beta}\left(u_\alpha u_\beta - h_\alpha h_\beta\right) = -\left[\frac{\partial u_\alpha}{\partial x_\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \frac{\partial h_\alpha}{\partial x_\beta}\frac{\partial h_\beta}{\partial x_\alpha}\right] . \tag{6.2.5}$$

In a conducting infinite fluid only the particular solution of the resulting equation (6.2.5) is relevant, and so we have

$$W = \frac{1}{4\pi}\int\left[\frac{\partial u'_\alpha}{\partial x'_\beta}\frac{\partial u'_\beta}{\partial x'_\alpha} - \frac{\partial h'_\alpha}{\partial x'_\beta}\frac{\partial h'_\beta}{\partial x'_\alpha}\right]\frac{dx'}{|x' - x|} \tag{6.2.6}$$

and hence, equation (6.2.1) becomes

$$\frac{\partial u_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta}\left(u_\alpha u_\beta - h_\alpha h_\beta\right) = -\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha}\int\left[\frac{\partial u'_\alpha \partial u'_\beta}{\partial x'_\beta \partial x'_\alpha} - \frac{\partial h'_\alpha \partial h'_\beta}{\partial x'_\beta \partial x'_\alpha}\right]\frac{dx'}{|x' - x|}$$

$$+ \nu\nabla^2 u_\alpha . \tag{6.2.7}$$

6.3. FORMULATION OF THE PROBLEM

Here we consider a large ensemble of identical incompressible reacting fluid in turbulent state. We also consider the turbulence and the concentration fields are homogeneous, the chemical reaction and the local mass transfer have no effect on the velocity field and the reaction rate and the diffusivity are constant. The fluid velocity $\hat{u}$, Alfven velocity $\hat{h}$, and concentration field C are randomly distributed functions of position and time and satisfy the equations of motion and continuity given by equations, (6.2.1)-(6.2.4). The only difference between members of the ensemble are the initial conditions that vary from member to member and our aim is to find a way by which we can determine the ensemble average at the initial time. In this regard, our present aim is to construct the distribution functions, study its properties and derive the equation for the evolution of these distribution functions.

6.4. DISTRIBUTION FUNCTIONS AND THEIR PROPERTIES

We may consider the fluid velocity $\hat{u}$, Alfven velocity $\hat{h}$ and concentration fluctuation C at each point of the flow field in MHD turbulent flow. Corresponding to each point of the flow field, we have three measurable characters: v, g and ϕ and denote the pairs of these three variables at the points $x^{(1)}, x^{(2)}, \ldots, x^{(n)}$ as $(v^{(1)}, g^{(1)}, \phi^{(1)}), (v^{(2)}, g^{(2)}, \phi^{(2)}), \ldots\ldots, (v^{(n)}, g^{(n)}, \phi^{(n)})$ at the fixed instant of time. It is possible that the same pairs may occur more than once, therefore we simplify the problem by making use of the assumption that the distribution is discrete (in the sense that no pairs occur more then once). Symbolically we can express the distribution as

$$(v^{(1)}, g^{(1)}, \phi^{(1)}), (v^{(2)}, g^{(2)}, \phi^{(2)}), \ldots\ldots\ldots, (v^{(n)}, g^{(n)}, \phi^{(n)}).$$

The distribution functions of the fluid velocity, the Alfven velocity and concentration field can be defined in terms of Dirac delta-functions.

The one point distribution function $F_1^{(1)}(v^{(1)}, g^{(1)}, \phi^{(1)})$ is defined in such a way that $F_1^{(1)}(v^{(1)}, g^{(1)}, \varphi^{(1)})dv^{(1)}dg^{(1)}d\phi^{(1)}$ is the probability that the fluid velocity, Alfven velocity and concentration field at a time t are in the element $dv^{(1)}$ about $v^{(1)}$, $dg^{(1)}$ about $g^{(1)}$, $d\phi^{(1)}$ about $\phi^{(1)}$, respectively; and is given as

$$F_1^{(1)}(v^{(1)}, g^{(1)}, \phi^{(1)}) = \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(C^{(1)} - \phi^{(1)}) \rangle \tag{6.4.1}$$

and two point distribution function is given by

$$F_1^{(1)}(v^{(1)}, g^{(1)}, \phi^{(1)}) = \langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(C^{(1)} - \phi^{(1)})$$

$$\times \delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(C^{(2)} - \phi^{(2)}) \rangle \tag{6.4.2}$$

Similarly, we can an infinite number of multi point distribution functions $F_3^{(1,2,3)}$, $F_4^{(1,2,3,4)}$, etc.

The distribution functions so constructed have the following properties:

6.4.1. REDUCTION PROPERTY

Integrating with respect to pairs of variables at one point lowers the order of distribution function by one, for example,

$$\iiint F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} = 1$$

$$\iiint F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} = F_1^{(1)} \qquad \text{etc.}$$

Also integration with respect to any one of the variables reduces the number of delta-functions in the distribution function by one as

$$\int F_1^{(1)} dv^{(1)} = \langle \delta(h^{(1)} - g^{(1)}) \delta(C^{(1)} - \phi^{(1)}) \rangle ,$$

$$\int F_1^{(1)} dg^{(1)} = \langle \delta(u^{(1)} - v^{(1)}) \delta(C^{(1)} - \phi^{(1)}) \rangle ,$$

$$\int F_1^{(1)} d\phi^{(1)} = \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \rangle$$

and

$$\int F_1^{(1,2)} dv^{(2)} = \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(C^{(1)} - \phi^{(1)}) \rangle$$

$$\times\, \delta(h^{(2)} - g^{(2)}) \delta(C^{(2)} - \phi^{(2)}) , \qquad \text{etc.}$$

6.4.2. SEPARATION PROPERTY

If two points are far apart from each other in the flow field, the pairs of variables at these points are statistically independent of each other – i.e.,

$$\lim_{\left|x^{(2)}\to x^{(1)}\right|\to\infty} F_2^{(1,2)} = F_1^{(1)} F_1^{(2)}$$

and similarly,

$$\lim_{\substack{\left|x^{(3)}\to x^{(1)}\right| \\ \left|x^{(3)}\to x^{(2)}\right|}\to\infty} F_3^{(1,2,3)} = F_1^{(1,2)} F_1^{(3)}, \qquad \text{etc.}$$

6.4.3. COINCIDENCE PROPERTY

When two points coincide in a flow field, the components at these points should be obviously the same, that is $F_2^{(1,2)}$ must be zero unless $v^{(2)} = v^{(1)}$, $g^{(2)} = g^{(1)}$, and $\phi^{(2)} = \phi^{(1)}$, but $F_2^{(1,2)}$ must also have the property

$$\iiint F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} = F_1^{(1)}$$

and, hence, it follows that

$$\lim_{x^{(2)}\to x^{(1)}} F_2^{(1,2)} = F_1^{(1)} \delta(v^{(2)} - v^{(1)}) \delta(g^{(2)} - g^{(1)}) \delta(\phi^{(2)} - \phi^{(1)}).$$

Similarly,

$$\lim_{x^{(3)}\to x^{(1)}} F_3^{(1,2,3)} = F_2^{(1,2)} \delta(v^{(3)} - v^{(1)}) \delta(g^{(3)} - g^{(1)}) \delta(\phi^{3)} - \phi^{(1)}).$$

6.5. CONTINUITY EQUATIONS EXPRESSED IN TERMS OF THE DISTRIBUTION FUNCTION

An infinite number of continuity equations can be derived for the convective MHD turbulence, which will be satisfied for the initial values of distribution functions and are obtained directly by $div\ \hat{u} = 0$. Taking ensemble average of equation (6.2.4), we have

$$0 = \left\langle \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \right\rangle = \langle \frac{\partial}{\partial x_\alpha^{(1)}} u_\alpha^{(1)} \iiint F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} \rangle$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \iiint \langle u_\alpha^{(1)} \rangle \langle F_1^{(1)} \rangle dv^{(1)} dg^{(1)} d\phi^{(1)}$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \iiint v_\alpha^{(1)} F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)}$$

$$= \iiint \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} v_\alpha^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} \qquad (6.5.1)$$

and, similarly,

$$0 = \iiint \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} g_\alpha^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} \qquad (6.5.2)$$

which are the first-order continuity equation in which only one point distribution function is involved. In a similar way, second-order continuity equations can be derived and are formed to be

$$\frac{\partial}{\partial x_\alpha^{(1)}} \iiint v_\alpha^{(1)} F_2^{(1,2)} dv^{(1)} dg^{(1)} d\phi^{(1)} = 0 \qquad (6.5.3)$$

and

$$\frac{\partial}{\partial x_\alpha^{(1)}} \iiint g_\alpha^{(1)} F_2^{(1,2)} dv^{(1)} dg^{(1)} d\phi^{(1)} = 0. \qquad (6.5.4)$$

The Nth order continuity equations are

$$\frac{\partial}{\partial x_\alpha^{(1)}} \iiint v_\alpha^{(1)} F_N^{(1,2,\ldots\ldots N)} dv^{(1)} dg^{(1)} d\phi^{(1)} = 0 \qquad (6.5.5)$$

and

$$\frac{\partial}{\partial x_\alpha^{(1)}} \iiint g_\alpha^{(1)} F_N^{(1,2,\ldots\ldots N)} dv^{(1)} dg^{(1)} d\phi^{(1)} = 0. \qquad (6.5.6)$$

The continuity equations are symmetric in their arguments --i.e.,

$$\frac{\partial}{\partial x_{\alpha}^{(1)}}\iiint g_{\alpha}^{(r)} F_N^{(1,2,\ldots,r,\ldots N)} dv^{(r)} dg^{(r)} d\phi^{(r)} =$$
$$= \frac{\partial}{\partial x_{\alpha}^{(s)}}\iiint g_{\alpha}^{(s)} F_N^{(1,2,\ldots,r,s,N)} dv^{(s)} dg^{(s)} d\phi^{(s)}. \tag{6.5.7}$$

Since the divergence property is an important property and it is easily verified by the use of the property of distribution as

$$\frac{\partial}{\partial x_{\alpha}^{(1)}}\iiint v_{\alpha}^{(1)} F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} = \frac{\partial}{\partial x_{\alpha}^{(1)}}\langle u_{\alpha}^{(1)}\rangle = \langle \frac{\partial u_{\alpha}^{(1)}}{\partial x_{\alpha}^{(1)}}\rangle = 0 \tag{6.5.8}$$

and all the properties of the distribution function obtained in section (6.4) can also be easily verified.

6.6 EQUATION FOR EVOLUTION OF DISTRIBUTION FUNCTION

The equations for distribution function are obtained from the definition of the constructed distribution functions and use of the equations, (6.2.2), (6.2.3) and (6.2.7). If we differentiate equation (6.4.1) partially with respect to time, we get

$$\frac{\partial F_1^{(1)}}{\partial t} = \frac{\partial}{\partial t}\langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(C^{(1)} - \phi^{(1)})\rangle$$

or

$$\frac{\partial F_1^{(1)}}{\partial t} = \langle \delta(h^{(1)} - g^{(1)})\delta(C^{(1)} - \phi^{(1)})\frac{\partial}{\partial t}\delta(u^{(1)} - v^{(1)})\rangle$$

$$+\langle \delta(u^{(1)} - v^{(1)})\delta(C^{(1)} - \phi^{(1)})\frac{\partial}{\partial t}\delta(h^{(1)} - g^{(1)})\rangle$$

$$+\langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\frac{\partial}{\partial t}\delta(C^{(1)} - \phi^{(1)})\rangle$$

$$= \langle -\delta(h^{(1)} - g^{(1)})\delta(C^{(1)} - \phi^{(1)})\frac{\partial u^{(1)}}{\partial t}\frac{\partial}{\partial v^{(1)}}\delta(u^{(1)} - v^{(1)})\rangle$$

$$+ \langle -\delta(u^{(1)} - v^{(1)})\delta(C^{(1)} - \phi^{(1)})\frac{\partial h^{(1)}}{\partial t}\frac{\partial}{\partial g^{(1)}}\delta(h^{(1)} - g^{(1)})\rangle$$

$$+ \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\frac{\partial C^{(1)}}{\partial t}\frac{\partial}{\partial \phi^{(1)}}\delta(C^{(1)} - \phi^{(1)})\rangle \quad . \qquad (6.6.1)$$

If we use equations, (6.2.2), (6.2.3) and (6.2.7) in equation (6.6.1), we have

$$\frac{\partial F_1^{(1)}}{\partial t} = \langle -\delta(h^{(1)} - g^{(1)})\delta(C^{(1)} - \phi^{(1)})\{-\frac{\partial u_\alpha^{(1)} u_\beta^{(1)}}{\partial x_\beta^{(1)}} + \frac{\partial h_\alpha^{(1)} h_\beta^{(1)}}{\partial x_\beta^{(1)}} - \frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}$$

$$\times \int \left[\frac{\partial u_\alpha^{(1)} \partial u_\beta^{(1)}}{\partial x_\beta^{(1)} \partial x_\alpha^{(1)}} - \frac{\partial h_\alpha^{(1)} \partial h_\beta^{(1)}}{\partial x_\beta^{(1)} \partial x_\alpha^{(1)}}\right]\frac{dx'}{|x' - x|} + \nu\nabla^2 u_\alpha^{(1)}\}\frac{\partial}{\partial v_\alpha^{(1)}}\delta(u^{(1)} - v^{(1)})\rangle$$

$$+ \langle -\delta(u^{(1)} - v^{(1)})\delta(C^{(1)} - \phi^{(1)})\{-\frac{\partial h_\alpha^{(1)} u_\beta^{(1)}}{\partial x_\beta^{(1)}} + \frac{\partial u_\alpha^{(1)} h_\beta^{(1)}}{\partial x_\beta^{(1)}} + \lambda\nabla^2 h_\alpha^{(1)}\}\frac{\partial}{\partial g_\alpha^{(1)}}\delta(h^{(1)} - g^{(1)})\rangle$$

$$+ \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\{-u_\beta^{(1)}\frac{\partial C^{(1)}}{\partial x_\beta^{(1)}} + D\nabla^2 C^{(1)}\}\frac{\partial}{\partial \phi^{(1)}}\delta(C^{(1)} - \phi^{(1)})\rangle \; .$$

or

$$\frac{\partial F_1^{(1)}}{\partial t} = \langle \delta(h^{(1)} - g^{(1)})\delta(C^{(1)} - \phi^{(1)})\frac{\partial u_\alpha^{(1)} u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta(u^{(1)} - v^{(1)})\rangle$$

$$+ \langle -\delta(h^{(1)} - g^{(1)})\delta(C^{(1)} - \phi^{(1)})\frac{\partial h_\alpha^{(1)} h_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta(u^{(1)} - v^{(1)})\rangle$$

$$+ \langle \delta(h^{(1)} - g^{(1)})\delta(C^{(1)} - \phi^{(1)})\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}\int \left[\frac{\partial u_\alpha^{(1)} \partial u_\beta^{(1)}}{\partial x_\beta^{(1)} \partial x_\alpha^{(1)}} - \frac{\partial h_\alpha^{(1)} \partial h_\beta^{(1)}}{\partial x_\beta^{(1)} \partial x_\alpha^{(1)}}\right]\frac{dx'}{|x' - x|}$$

$$\times \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle + \langle -\delta(h^{(1)} - g^{(1)}) \delta(C^{(1)} - \phi^{(1)}) \nu \nabla^2 u_\alpha^{(1)} \frac{\partial}{\partial v^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle$$

$$+ \langle \delta(u^{(1)} - v^{(1)}) \delta(C^{(1)} - \phi^{(1)}) \frac{\partial h_\alpha^{(1)} u_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial g_\alpha^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle$$

$$+ \langle -\delta(u^{(1)} - v^{(1)}) \delta(C^{(1)} - \phi^{(1)}) \frac{\partial u_\alpha^{(1)} h_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial g_\alpha^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle$$

$$+ \langle -\delta(u^{(1)} - v^{(1)}) \delta(C^{(1)} - \phi^{(1)}) \lambda \nabla^2 h_\alpha^{(1)} \frac{\partial}{\partial g_\alpha^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle$$

$$+ \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) u_\beta^{(1)} \frac{\partial C^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial \phi^{(1)}} \delta(C^{(1)} - \phi^{(1)}) \rangle$$

$$+ \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) D \nabla^2 C^{(1)} \} \frac{\partial}{\partial \phi^{(1)}} \delta(C^{(1)} - \phi^{(1)}) \rangle$$

$$+ \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) R C^{(1)} \} \frac{\partial}{\partial \phi^{(1)}} \delta(C^{(1)} - \phi^{(1)}) \rangle . \tag{6.6.2}$$

Various terms in equation (6.6.2) can be simplified as that they may be expressed in terms of one point and two point distribution functions. For example, the first term on the right -hand side of the equation is simplified as

$$\langle \delta(h^{(1)} - g^{(1)}) \delta(C^{(1)} - \phi^{(1)}) \frac{\partial u_\alpha^{(1)} u_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle$$

$$= \langle u_\beta^{(1)} \delta(h^{(1)} - g^{(1)}) \delta(C^{(1)} - \phi^{(1)}) \frac{\partial u_\alpha^{(1)}}{\partial v_\alpha^{(1)}} \frac{\partial}{\partial x_\beta^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle$$

(since $\partial u_\alpha^{(1)} / \partial v_\alpha^{(1)} = 1$)

$$= \langle -\delta(h^{(1)} - g^{(1)})\delta(C^{(1)} - \phi^{(1)}) u_\beta^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle . \tag{6.6.3}$$

Similarly, the 5[th] and 8[th] terms on the right-hand side of equation (6.6.2) can reduce as

$$\langle \delta(u^{(1)} - v^{(1)})\delta(C^{(1)} - \phi^{(1)}) \frac{\partial h_\alpha^{(1)} u_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial g_\alpha^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle$$

$$= \langle -\delta(u^{(1)} - v^{(1)})\delta(C^{(1)} - \phi^{(1)}) u_\beta^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \tag{6.6.4}$$

and

$$\langle \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)}) u_\beta^{(1)} \frac{\partial C^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial \phi^{(1)}} \delta(C^{(1)} - \phi^{(1)}) \rangle$$

$$= \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)}) u_\beta^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} \delta(C^{(1)} - \phi^{(1)}) \rangle . \tag{6.6.5}$$

If we add equations, (6.6.3), (6.6.4) and (6.6.5), we get

$$= \langle -\delta(h^{(1)} - g^{(1)})\delta(C^{(1)} - \phi^{(1)}) u_\beta^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle$$

$$+ \langle -\delta(u^{(1)} - v^{(1)})\delta(C^{(1)} - \phi^{(1)}) u_\beta^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle$$

$$+ \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)}) u_\beta^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} \delta(C^{(1)} - \phi^{(1)}) \rangle$$

$$= -\langle u_\beta^{)1)} \rangle \langle \frac{\partial}{\partial x_\beta^{(1)}} \{ \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(C^{(1)} - \phi^{(1)}) \} \rangle = -v_\beta^{(1)} \frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}} . \tag{6.6.6}$$

Similarly, the 2[nd] and 6[th] terms on the right-hand of equation (6.6.2) can be simplified as

$$\langle -\delta(h^{(1)} - g^{(1)})\delta(C^{(1)} - \phi^{(1)}) \frac{\partial h_\alpha^{(1)} h_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)})\rangle$$

$$= -g_\beta^{(1)} \frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}} \frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}} \tag{6.6.7}$$

and

$$= -g_\beta^{(1)} \frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}} \frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}} . \tag{6.6.8}$$

We can reduce the 4th term

$$\langle -\delta(h^{(1)} - g^{(1)})\delta(C^{(1)} - \phi^{(1)}) \nu \nabla^2 u_\alpha^{(1)} \frac{\partial}{\partial v^{(1)}} \delta(u^{(1)} - v^{(1)})\rangle$$

$$= -\nu \frac{\partial}{\partial v_\alpha^{(1)}} \langle \nabla^2 u_\alpha^{(1)} \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(C^{(1)} - \phi^{(1)})\rangle$$

$$= -\nu \frac{\partial}{\partial v_\alpha^{(1)}} \frac{\partial^2}{\partial x_\beta^{(1)} \partial x_\beta^{(1)}} \langle u_\alpha^{(1)} \{\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(C^{(1)} - \phi^{(1)})\}\rangle$$

$$= -\nu \frac{\partial}{\partial v_\alpha^{(1)}} \lim_{x^{(2)} \to x^{(1)}} \frac{\partial^2}{\partial x_\beta^{(2)} \partial x_\beta^{(2)}} \langle u_\alpha^{(2)} \{\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(C^{(1)} - \phi^{(1)})\}\rangle$$

$$= -\nu \frac{\partial}{\partial v_\alpha^{(1)}} \lim_{x^{(2)} \to x^{(1)}} \frac{\partial^2}{\partial x_\beta^{(2)} \partial x_\beta^{(2)}} \langle \iiint u_\alpha^{(2)} \times \delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(C^{(2)} - \phi^{(2)})$$

$$\times \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(C^{(1)} - \phi^{(1)})\rangle dv^{(2)} dg^{(2)} d\phi^{(2)}$$

$$= -\nu \frac{\partial}{\partial v_\alpha^{(1)}} \lim_{x^{(2)} \to x^{(1)}} \frac{\partial^2}{\partial x_\beta^{(2)} \partial x_\beta^{(2)}} \iiint v_\alpha^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} \phi^{(2)} . \tag{6.6.9}$$

Similarly, 7th and 9th terms of equation (6.6.2) reduce to

$$= -\lambda \frac{\partial}{\partial g_\alpha^{(1)}} \lim_{x^{(2)} \to x^{(1)}} \frac{\partial^2}{\partial x_\beta^{(2)} \partial x_\beta^{(2)}} \iiint g_\alpha^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} \phi^{(2)} \tag{6.6.10}$$

$$= -D \frac{\partial}{\partial \phi_\alpha^{(1)}} \lim_{x^{(2)} \to x^{(1)}} \frac{\partial^2}{\partial x_\beta^{(2)} \partial x_\beta^{(2)}} \iiint \phi_\alpha^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} \phi^{(2)}. \tag{6.6.11}$$

Now, the 3[rd] term of the equation (6.6.2) reduces to

$$\langle \delta(h^{(1)} - g^{(1)}) \delta(C^{(1)} - \phi^{(1)}) \frac{1}{4\pi} \frac{\partial}{\partial x_\alpha^{(1)}} \int \left[\frac{\partial u_\alpha^{(1)} \partial u_\beta^{(1)}}{\partial x_\beta^{(1)} \partial x_\alpha^{(1)}} - \frac{\partial h_\alpha^{(1)} \partial h_\beta^{(1)}}{\partial x_\beta^{(1)} \partial x_\alpha^{(1)}} \right]$$

$$\times \frac{dx'}{|x' - x|} \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle$$

$$= \frac{\partial}{\partial v_\alpha^{(1)}} [\frac{1}{4\pi} \iiiint \frac{\partial}{\partial x_\alpha^{(1)}} (\frac{1}{|x^{(2)} - x^{(1)}|})(\frac{\partial v_\alpha^{(2)} \partial v_\beta^{(2)}}{\partial x_\beta^{(2)} \partial x_\alpha^{(2)}} - \frac{\partial g_\alpha^{(2)} \partial g_\beta^{(2)}}{\partial x_\beta^{(2)} \partial x_\alpha^{(2)}})$$

$$\times F_1^{(1,2)} dx^{(2)} dv^{(2)} dg^{(2)} d\phi^{(2)}. \tag{6.6.12}$$

And, the last term of the equation (6.6.2) reduces to

$$+ \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) R C^{(1)} \frac{\partial}{\partial \phi^{(1)}} \delta(C^{(1)} - \phi^{(1)}) \rangle$$

$$= -R\phi^{(1)} \frac{\partial}{\partial \phi^{(1)}} F_1^{(1)}. \tag{6.6.13}$$

Now, summing up the whole process, the equation for the one point distribution function $F_1^{(1)}(v, g, \phi)$ is obtained as

$$\frac{\partial F_1^{(1)}}{\partial t} + v_\beta^{(1)} \frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}} + g_\beta^{(1)} (\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}} + \frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}) \frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}$$

$$-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\iiiint\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|x^{(2)}-x^{(1)}\right|}\right)\left(\frac{\partial v_\alpha^{(2)}\partial v_\beta^{(2)}}{\partial x_\beta^{(2)}\partial x_\alpha^{(2)}}-\frac{\partial g_\alpha^{(2)}\partial g_\beta^{(2)}}{\partial x_\beta^{(2)}\partial x_\alpha^{(2)}}\right)F_1^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}\right.$$

$$+\nu\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{x^{(2)}\to x^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiint v_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}$$

$$+\lambda\frac{\partial}{\partial g_\alpha^{(1)}}\lim_{x^{(2)}\to x^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiint g_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}\phi^{(2)}$$

$$+D\frac{\partial}{\partial \phi_\alpha^{(1)}}\lim_{x^{(2)}\to x^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiint \phi_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}\phi^{(2)}$$

$$-R\phi^{(1)}\frac{\partial}{\partial\phi^{(1)}}F_1^{(1)}=0\quad, \tag{6.6.14}$$

Similarly, an equation for two-point distribution function $F_2^{(1,2)}$ can be derived by differentiating equation (6.4.2) and use of equations, (6.2.2), (6.2.3) and (6.2.7 and simplifying in the same manner written as

$$\frac{\partial F_2^{(1,2)}}{\partial t}+\left(v_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}+v_\beta^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\right)F_2^{(1,2)}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial}{\partial x_\beta^{(1)}}F_2^{(1,2)}$$

$$+g_\beta^{(2)}\left(\frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}}+\frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}}\right)\frac{\partial}{\partial x_\beta^{(2)}}F_2^{(1,2)}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\iiiint\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|x^{(2)}-x^{(1)}\right|}\right)\right.$$

$$\times\left(\frac{\partial v_\alpha^{(3)}\partial v_\beta^{(3)}}{\partial x_\beta^{(3)}\partial x_\alpha^{(3)}}-\frac{\partial g_\alpha^{(3)}\partial g_\beta^{(3)}}{\partial x_\beta^{(3)}\partial x_\alpha^{(3)}}\right)F_3^{(1,2,3)}dx^{(3)}dv^{(3)}dg^{(3)}d\phi^{(3)}\Big]-\frac{\partial}{\partial v_\alpha^{(2)}}\left[\frac{1}{4\pi}\iiiint\frac{\partial}{\partial x_\alpha^{(2)}}\right.$$

$$\times\left(\frac{1}{\left|x^{(3)}-x^{(2)}\right|}\right)\left(\frac{\partial v_\alpha^{(3)}\partial v_\beta^{(3)}}{\partial x_\beta^{(3)}\partial x_\alpha^{(3)}}-\frac{\partial g_\alpha^{(3)}\partial g_\beta^{(3)}}{\partial x_\beta^{(3)}\partial x_\alpha^{(3)}}\right)F_3^{(1,2,3)}dx^{(3)}dv^{(3)}dg^{(3)}d\phi^{(3)}\Big]$$

$$+\nu\left(\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{x^{(3)}\to x^{(1)}}+\frac{\partial}{\partial v_\alpha^{(2)}}\lim_{x^{(3)}\to x^{(1)}}\right)\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}$$

$$\times\iiint v_\alpha^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}+\lambda\left(\frac{\partial}{\partial g_\alpha^{(1)}}\lim_{x^{(3)}\to x^{(1)}}+\frac{\partial}{\partial g_\alpha^{(2)}}\lim_{x^{(3)}\to x^{(1)}}\right)$$

$$\times\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\iiint g_\alpha^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}+\gamma\left(\frac{\partial}{\partial \phi_\alpha^{(1)}}\lim_{x^{(3)}\to x^{(1)}}+\frac{\partial}{\partial \phi_\alpha^{(2)}}\lim_{x^{(3)}\to x^{(1)}}\right)$$

$$\times\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\iiint \phi_\alpha^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}-R\phi^{(1)}\frac{\partial}{\partial \phi^{(1)}}F_1^{(1)}=0. \qquad (6.6.15)$$

Containing this way we can derive the equations of $F_3^{(1,2,3)}, F_4^{(1,2,3,4)}$.......... etc. Logically, it is possible to have an equation for every F_n(n is an integer) but the system of equations so obtained are not closed. It seems that certain approximations will be required for the closure of the system of equations thus obtained.

6.7. DISCUSSION AND CONCLUSION

The first equation of BBGKY hierarchy in the kinetic theory of gases given by Ta-You Wu [102] as

$$\frac{\partial f}{\partial t}+\frac{1}{m}v_\alpha^{(1)}\frac{\partial f_1^{(1)}}{\partial x_\alpha^{(1)}}=n\iint\frac{\partial\psi_{1,2}}{\partial x_\alpha^{(1)}}\frac{\partial f_2^{(1,2)}}{\partial v_\alpha^{(1)}}dx^{(2)}dv^{(2)} \qquad (6.7.1)$$

where $\psi_{1,2}=\psi\left(\left|v_\alpha^{(2)}-v_\alpha^{(1)}\right|\right)$ is intermolecular potential energy. If we drop the viscous, magnetic and contaminant diffusive and constant reaction terms from our one-point hierarchy equation (6.6.14), we have

$$\frac{\partial F_1^{(1)}}{\partial t}+v_\beta^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\times\right.$$

$$\times\iiiint\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|x^{(2)}-x^{(1)}\right|}\right)\left(\frac{\partial v_\alpha^{(2)}\partial v_\beta^{(2)}}{\partial x_\beta^{(2)}\partial x_\alpha^{(2)}}-\frac{\partial g_\alpha^{(2)}\partial g_\beta^{(2)}}{\partial x_\beta^{(2)}\partial x_\alpha^{(2)}}\right)F_1^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}$$

$$=0 \qquad (6.7.2)$$

which strongly resembles with equation (6.7.1) in BBGKY hierarchy. The existence of the term

$$\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}$$

can be explained on the basis that two characteristics of the flow field are related to each other and describe the interaction between two modes(velocity and magnetic) at a single point.

CHAPTER-VII

EFFECT OF VERY STRONG MAGNETIC FIELD ON ACCELERATION COVARIANCE IN MHD TURBULENCE OF DUSTY FLUID IN A ROTATING SYSTEM.

7.1 INTRODUCTION

Taylor [104] pointed out that the equation of motion of turbulence relates the pressure gradient and acceleration of the fluid particles and that the mean-square acceleration can be determined from the observation of the diffusion of the marked fluid particles. The behavior of dust particles in a turbulent flow depends on the concentration and size of the particles with respect to scale of turbulent fluid. A good deal of theoretical studies of MHD turbulent has been made during last fifteen years. Some authors (e.g. Ohji, [71]) have considered MHD turbulence in the absence of an external magnetic field in order to gain a basic understanding of a half adjusting process of the mechanical and magnetic mode of turbulence. The essential effect in presence of an imposed magnetic field is that the mechanical and magnetic mode of turbulence interacts not only with each other through the self-adjusting process but also with external magnetic field. If the external magnetic field is very strong, the effect of the later interaction will predominance that of the self-adjusting process. Ohji [71] presented a first order theory of turbulence of an electrically conducting fluid in the presence of a uniform magnetic field which is so strong that the non-linear mechanism as well as the dissipation terms are of minor important when comparing with the external coupling terms. Ohji [72], discussed the effect of a very strong uniform magnetic field on incompressible MHD turbulence in presence of a constant angular velocity and Hall effect. Kishore and Dixit [44] studied the effect of a uniform magnetic field on acceleration covariance in MHD turbulence. Dixit [23] discussed the effect of uniform magnetic field on acceleration covariance in MHD dusty fluid turbulence.

In this paper, we have discussed the effect of a strong magnetic field on acceleration covariance in MHD turbulence of dusty fluid in a rotating system. Due to rotation, coriolis force is produced which plays an important role in a rotating system of turbulent flow, while the centrifugal force with the potential is incorporated into the pressure.

7.2. FUNDAMENTAL EQUATIONS

If $\hat{U}$ denotes the velocity, $\hat{B}$ the magnetic induction, $\hat{P}$ the pressure, ρ the density, ν the kinetic viscosity, σ the conductivity and μ the permeability, the MHD equation are written in M. K. S. units as M. Ohji [71]

$$\frac{\partial \hat{U}}{\partial t} + (\hat{U}\cdot grad)\hat{U} - \frac{1}{\rho r}(\hat{B}\cdot grad)\hat{B} = -\frac{1}{\rho} grad(\hat{P} + \frac{1}{2r}\hat{B}^2) + \nu\nabla^2\hat{U} \tag{7.2.1}$$

for the momentum, and

$$\frac{\partial \hat{B}}{\partial t} + (\hat{U}\cdot grad)\hat{B} - (\hat{B}\cdot grad)\hat{U} = -\frac{1}{\pi\sigma}\nabla^2\hat{B} \tag{7.2.2}$$

for the induction respectively, together with the supplementary equations

$$\nabla \cdot \hat{U} = 0 \quad \text{and} \quad \nabla \cdot \hat{B} = 0 \tag{7.2.3}$$

where ρ, ν, σ and μ are assumed constants.

Further, for convenient we introduce the Alfven velocity

$$\hat{H} = \frac{\hat{B}}{\sqrt{\mu\rho}} \tag{7.2.4}$$

and the magnetic viscosity

$$\lambda = \frac{1}{\mu\sigma} \tag{7.2.5}$$

For a turbulent flow we can put $\hat{U} = U + u$, $\hat{H} = H + h$, $\hat{P} = P + p$

where U, H and P are the mean values and u, h and p represents the fluctuating components. Then, taking the statistical average (expressed by an overbar) of equations, (7.2.1)-(7.2.3), we have

$$\frac{\partial U_i}{\partial t} + \frac{\partial}{\partial x_k}(U_i U_k - H_i H_k + \overline{u_i u_k} - \overline{h_i h_k}) =$$

$$= -\frac{1}{\rho}\frac{\partial}{\partial x_i}(P + \frac{\rho}{2}(H^2 \bar{h}^2)) + \nu\nabla^2 U_i \tag{7.2.6}$$

$$\frac{\partial H_i}{\partial t} + \frac{\partial}{\partial x_k}(H_i U_k - U_i H_k + \overline{h_i u_k} - \overline{u_i h_k}) = \lambda\nabla^2 H_i \tag{7.2.7}$$

and

$$\frac{\partial U_i}{\partial x_i} = \frac{\partial H_i}{\partial x_i} = 0 \tag{7.2.8}$$

for the mean fields, and subtracting these from equations (7.2.1)-(7.2.3), we get

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) - U_k \frac{\partial u_i}{\partial x_k} - H_k \frac{\partial h_i}{\partial x_k} =$$

$$=-\frac{1}{\rho}\frac{\partial}{\partial x_i}[P+\frac{\rho}{2}(h^2+2H_k h_k)]+\nu\nabla^2 u_i+\{\frac{\partial}{\partial x_k}(\overline{u_i u_k}-\overline{h_i h_k})-$$

$$-u_k\frac{\partial U_i}{\partial x_k}+h_k\frac{\partial H_i}{\partial x_k}+\frac{1}{2}\frac{\partial \overline{h}^2}{\partial x_i}\},\qquad (7.2.9)$$

$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_i}(h_i u_k-u_i h_k)+U_k\frac{\partial h_i}{\partial x_k}-H_k\frac{\partial u_i}{\partial x_k}=$$

$$=\lambda\nabla^2 h_i+\{\frac{\partial}{\partial x_k}(\overline{h_i u_k}-\overline{u_i h_k}-u_k\frac{\partial H_i}{\partial x_k}+h_k\frac{\partial U_i}{\partial x_k}\}\qquad (7.2.10)$$

and

$$\frac{\partial u_i}{\partial x_i}=\frac{\partial h_i}{\partial x_i}=0\qquad (7.2.11)$$

for the fluctuating fields. Specially, if both $\hat{U}$ and $\hat{H}$ are steady, uniform and the turbulence is spatially homogenous, the average equations, (7.2.6) and (7.2.7) are satisfied identically and it is seen that in the equations (7.2.9) and (7.2.10) the terms in the curly brackets vanish. Equations, (7.2.9)-(7.2.10) becomes

$$\frac{\partial u_i}{\partial t}+U_k\frac{\partial u_i}{\partial x_k}-H_k\frac{\partial h_i}{\partial x_k}=-\frac{\partial W}{\partial x_i}+\nu\nabla^2 u_i,\qquad (7.2.12)$$

$$\frac{\partial h_i}{\partial t}+U_k\frac{\partial h_i}{\partial x_k}-H_k\frac{\partial u_i}{\partial x_k}=\lambda\nabla^2 h_i\qquad (7.2.13)$$

where $W=P/\rho+H_k h_k$.

Now, the equations of MHD dusty turbulence with effect of very strong magnetic field in a rotating frame are

$$\frac{\partial u_i}{\partial t} + U_k \frac{\partial u_i}{\partial x_k} - H_k \frac{\partial h_i}{\partial x_k} = -\frac{\partial W}{\partial x_i} + \nu\nabla^2 u_i -$$

$$-2 \in_{mik} \Omega_m u_i + f(u_i - v_i) \tag{7.2.14}$$

$$\frac{\partial h_i}{\partial t} + U_k \frac{\partial h_i}{\partial x_k} - H_k \frac{\partial u_i}{\partial x_k} = \lambda\nabla^2 h_i \tag{7.2.15}$$

and

$$\frac{\partial v_i}{\partial t} + v_k \frac{\partial v_i}{\partial x_k} = -\frac{k}{m_s}(v_i - u_i) \tag{7.2.16}$$

where $W = P/\rho + H_k h_k + \frac{1}{2}|\hat{\Omega} \times \hat{x}|$, $\hat{H}$ is the external applied magnetic field, $\hat{\Omega}$, the angular velocity vector of a uniform rotation; u_i's and v_i's represents the turbulent component of velocity and magnetic field respectively. $m_s = \frac{4}{3}\pi R_s{}^3 \rho_s$ is the mass of a single spherical dust particle of radius R_s; $f = \frac{KN}{\rho}$, has dimension of frequency; K is the stock resistance coefficient; N is the number density of the dust particles ; $v_i(\hat{x},t)$ is the fluctuating velocity of the dust particles. The third term of the right hand side of the equation (7.2.14) represent the coriolies force which plays an important role in a rotating system.

7.3. MATHEMATICAL MODEL OF THE PROBLEM

Let $A_i(x_i,t)$ denotes the ith component of the acceleration of fluid particles, which is instantaneously at point $P'(x_i{}',t)$, then

$$A_i(x_i',t) = \frac{Du_i'}{Dt'} = \frac{\partial u_i'}{\partial t'} + U_k \frac{\partial u_i'}{\partial x_k'} = -\frac{\partial W'}{\partial x_i'} + \nu \nabla_{x'}{}^2 u_i' +$$

$$+ H_k \frac{\partial h_i'}{\partial x_k'} - 2 \in_{mik} \Omega_m u_i' + f(u_i' - v_i') \qquad (7.3.1)$$

Similarly, if $A_j''(x_j'',t)$ denote the jth component of the acceleration of another fluid particle which is instantaneously at point $P''(x_j'',t)$, we can write

$$A_j''(x_j'',t) = -\frac{\partial W''}{\partial x_j''} + \nu \nabla_{x''}{}^2 u_j'' + H_l \frac{\partial h_j''}{\partial x_l''} - 2 \in_{njl} \Omega_n u_j'' + f(u_j'' - v_j'') \qquad (7.3.2)$$

Therefore,

$$A_i' A_j'' = \frac{\partial^2 W' W''}{\partial x_i' \partial x_j''} - \nu \frac{\partial}{\partial x_j''} \nabla_{x'}{}^2 u_i W'' - H_k \frac{\partial^2 h_i' W''}{\partial x_k' x_j''} - \nu \frac{\partial}{\partial x_i'} \nabla_{x''} W' u_j''$$

$$- H_l \frac{\partial^2}{\partial x_i' \partial x_l''} W' h_j'' + \nu^2 \nabla_{x'}{}^2 \nabla_{x''}{}^2 u_i' u_j'' + \nu H_k \nabla_{x''}{}^2 \frac{\partial}{\partial x_k} h_i' u_j'' +$$

$$+ \nu H_l \nabla_{x'}{}^2 \frac{\partial}{\partial x_l''} h_j'' u_i' + H_k H_l \frac{\partial^2 h_i' h_j''}{\partial x_k \partial x_l} + 2 \in_{mik} \Omega_m \frac{\partial W'' u_i'}{\partial x_j} -$$

$$- 2\nu \nabla_{x''}{}^2 \in_{mik} \Omega_m u_i' u_j'' + f\left(\frac{\partial v_i' W''}{\partial x_j''} - \frac{\partial u_i' W''}{\partial x_j''} - \frac{\partial W' u_j''}{\partial x_i'} + \frac{\partial W' v_j''}{\partial x_i}\right)$$

$$+ \nu f(\nabla_{x''}{}^2 u_i' u_j'' - \nabla_{x''}{}^2 v_i' u_j'' + \nabla_{x'}{}^2 u_i' u_j'' - \nabla_{x'}{}^2 u_i' v_j'') -$$

$$- 2 \in_{mik} \Omega_m H_l \frac{\partial u_i' h_j''}{\partial x_l} - 2 \in_{njl} \Omega_n H_k \frac{\partial h_i u_j''}{\partial x_k'} + 2 \in_{njk} \Omega_n \frac{\partial W' u_j''}{\partial x_i'} -$$

$$-2\nu\nabla_{x'}^{2}\in_{njl}\Omega_n u_i' u_j'' + f(H_l\frac{\partial u_i' h_j''}{\partial x_l''} - H_l\frac{\partial v_i' h_j''}{\partial x_l''} + H_k\frac{\partial h_i' u_j''}{\partial x_k'} -$$

$$-H_k\frac{\partial h_i' v_j''}{\partial x_k'}) + 2f\in_{mik}\Omega_m(u_i' v_j'' - u_i' u_j'') + 2f\in_{njl}\Omega_n(v_i' u_j'' - u_i' u_j'') +$$

$$+\in_{mik}\in_{njl}\Omega_m\Omega_n u_i' u_j'' + f^2(u_i' u_j'' - v_i' u_j'' - u_i' v_j'' + v_i' v_j''). \qquad (7.3.3)$$

Taking the average of the equation (7.3.3) and using the condition of spatial homogeneity and by virtue of the solenoidal relation's (7.2.11), $\overline{W'u_j''}, \overline{W''u_i'}$, etc. do not exist. Since the dust particles are taken as non-conducting, therefore, $\overline{h_i' v_j''}, \overline{h_j'' v_i'} = 0$ and assuming that the instantaneous velocities at one point remain unaffected by the dust particles of the other points i.e. $\overline{u_i' v_j''}, \overline{u_j'' v_i'} = 0$, we have

$$\overline{A_i' A_j''} = -\frac{\partial^2 \overline{W'W''}}{\partial r_i \partial r_j} + \nu^2\frac{\partial^2}{\partial r_k \partial r_k}\frac{\partial^2}{\partial r_l \partial r_l}\overline{u_i' u_j''} - \nu H_k\frac{\partial^2}{\partial r_l \partial r_l}\frac{\partial}{\partial r_k}\overline{h_i' u_j''} +$$

$$+\nu H_l\frac{\partial^2}{\partial r_k \partial r_k}\frac{\partial}{\partial r_l}\overline{h_j'' u_i'} - H_k H_l\frac{\partial^2 \overline{h_i' h_j''}}{\partial r_k \partial r_l} - 2\nu\frac{\partial^2}{\partial r_k \partial r_k}\in_{mik}\Omega_m\overline{u_i' u_j''} -$$

$$-2\nu\frac{\partial^2}{\partial r_k \partial r_k}\in_{njl}\Omega_n\overline{u_i' u_j''} + 2\nu f\frac{\partial^2}{\partial r_l \partial r_l}\overline{u_i' u_j''} - 2\in_{mik}\Omega_m H_l\frac{\partial \overline{u_i' h_j''}}{\partial r_l} +$$

$$+2\in_{njl}\Omega_n H_k\frac{\partial}{\partial r_k}\overline{h_i' u_j''} + f(H_l\frac{\partial}{\partial r_k}\overline{u_i' h_j''} - H_k\frac{\partial}{\partial r_k}\overline{h_i' u_j''}) -$$

$$-2\in_{mik}\Omega_m\overline{h_i' u_j''} - 2f\in_{njl}\Omega_n\overline{u_i' h_j''} + 4\in_{mik}\in_{njl}\Omega_m\Omega_n\overline{u_i' u_j''} +$$

$$+f^2(\overline{u_i' u_j''} + \overline{v_i' v_j''}) \qquad (7.3.4)$$

where $x_i'' - x_i' = r_i$ and $\frac{\partial}{\partial r_i} = \frac{\partial}{\partial x_i''} = -\frac{\partial}{\partial x_i'}$, $\nabla_{x'}^2 = \nabla_{x''}^2 = \nabla^2$.

The Fourier transform of various correlation tensors appearing in (7.3.4) are expressed as spectral tensors:

$$\overline{u_i' u_j''} = \int \phi_{ij}(\hat{k},t) e^{i\hat{k}\cdot\hat{r}} d\hat{k}, \qquad \overline{h_i' h_j''} = \int \psi_{ij}(\hat{k},t) e^{i\hat{k}\cdot\hat{r}} d\hat{k}$$

$$\overline{u_i' h_j''} = \int \Gamma_{ij}(\hat{k},t) e^{i\hat{k}\cdot\hat{r}} d\hat{k}, \quad \overline{h_i' u_j''} = \int \gamma_{ij}(\hat{k},t) e^{i\hat{k}\cdot\hat{r}} d\hat{k} \tag{7.3.5}$$

$$\overline{W_i' u_j''} = \int \phi_{ij}(\hat{k},t) e^{i\hat{k}\cdot\hat{r}} d\hat{k}, \qquad \overline{v_i' v_j''} = \int M_{ij}(\hat{k},t) e^{i\hat{k}\cdot\hat{r}} d\hat{k}$$

It is noted that ϕ_{ij}, ψ_{ij} and M_{ij} are true tensors but Γ_{ij} and γ_{ij} are skew tensors, and from homogeneity,

$$\phi_{ij}(\hat{k}) = \phi_{ji}(\hat{k}), \qquad \psi_{ij}(\hat{k}) = \psi_{ji}(-\hat{k}), \qquad M_{ij}(\hat{k}) = M_{ji}(-\hat{k}),$$

$$\Gamma_{ij}(\hat{k}) = \gamma_{ji}(-\hat{k}) \tag{7.3.6}$$

and from solenoidality, $k_i\phi_{ij}(\hat{k}) = k_i\psi_{ij}(\hat{k}) = k_i M_{ij}(\hat{k}) = k_i\Gamma_{ij}(\hat{k}) = k_i\gamma_{ij}(\hat{k}) = 0$

$$k_j\phi_{ij}(\hat{k}) = k_j\psi_{ij}(\hat{k}) = k_j M_{ij}(\hat{k}) = k_j\Gamma_{ij}(\hat{k}) = k_j\gamma_{ij}(\hat{k}) = 0 \tag{7.3.7}$$

Again,

$$-\frac{\partial^2 \overline{W'W''}}{\partial r_i \partial r_j} = \frac{\partial^2}{\partial r_i \partial r_j} \int R(\hat{k},t) e^{i\hat{k}\cdot\hat{r}} d\hat{k} = -\frac{\partial}{\partial r_i} \int i k_j R(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k}$$

$$= -i^2 k_i k_j \int R(\hat{k},t) e^{i(\hat{k}\cdot\hat{r})} d\hat{k} = k_i k_j \int R(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k}, \tag{7.3.8}$$

and

$$-H_k H_l \frac{\partial^2 \overline{h_i' h_j''}}{\partial r_k \partial r_l} - H_k \frac{\partial}{\partial r_k} H_l \frac{\partial \overline{h_i' h_j''}}{\partial r_l} = -iH_k \frac{\partial}{\partial r_k} H_l k_i \int \psi_{ij} e^{i\hat{k}\cdot\hat{r}} d\hat{k}$$

$$= -ik\mu H H_k \frac{\partial}{\partial r_k} \int \psi_{ij}(\hat{k},t) e^{i\hat{k}\cdot\hat{r}} d\hat{k} = -i^2 k^2 \mu^2 H^2 \int \psi_{ij} e^{i\hat{k}.\hat{r}} d\hat{k}$$

$$= k^2 \mu^2 H^2 \int \psi_{ij} e^{i\hat{k}.\hat{r}} d\hat{k} \qquad (7.3.9)$$

where μ denotes the cosine of the angle k and H i.e. $k\mu H = k_k H_k$.

$$\nu^2 \frac{\partial^2}{\partial r_k \partial r_k} \frac{\partial^2}{\partial r_l \partial r_l} \overline{u_i' u_j''} = \nu^2 \frac{\partial^2}{\partial r_k \partial r_k} \frac{\partial^2}{\partial r_l \partial r_l} \int \phi_{ij}(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k}$$

$$= \nu^2 \frac{\partial^2}{\partial r_k \partial r_k} i^2 k_l k_l \int \phi_{ij}(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k} = -\nu^2 k^2 i^2 k_l k_l \int \phi_{ij}(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k}$$

$$= -\nu^2 k^4 \int \phi_{ij}(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k} \qquad (7.3.10)$$

$$-\nu H_k \frac{\partial^2}{\partial r_l \partial r_l} \frac{\partial}{\partial r_k} \overline{h_i' u_j''} = -\nu \frac{\partial^2}{\partial r_l \partial r_l} H_k \frac{\partial}{\partial r_k} \int \gamma_{ij}(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k}$$

$$= -\nu \frac{\partial^2}{\partial r_l \partial r_l} H_k k_k \int \gamma_{ij}(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k} = i\nu k^3 \mu H \int \gamma_{ij} e^{i\hat{k}.\hat{r}} d\hat{k} \qquad (7.3.11)$$

$$-\nu H_l \frac{\partial^2}{\partial r_k \partial r_k} \frac{\partial}{\partial r_l} \overline{u_i' h_j''} = -\nu \frac{\partial^2}{\partial r_k \partial r_k} H_l \frac{\partial}{\partial r_l} \int \Gamma_{ij}(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k}$$

$$= -\nu \frac{\partial^2}{\partial r_k \partial r_k} iH_l k_l \int \Gamma_{ij}(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k} = -i\nu k^3 \mu H \int \Gamma_{ij} e^{i\hat{k}.\hat{r}} d\hat{k} \qquad (7.3.12)$$

$$-2\nu f \frac{\partial^2}{\partial r_l \partial r_l} \int \phi_{ij}(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k} = -2\nu f k^2 \int \phi_{ij} e^{i\hat{k}.\hat{r}} d\hat{k}. \qquad (7.3.13)$$

With the help of equations, (7.3.8)—(7.3.13), equation (7.3.4) becomes

$$\overline{A_i^{'}A_j^{''}} = \nu^2 k^4 \int \phi_{ij}(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k} + k_i k_j \int R(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k} +$$

$$+ k^2 \mu H^2 \int \psi_{ij}(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k} + i\nu k^3 \mu H [\int \{\gamma_{ij}(\hat{k},t) - \Gamma_{ij}(\hat{k},t)\} e^{i\hat{k}.\hat{r}} d\hat{k}$$

$$- 2\nu k^2 [(\in_{mik} \Omega_m + \in_{njl} \Omega_m) \int \phi_{ij}(k,t) e^{i\hat{k}.\hat{r}} d\hat{k} - 2\nu f k^2 \int \phi_{ij}(\hat{k},t) e^{\hat{k}.\hat{r}} d\hat{k}$$

$$+ 2ik\mu H [\in_{njl} \Omega_n \int \gamma_{ij}(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k} - \in_{mik} \Omega_m \int \Gamma_{ij}(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k}]$$

$$+ fk\mu H [\int \{\Gamma_{ij}(\hat{k},t) - \gamma_{ij}(\hat{k},t)\} e^{i\hat{k}.\hat{r}} d\hat{k}] - 2f[(\in_{mik} \Omega_m + \in_{njl} \Omega_n)$$

$$\times \int \phi_{ij}(\hat{k},l) e^{i\hat{k}.\hat{r}} d\hat{k}] + 4 \in_{mik} \in_{njl} \Omega_m \Omega_n \int \phi_{ij}(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k}$$

$$+ f^2 [\int \phi_{ij}(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k} + \int M_{ij}(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k}]. \qquad (7.3.14)$$

Let us assume that

$\overline{A_i^{'}A_j^{''}} = \int A_{ij}(\hat{k},t) e^{i\hat{k}.\hat{r}} d\hat{k}$, then the spectral equations in this context become

$$A_{ij} = \nu^2 k^4 \phi_{ij} + k_i k_j R(\hat{k},t) + k^2 \mu^2 H^2 \psi_{ij} + i\nu k^3 \mu H (\gamma_{ij} - \Gamma_{ij}) -$$

$$- 2\nu k^2 (\in_{mik} \Omega_m - \in_{njl} \Omega_n) \phi_{ij} - 2\nu f k^2 \phi_{ij} + 2ik\mu H (\in_{njl} \Omega_n \gamma_{ij} -$$

$$- \in_{mik} \Omega_m \Gamma_{ij}) + fk\mu H (\Gamma_{ij} - \gamma_{ij}) - 2f(\in_{mik} \Omega_m + \in_{njl} \Omega_n) \phi_{ij}$$

$$+ 4\Omega_m{}^2 (\delta_{kl} - \phi_{kl}) + f^2 (\phi_{ij} + M_{ij}). \qquad (7.3.15)$$

This equation displays the effect of magnetic field on acceleration covariance in MHD dusty turbulence in a rotating system.

For the axi-symmetric case we can put

$$\phi_{ij}(\hat{k},t)=\phi^{(1)}D_{ij}(\hat{k})+\phi^{(2)}\theta_{ij}(\hat{s},\hat{k})\,;\quad \psi_{ij}(\hat{k},t)=\psi^{(1)}D_{ij}(\hat{k})+\psi^{(2)}\theta_{ij}(\hat{s},\hat{k})$$

$$\Gamma_{ij}(\hat{k},t)=\Gamma^{(1)}D_{ij}(\hat{k})+\Gamma^{(2)}\theta_{ij}(\hat{s},\hat{k})\,;\quad \gamma_{ij}(\hat{k},t)=\gamma^{(1)}D_{ij}(\hat{k})+\gamma^{(2)}\theta_{ij}(\hat{s},\hat{k})$$

$$M_{ij}(\hat{k},t)=M^{(1)}D_{ij}(\hat{k})+M^{(2)}\theta_{ij}(\hat{s},\hat{k})\,;\quad k_ik_j=k^2D_{ij}(\hat{k}) \tag{7.3.16}$$

where s is the unit vector in the direction of H, and

$$D_{ij}(\hat{k})=(k^2\delta_{ij}-\frac{k_ik_j}{k^2}) \tag{7.3.17}$$

$$\theta_{ij}(\hat{s},\hat{k})=(1-\mu^2)\delta_{ij}-\frac{k_ik_j}{k^2}-s_is_j+\frac{\mu}{k}(s_ik_j+k_is_j)\ . \tag{7.3.18}$$

While the defining scalars $\phi^{(2)}$ ----$M^{(2)}$ are the functions of k, $k\mu$ and time.

It follows from the homogeneity condition that

$$\phi^{(1,2)}(k,k\mu)=\phi^{(1,2)}(k,-k\mu)\,;\qquad \psi^{(1,2)}(k,k\mu)=\psi^{(1,2)}(k,-k\mu)$$

$$M^{(1,2)}(k,k\mu)=M^{(1,2)}(k,-k\mu) \tag{7.3.19}$$

for true tensors, and

$$\Gamma^{(1,2)}(k,k\mu)=\Gamma^{(1,2)}(k,-k\mu) \tag{7.3.20}$$

for skew tensors.

Under these conditions, equation (7.3.15) becomes

$$A^{(1)}=\nu^2k^4\phi^{(1)}+k^2R(k,t)+k^2\mu^2H^2\psi^{(1)}+i\nu k^3\mu H(\gamma^{(1)}-\Gamma^{(1)})$$

$$-2\nu k^2\phi^{(1)}(\in_{mik}\Omega_m-\in_{njl}\Omega_n)-2\nu fk^2\phi^{(1)}+2ik\mu H(\in_{njl}\Omega_n\gamma^{(1)}$$

$$- \in_{mik} \Gamma^{(1)}) + fk\mu H(\Gamma^{(1)} - \gamma^{(1)}) - 2f\phi^{(1)}(\in_{mik} \Omega_m + \in_{njl} \Omega_n)$$

$$+ f^2(\phi^{(1)} + M^{(1)}) \qquad (7.3.21)$$

and

$$A^{(2)} = \nu^2 k^4 \phi^{(2)} + k^2 \mu^2 H^2 \psi^{(2)} + i\nu k^3 \mu H(\gamma^{2)} - \Gamma^{(2)})$$

$$- 2\nu k^2 \phi^{(2)}(\in_{mik} \Omega_m - \in_{njl} \Omega_n) - 2\nu f k^2 \phi^{(2)} + 2ik\mu H(\in_{njl} \Omega_n \gamma^{2)}$$

$$- \in_{mik} \Gamma^{(2)}) + fk\mu H(\Gamma^{(2)} - \gamma^{(2)}) - 2f\phi^{(2)}(\in_{mik} \Omega_m + \in_{njl} \Omega_n)$$

$$+ f^2(\phi^{(2)} + M^{(2)}) \qquad (7.3.22)$$

7.4. CONCLUSION

Here we discussed the effect of very strong magnetic field on acceleration covariance in MHD turbulence of dusty fluid in a rotating system.

Defining scalars of acceleration covariance have been obtained in terms of the defining scalars of various spectrum functions in the simplest form.

If we put $f = 0$, in equations, (7.3.21) and (7.3.22), we get the effect of magnetic field on acceleration covariance in MHD turbulence in a rotating system for clean flow.

Again, if the fluid is clean and the system is non- rotating (i.e. for $f = 0$, $\Omega = 0$), we have

$$A^{(1)} = \nu^2 k^4 \phi^{(1)} + k^2 R(k,t) + k^2 \mu^2 H^2 \psi^{(1)} + i\nu k^3 \mu H(\gamma^{(1)} - \Gamma^{(1)}) \qquad (7.4.1)$$

and

$$A^{(2)} = \nu^2 k^4 \phi^{(2)} + k^2 \mu^2 H^2 \psi^{(2)} + i\nu k^3 \mu H(\gamma^{2)} - \Gamma^{(2)}) \qquad (7.4.2)$$

which is obtained earlier by Kishore & Dixit [44].

In order to estimate the order of magnitude of various terms in (7.3.21) and (7.3.22), we introduce $a = (u^2 + h^2/3)^{1/2}$ as the level of turbulence and the characteristic length l to derive a relation of the form,

$$\frac{\textit{viscous dissipation terms}}{\textit{external coupling terms}} \approx \frac{\nu a^2/l^2}{Ha^2 l} = \frac{\nu}{Hl} = \frac{1}{R_H},$$

where R_H is the Reynolds number. If the imposed magnetic field is sufficiently strong, $\frac{1}{R_H}$ is very small in comparison with 1, and, hence equations (7.3.21 & 7.3.22) becomes

$$A^{(1)} = k^2\mu^2H^2\psi^{(1)} + k^2R(k,t) + 2ik\mu H(\in_{njl}\Omega_n\gamma^{(1)} - \in_{mik}\Omega_m\Gamma^{(1)})$$
$$+ fk\mu H(\Gamma^{(1)} - \gamma^{(1)}) - 2f\phi^{(1)}(\in_{mik}\Omega_m + \in_{njl}\Omega_n) + f^2(\phi^{(1)} + M^{(1)}) \quad (7.4.3)$$

and

$$A^{(2)} = k^2\mu^2H^2\psi^{(2)} + 2ik\mu H(\in_{njl}\Omega_n\gamma^{2)} - \in_{mik}\Omega_m\Gamma^{(2)} + fk\mu H(\Gamma^{(2)}$$
$$-\gamma^{(2)}) - 2f\phi^{(2)}(\in_{mik}\Omega_m + \in_{njl}\Omega_n) + f(\phi^{(2)} + M^{(2)}) \quad (7.4.4)$$

which show the predominance of the external magnetic field over other forces on acceleration covariance in MHD turbulence of dusty fluid in a rotating system.

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