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A Study of Some Non-linear Stochastic Models on Renewable Resources Management: Application to Forestry

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University of Rajshahi

A Study of Some Non-linear Stochastic Models on Renewable Resources Management: Application to Forestry

A Dissertation
Submitted to the University of Rajshahi in
Fulfillment of the Requirements for the Degree of Master of
Philosophy

By
Md. Mowazzem Hossain

University Of Rajshahi
June, 2010

Department of Statistics
University of Rajshahi
Rajshahi - 6205
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Under the Supervision of

Professor Dr. Md. Asaduzzaman Shah

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CERTIFICATE

This to certify that Md. Mowazzem Hossain, Lecturer, Department of Mathematics, Birampur Degree College, Dinajpur, has worked for the Degree of Master of Philosophy in Statistics under my supervision. His topic of research is "A Study of Some Non-linear Stochastic Models on Renewable Resources Management: Application to Forestry". The work is genuinely his own and it is original. No part of this study has been submitted in substance for any higher degree or diploma. The work is fit for submission.

I wish him every success in his life.

Supervisor

29.06.10

(Dr. Md. Asaduzzaman Shah)
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STATEMENT OF ORIGINALITY

This dissertation does not incorporate any part without acknowledgement of any material previously submitted for a higher degree or diploma in any University/Institute and to best of my knowledge and belief, does not contain any material previously published or written by another person except where due reference is made in the text.

(Md. Mowazzem Hossain)
M. Phil. Research Fellow
Roll: 110
Session: July - 2005

DEDICATED

TO

MY BELOVED LATE

PARENTS

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June 2010 The Author

CONTENTS

BIBLIOGRAOHY

LIST OF FIGURE

LIST OF TABLE

3.3(b) 93

The relationship between $\phi(\eta; R, Q)$ and η for a particular set of control parameters (R, Q) when the higher root, ψ_3 of the cubic equation, is to be exercised and η is greater than ψ_3.

A Study of Some Non-linear Stochastic Modelson Renewable Resources Management: Application to Forestyy

Abstract : Rapid industrialization and rising energy and resource consumptions have led scientists to believe that the optimal exploitation and efficient management of the resource –renewable as wel l as non-renewable, are the impelling needs of the day, and demand urgent attention of all concerned. Every item of the human utility involves energy consumption. According to the recent estimates, at the current rate of consumption and trend of demands, our reserves of oil and natural gas will last for some eighty odd years, and the coal, some two hundred years. Keeping in view the growth of human population at an alarming rate, the efficient management and conservation of our resources is no longer a soft option, rather it is a pressing hard core necessity. Further, the planning, conservation and management of renewable resources becomes all the more important as the non-renewable resources are limited.

Resources generated through bio-reproduction processes and through photo-bio-chemical processes constitutes a rare and unique gift of the nature to the human race. These resources are renewable by the very nature of biological processes. Fishery and forestry are prime examples of renewable resources that the human race has been exploiting for its survival and shelter since the time immemorial. Fresh water resources are repeatedly renewed our

agricultural produce and hydro-electric power generation. The area of renewable resources is vast and unfathomable, and thus will remain a subject of research for ages. Keeping our limited objective, for the present, in mind, the proposed thesis is confines to the study of some nonlinear stochastic models pertaining to the management of forestry.

This thesis consists of five chapters. To cater to the needs of non-specialist, and to make this piece of work self-sufficient, the first chapter is intended to be a self-contained module of concepts, population growth model, equilibrium, notions about management of forestry. Further, uncertainties arise in several ways in biological growth of these resources, and also in the management due to involvement of unpredictable human behaviour, and thus render these problems as stochastic in nature. Therefore, we have given a brief description of stochastic processes and stochastic differential equation in terms of which the models have been developed. A modified version of van Kampen's method of system-size expansion, which is the basic mathematical tool for our investigation is also introduced. In our first problem, which comprises the second chapter of this thesis, we have studied the deterministic version of the Ludwig, Jones and Holloing's non-linear pest model. Forestry have potential impact on environment so it is evident to research forestry management. The spruce budworm is a pest and it is very harmful for the

Coniferous forest of Canada. In this direction we have discussed the life history of the spruce budworm, details of its nature and development, food habits and specially the higher reproductive activity. Initially the budworms are so few that the birds cannot easily locate them and at a later stage, the pest population attains a good size, as the size of the population tends towards the carrying capacity of the system (the available leaf-area) their rate of reproduction decline due to defoliation. In view of this facts the marvelous and wonderful mathematical tricky tools applied to locate and describe the equilibrium. The relationship between straight line and curve represents the qualitative nature of the pest. To understand and explain the critical patch width and critical patch size we have utilized the linear and logistic models and have highlighted in the third chapter. The linearity and stability of the process have also been verified.

In the fourth chapter, we have developed a stochastic quantative version of the Wright's model and have carried out a detailed deterministic analysis of the model. The purpose of this work is to explore possible mathematical structures in the model by incorporating `fluctuation` components. The mean evolution of the process and the stead-state solution along with the minimum and maximum values of the dominant control parameters have been analyzed in this direction. The qualitative management strategy for controlling the pest by choosing a combination of alternatives: (i) tree-felling, and (ii) spraying with pesticide are also addressed herewith.

In the last chapter, continuing with the models developed on stochastic version formulated on the basis of birth-and-death process and have carried out a detailed stochastic study of the model in three regions of interest and of importance. It has been shown that the system exhibits a first-order phase transition, and at the critical point and its close neighborhood, the system undergoes large fluctuations as compared to those predicted by the Central Limit Theorem. Tremendously enhanced fluctuation play a very important role in bring out the unfolding of the cusp catastrophe which we have discussed.

CHAPTER-ONE

INTRODUCTION

CHATER ONE

INTRODUCTION

Management is a vibrant and multiphase dynamic process, just like social, scientific and engineering processes, that involves a wide spectrum of activities such as planning, coordination, communication, policy framing, decision making and their implementation. During the last three decades, the management of natural resources in general and that of renewable resources, in particular, has invited the attention of a large segment of researchers in various field [1-5].

In order to maintain the ecological balance, as well as, meet the economic needs, the forest can play a vital role. Our government also taking special initiative to carry forward a forestation program throughout the country. Also to pollution free environment a forestation has no alternative. In this direction government organizations came forward to utilize the renewable resources for economic prosperity of the nation.

Coyle studied the dynamics of management system [6] and of capital expenditure [7]. However, it is difficult to define precisely the dynamic nature of management problems as it involves in important factor of uncertainty arising from various sources, intrinsic as well as extrinsic to a management system under consideration. Further, most of the management problems involve very sensitive control parameters, sensitive in the sense that very small changes in the parameters lead to unexpected large effects reflected in the response. When sensitive parameters are present, the system under study may exhibit a catastrophic behaviour over certain regions in the parameter space. Recently, Wright [8], has carried a detailed qualitative analysis of management problems related to forestry by employing concepts of Thom's Catastrophe Theory [9, 10].

Resources generated through bio-reproduction processes and through photo-biochemical processes are a rare gift of nature to the human race. These resources are renewable by the very nature of biological processes. Forestry is prime example of renewable resources that the human race has been exploiting for its survival and shelter since the time immemorial. In the management of renewable resources, both biology and economics play important roles. The basic problem is the resource conservation, and hence a problem of the optimal use of resource stocks overtime. Thus, the resource conservation theory is to be founded on the explicit dynamic mathematical modes of biological processes coupled with objective functions framed with economic concepts, and therefore, must concern itself with the problem of dynamic optimization. Further, uncertainties crop out in several ways in the biological growth of these resources, and also arise in the management due to involvement of unpredictable human behaviour, and consequently these problems become stochastic in nature. In the proposed thesis, we shall confine to the study of some non-linear stochastic models pertaining to the management of forestry.

1.1.1 EQUILIBRIUM:

Equilibrium is a state of a system which does not change with respect to time. Therefore, the dynamics of a system is described by a differential (or a system of differential equations), then equilibrium can be estimated by setting a derivative (all derivatives) to zero.

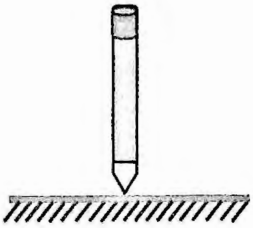
Example: Logistic model

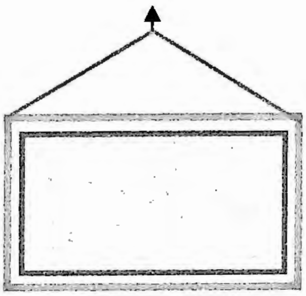
$$\frac{dn}{dt} = r_0 n(1 - \frac{n}{k}).$$

In order to find equilibrium, we have to solve the equation : $\frac{dn}{dt} = 0$ gives

$$r_0 n(1 - \frac{n}{k}) = 0.$$

This equation has two roots ; $n = 0$ and $n = k$. An equilibrium may be stable or unstable. For example, the equilibrium of a pencil standing on its tip is unstable, however equilibrium of a picture on the wall is (usually) stable.





An equilibrium is considered stable (for simplicity we will consider asymptotic stability only) if the system always returns to it after small disturbances. If the system moves away from the equilibrium after small disturbances, then the equilibrium is unstable.

1.1.2 BIFURCATION:

When a non-linear dynamic system develops twice the possible solutions that it had before it passed its critical level. A bifurcation cascade is often called the period doubling route to chaos because the transition from an orderly system to a chaotic system often occurs when the number of possible solutions begins increasing, doubling each time.

1.2 POPULATION GROWTH MODELS:

Lel $n(t)$ denoted the size of a population at time t. Considering the largeness of biological populations, we shall treat $n(t)$ continuous. A class of density dependent growth model is dynamically represented by the differential equation

$$\frac{dn}{dt} = f(n) \qquad (1.1)$$

subject to the condition of initial population,

$$n(0) = n_0 \qquad (1.2)$$

where $f(n)$ is an appropriate function of $n(t)$ of degree≥ 2. The growth function per unit of the population is given by

$$r(n) = \frac{1}{n}\frac{dn}{dt} = \frac{f(n)}{n} \qquad (1.3)$$

and sometimes referred to as the proportion growth rate. Natural populations have been investigate through a large numbers of models[11-16].

A growth model for which $r(n)$ is a non-increasing function of n is referred to as a 'pure compensation model'. If $r(n)$ is a non-decreasing function of n for certain values of n, then the model is said to be a 'dispensation model'. We would like to mention that $r(n)$ may be such that it increases for $0 < n < N^*$, and the decreases for $n > N^*$, even in this situation, the model will be referred to as a dispensation model.

A dispensation model with an addition property $f(n) < 0$ for certain values of n, near $n = 0$ is termed as a 'critical dispensation model'. For such models an unstable equilibrium N_0 exists, such that whenever the initial population $n_0 < N_0$, then $n(t) \to 0$ as $t \to \infty$. For obvious reasons, $n = N_0$ is called the 'minimum viable' population.

1.3 STOCHASTIC PROCESSES :

For a deterministic dynamic system given the initial conditions and the laws of the dynamics of system, its future course can be easily predicted at any subsequent instant of time. However, there is an inherent weakness of deterministic system. The theory of determinism is based on three classical assumptions of exact, instantaneous and free recording of observations pertaining to the evolution of the system. For real life problems these three axioms are too harsh and are hardly met by a dynamic system. Uncertainties creep into the course of the system and influence it remarkably. Thus, in realistic situations what we can do at best is that, given the initial probability distribution of the system in different possible states and the laws of dynamics driving the system, we may predict only the probabilities of the system being in a particular state at a given subsequent time. The process associated with the evolution of such a system is called a stochastic process. Summarily, we may state that a process that evolves in time or space according to certain laws of probability is referred to as a stochastic process. Speaking mathematically, it is a family of random variable $\{X(t), t \in T\}$ indexed by a real parameter t, and defined on a common probability space (Ω, S, P). When T is a discrete set of finite or countable indexing numbers, the stochastic process is said to be a discrete parameter process, when T is an interval of the real number system, then it is called a continuous parameter process. Further, the process is referred to discrete or continuous state process according to as it assumes values over a discrete set or a continuous set (interval) of the real number system. For an axiomatic rigorous definition, reader may consult Sharma [17].

A discrete parameter stochastic process $\{X(t)\}$ $t = 0, 1, 2, ...$ or a continuous parameter stochastic process $\{X(t)\}$, $t \geq 0$, is called a 'Markovian process' if, for any set of $n+1$ point $t_1 < t_2 < t_3 <,...,< t_n < t_{n+1}$ in the index set T of the process, the conditional distribution of $X(t)$, for given values of $X(t_1), X(t_2), \ldots, X(t_n)$ depends only on the value of $X(t_n)$, the most recent known value, and is independent of the remote past. Put mathematically,

$$P[x(t) \leq x | X(t_1) = x_1, ..., X(t_n) = x_n] = P[X(t) \leq x | X(t_n) = x_n]$$

A stochastic process $\{X(t), t \in T\}$, continuous in parameter as well as the state space is called a normal or Gaussian process if, for any $n \geq 1$, and any finite sequence $t_1 < t_2 <,...,< t_n$ from T, the random variable $X(t_1), X(t_2),..., X(t_n)$ are jointly normally distributed with its joint probability density function

$$f(\vec{x}) = (2\pi)^{-\frac{n}{2}} |\det \Sigma|^{-\frac{1}{2}} \exp[-(\vec{x} - \vec{\mu} \Sigma^{-1} (\vec{x} - \vec{\mu})] \quad \text{.....(1.4)}$$

where $\vec{X}(t) = [X_1(t), X_2(t),..., X_n(t)$, and Σ and $\vec{\mu}$ are the covariance matrix and the expectation of $\vec{X}(t)$ respectively.

Further a stochastic process whose probability laws are invariant under transition/shift in time is called a stationary process and it is called an independent increment process if $X(0)$, and for all choices $t_0 < t_1 < t_2 <,...,< t_n$, the random variables $X(t_{i+1}) - X(t_i), i = 0,1,....,n-1$ are stochastically independent.

An independent increment stationary process $\{X(t)\}$ $t \geq 0$ is called a 'Wiener process' if, for every t>0, $X(t)$ is normally distributed with expectation $E[x(t)] = 0$ and variance $\sigma^2 t$. The parameter σ^2 is an empirical characteristic of the process and must be determined from observations in respect of the process.

From the definition of a Wiener process, it follows immediately that for $t > s$, the probability density function of $X(t) - X(s)$ will be

$$f_{X(t)-X(s)} = \frac{1}{\sigma\sqrt{2\pi(t-s)}} \exp[-x^2 / 2\sigma^2 (t-s)] \quad \text{.....(1.5)}$$

In 1828, Robert Brown, an English Botanist, observed that small pollen particles of size a few micron immersed in a liquid executed an irregular motion. This phenomenon, which is referred to as Brownian motion, arises from incessant collisions of the molecules of liquid on the pollen particles suspended in it. After about a three quarter of a century, using the the concept of a simple random walk, in 1905 Einstein [18] gave scientific interpretation of the Brownian motion, Wiener [19] and Levy [20] independently gave first rigorous treatment of the Brownian motion. For this reason, the Brownian motion is sometimes called the Wiener process or the Wiener-Levy process.

1.4 DIFFUSION PROCESS AND FOKKER-PLANK EQUATION:

A Markov process $\{X(t)\}$, $t \geq 0$, with state space R is called a 'diffusion process' if there exist real valued function $M(x,t)$ and $S(x,t)$ such that, for any arbitrary fixed $\varepsilon > 0$, the following condition are satisfied:

(i) $$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{|y-x|>\varepsilon} G(x,t;y,t+\Delta t)dy = 0 \qquad \text{(1.6a)}$$

(ii) $$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{|y-x|\leq\varepsilon} (y-x)G(x,t;y,t+\Delta t)dy = M(x,t) \qquad \text{(1.6b)}$$

(iii) $$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{|y-x|\leq\varepsilon} (y-x)^2 G(x,t;y,t+\Delta t)dy = S(x,t) \qquad \text{(1.6c)}$$

and for $n \geq 3$,

$$\text{(iv)} \quad \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{|y-x| \leq \varepsilon} (y-x)^n G(x,t;y,t+\Delta t)dy = M_n(x,t) \quad \text{.........................(1.6d)}$$

where $G(x,t;y,s)$ is given by

$$G(x,t;y,s) = P[X(s) = y | X(t) = x]$$

The functions $M(x,t)$ and $S(x,t)$ are called the 'drift' and 'diffusion' coefficients of the process, and $M_n(x,t)$ the jump moments of order n.

We shall now state a theorem which embodies the Fokker-Plank equation. Let $\{X(t)\}$, $t \geq 0$, be a diffusion process with drift coefficient $M(x,t)$ and diffusion coefficient $S(x,t)$ and transition distribution $G(x,t;y,s)$ such that the following partial derivatives exist and are continuous:

$$g(y,s;x,t) = \frac{\partial}{\partial x} G(y,s;x,t), \quad \frac{\partial}{\partial t} G(y,s;x,t), \quad \frac{\partial}{\partial y} G(y,s;x,t),$$

$$\frac{\partial}{\partial x}[M(x,t) \quad g(y,s;x,t)], \quad \frac{\partial^2}{\partial x^2}[S(x,t) \quad g(y,s;x,t)] \quad \text{..............................(1.7)}$$

Then $g(x,t;y,s)$ satisfies the 'Kolmogorov forward' equation

$$\frac{\partial g(y,s;x,t)}{\partial t} = -\frac{\partial}{\partial x}[M(x,t) \quad g(y,s;x,t)] + \frac{1}{2}\frac{\partial^2}{\partial x^2}[S(x,t) \quad g(y,s;x,t)] \quad \text{....................(1.8)}$$

which in the literature is referred to as the Fokker-Plank equation, especially by Physicists and Chemists.

1.5 STOCHASTIC DIFFERERNTIAL EQUATIONS AND WHITE NOISE:

Following Einstein's interpretation of the Brownian motion in 1905 [21], Langevin and others [22-27] formulated the dynamics of such motions in terms of the so-called stochastic differential equations. Speaking intuitively, any differential equation in which stochasticity involves any form is called a stochastic differential equation. Obviously the stochasticity may appear through any one of the following mechanisms or through any combination them.

1. Initial conditions are stochastic
2. Boundary conditions are stochastic
3. Coefficients appearing therein are stochastic variables
4. The driving force is stochastic in nature
5. Fluctuations prevailing in the reservoir in which the system under consideration is embedded contributed stochastic perturbations.

In case of the Brownian motion, the stochasticity arises from incessant random impact of water molecules on the Brownian particles (pollen particles). Thus the source of stochasticity is of the last type. If $X(t)$ denotes the random position of the Brownian particle at time t, when the motion being considered along a straight line, the resulting equations were written in the form:

$$dX(t) = f(t; x(t))\,dt + g(t; x(t))\,h(t)\,dt \quad \text{......} (1.9)$$

With a deterministic or average drift term

$$\frac{dX}{dt} = f(t\,;\,x) \qquad (1.10)$$

perturbed by a noise diffusion term $g(t; x(t))\,h(t)$, where $h(t)$ were standard Gaussian random variables for each t, and $g(t; x(t))$, a space- time dependent intensity factor. The symbolic differential equation, Eq.-(1.9) was rewritten and interpreted as an integral equation

$$X(t,\omega) = X(t_0,\ \omega) + \int_{t_0}^{t} f(\tau, x(q,\omega) d\tau + \int_{t_0}^{t} g(\tau, x(q,\omega))\, h(\tau,\omega) d\tau \qquad (1.11)$$

For each realization (sample path) ω. Since Brownian motion is not differentiable in the classical sense, some extrapolations to a limit were carried out (see Kloeden and platen [28] Schuss [29]. Arnold [30]. Ito [31,32]. Soong [33]. The observations of Brownian motion seemed to suggest that the covariance

$$\Gamma(t) = E[h(s)\ h(s+t)] \qquad (1.12)$$

of the process $h(t)$ had a constant spectral density, which is, with all time frequencies, equally weighted in any Fourier transform of $\Gamma(t)$. Such a process has been referred to as Gaussian White noise, particularly in the engineering literature. The obvious reason for this nomenclature being the fact that when a large number of stochastically independent factors cause fluctuations (noise) in the system then the fluctuation are uncorrelated in time and bear an analogy with white light which arises from the superposition of waves of different wavelengths (colours)

In integral equation, Eq.- (1.11), if we set

$$f(t\,;(x)) \equiv 0, \text{ and } g(t\,;x(t)) = 1 \qquad (1.13)$$

We obtain

$$X(t,\omega) = X(t_0,\ \omega) + \int_{t_0}^{t} h(\tau,\omega)d\tau \qquad (1.14)$$

which suggests that $h(t)$ may be regarded as formal derivative of pure Brownian motion, that is, the derivative of a Wiener process, thereby suggesting that we could write Eq.-(1.11) in the following form

$$X(t,\omega) = X(t_0,\ \omega) + \int_{t_0}^{t} f(\tau, x(\tau,\omega)d\tau + \int_{t_0}^{t} g(\tau, x(\tau,\omega))\, h(\tau,\omega)dw(\tau,\omega) \qquad (1.15)$$

Here. we would like to caution that, since a Wiener process $W(t)$ is nowhere differentiable, therefore strictly speaking the White noise process $h(t)$ dose not exist as a conventional function of t. Thus, the last integral of Eq.-(1.15) cannot be interpreted as an ordinary Riemann or Lebesque integral. Still worse, the continuous sample path of a Wiener process are not of bounded variation on any bounded time interval, and therefore, the second integral in Eq.-(1.15) cannot even be interpreted as a Riemann Stieltjes integral for each sample path.

In view of the foregoing discussion, we characterize $h(t)$ by two empirical parameters m and σ^2 defined by

$$< h(t) > = m \quad \text{.......... (1.16)}$$

and

$$\Gamma(t) = < (h(t) - m)(h(t') - m) > = \sigma^2 \delta(t - t'), \quad \text{.......... (1.17)}$$

with all the correlations of order≥ 3 being zero. Here $\delta(t)$ is Dirac's delta function, and symbols $<.>$ denote the ensemble average, that is the average taken over a number of repetitive observations on the system.

$h(t)$ defined by Eq.-(1.16) and Eq.-(1.17) is said to be generated by a Gaussian process, and σ^2 termed as an incremental variance or the intensity of the input process. We define a new function $V(t)$ by setting,

$$V(t) = [h(t) - m] / \sigma \quad \text{.......... (1.18)}$$

So that

$$< V(t) > = 0 \quad \text{.......... (1.19)}$$

and

$$< V(t)\, V(t') > = \delta(t - t') \quad \text{.......... (1.20)}$$

and with vanishing correlation function of $V(t)$ of order ≥ 3. $V(t)$, so defined is called the 'White noise' with zero mean and intensity one. Obviously, $h(t)$ is a white noise with mean m and intensity δ^2.

Using Eq.-(1.18), we rewrite Eq.-(1.19) as follows

$$dX(t) = \alpha(x)dt + \beta(x)\,dW(t) \quad \text{.......} \quad (1.21)$$

where

$$\alpha(x) = f(x) + m\,g(x) \quad \text{and} \quad \beta(x) = \sigma\,g(x)$$

Following Cox and Miller [34], one can shown that for the solution process of (1.21)

$$N(x,t) = N_1(x) = \alpha(x) + \frac{1}{4}.\frac{\partial}{\partial x}\left[\beta^2(x)\right] \quad \text{.......} \quad (1.22)$$

$$S(x) = M_2(x) = \beta^2(x) \quad \text{.......} \quad (1.23)$$

and

$$M_n(x) = 0 \quad \text{for all} \quad n \geq 3 \quad \text{.......} \quad (1.24)$$

and the corresponding probability density function $P(x, t)$ satisfies the Fokker-Plank equation, Eq.- (1.18)

1.6 van KAMPEN'S SYSTEM-SIZE EXPANSION METHOD: MODIFID VERSION:

A large cross section of the real world problems related to several disciplines of learning such as Physics, Chemistry, Biology, Engineering. Economics, Sociology, Management etc, have been modeled through Markovian stochastic processes. The temporal evolution of the conditional probabilities for such a stochastic process is described by the master equation (ME), which is an equivalent form of the Chapman-Kolmogorov of the process. Let $X(t)$ be a discrete one dimensional Markov process whose states are labeled by n. Further, let $W_{nn'}$ be the transition probability per unit time from state n' to state n, that is

$$W_{n\,n'} = P\left[X = n \middle| X = n'\right] \quad \text{(1.25)}$$

and let $P_n(t)$ be the probability that the system is in the state n at time, then its ME is given by

$$\frac{dP_n(t)}{dt} = \sum_{n'}\left[W_{n\,n'} P_{n'}(t) - W_{n'n} P_n(t)\right] \quad \text{(1.26)}$$

In this form the master equation becomes specially meaningful as it is now a gain-loss equation for the probability of each state n. In the language of fluid mechanics it is the equation of continuity for the flow of probabilities.

In the process $X(t)$ involves only one step transition from the neighboring states $n-1$ and $n+1$, then writing

$$W_{n+1,n}=\lambda_n \text{ and } W_{n-1,n}=\mu_n \qquad (1.27)$$

the ME reduces to the form

$$\frac{dP_n(t)}{dt}=\lambda_{n-1}P_{n-1}(t)+\mu_{n+1}(t)P_{n+1}(t)-(\lambda_n+\mu_n)P_n(t) \qquad (1.28)$$

In terms of the transition or Shift operator E defined by

$$E^{\pm 1}f(n)=f(n\pm 1), \qquad (1.29)$$

The ME assumes the form

$$P_n(t)=\left[(E-1)\mu_n+(E^{-1}-1)\lambda_n\right]P_n(t) \qquad (1.30)$$

where

$$P_n(t)=\frac{dP_n(t)}{dt}$$

However, only in the rare cases, the ME is solvable in an explicit form. For all those master equations that cannot be solved exactly, it become necessary to develop a systematic approximation method rather than Fokker-Plank equation [FPE] based on the Langevin's intuitive approach [35]. van Kampen [36] remedied the situation by providing with a systematic approximation method in the form of an expansion in powers of a small parameter ε, related to the overall size of the system that again converted the ME into FPE. Pawula [37], with all mathematical rigors, established the important fact that it is impossible to find a partial differential equation other than the FPE that approximates the ME and leads to a non-negative solution as well. The method is provided on splitting the random variable $n(t)$ in the form

$$n(t) = \Omega\Phi(t) + \Omega^{\frac{1}{2}} x \quad (1.31)$$

where Ω is a measure of the overall-size of the system in which the Markov process evolves; $\Phi(t)$ is a smoothly varying function of time that governs the phenomenological development of the process and x is a purely random function such that $0(x) = 1$ represents the fluctuations around the mean trajectory. Further, for large system, $n(t)$ has been treated as a continuous random variable.

The splitting given in Eq.-(1.31) is based on the implications of the Central limit theorem. One expects that in view of the Central limit theorem, $P(n,t)(\equiv P_n(t))$, will have a sharp maximum around the expected value $E[n(t)] = \Omega\Phi(t)$, with a width of order $n^{\frac{1}{2}} = \Omega^{\frac{1}{2}}$.

Subsequently, it was felt that for the Markov processes which can be described by a birth-and-death type of model, this approach is not adequate enough to provide with a proper description of fluctuations when a system is in an unstable equilibrium or away from equilibrium [38-40]. However, Fox

[41] and Dekker [42] independently suggested a modification to the van Kampen's method. The modified version of this method has been successfully applied by Sharma et al. [43] and Sharma and Pathria [44] for analyzing the stochastic behaviour of non-linear birth-and- death type models both for equilibrium as well as non-equilibrium system in their critical regions. The modified version runs as follows:

Considering the observation of the several researchers that fluctuations are immensely large at the critical point and in its immediate neighborhood, of a system the stochastic variable $n(t)$ is split as

$$n(t) = \Omega\Phi(t) + \Omega^{\nu} x, \qquad 0 < \nu < 1 \quad \text{(1.32)}$$

instead of Eq.-(1.31). The probability distribution $p(n,t)$ now transform into $q(x,t)$, where

$$q(x,t) = \Omega^{\nu} P\left[\Omega\Phi(t) + \Omega^{\nu}(x,t)\right] \quad \text{(1.33)}$$

while the translation operator E becomes a differential operator such that

$$E = \exp(D), \quad D = \Omega^{-\nu}\frac{\partial}{\partial x}. \quad \text{(1.34)}$$

Therefore

$$E^{\pm 1} - 1 = \left[\pm\frac{\Omega^{-\nu}}{1!}\frac{\partial}{\partial x} + \frac{\Omega^{-2\nu}}{2!}\frac{\partial^2}{\partial x^2} \pm \frac{\Omega^{-3\nu}}{3!}\frac{\partial^3}{\partial x^3} + \right] \quad \text{(1.35)}$$

The jump moments or derivative moments [45], defined by

$$A_k(n) = \sum_{n} \left(n' - n\right)^k W_{nn'} \quad (1.36)$$

then assume the form

$$A_k(n) = \lambda_n + (-1)^k \mu_n \quad (1.37)$$

and, in view of Eq.-(1.32) and Eq.-(1.35), can be written as

$$A_k = f(\Omega)\alpha_r(\Phi + \Omega^{\nu-1}x) \quad (1.38)$$

Substituting Eq.-(1.32) and Eq.-(1.35) into Eq.-(1.30) and using Eq.-.(1.38), we obtain

$$\frac{\partial q}{\partial t} - \Omega^{1-\nu}\frac{d\Phi}{dt}.\frac{\partial q}{\partial x}$$
$$= f(\Omega)\left[-\Omega^{-\nu}\frac{\partial}{\partial x}[\alpha_1(\Phi,x)q] + \Omega^{-2\nu}\frac{\partial^2}{\partial x^2}[\alpha_2(\Phi,x)q] - \Omega^{-3\nu}\frac{\partial^3}{\partial x^3}[\alpha_3(\Phi,x)q] + \ldots\right] \quad (1.39)$$

Redefining the time scale by setting

$$f(\Omega)t = \Omega\,\tau \quad (1.40)$$

$$\frac{\partial q}{\partial t} - \Omega^{1-\nu}\frac{d\Phi}{d\tau}.\frac{\partial q}{\partial x}$$
$$= -\frac{\Omega^{1-\nu}}{1!}\frac{\partial}{\partial x}[\alpha_1(\Phi,x)q] + \frac{\Omega^{1-2\nu}}{2!}\frac{\partial^2}{\partial x^2}[\alpha_2(\Phi,x)q] - \frac{\Omega^{1-3\nu}}{3!}\frac{\partial^3}{\partial x^3}[\alpha_3(\Phi.x)q] + \ldots \quad (1.41)$$

Since the leading term in Eq.-(1.41) are of order $\Omega^{1-\nu}$, equating the terms of this order on both sides, we get

$$\frac{d\Phi}{d\tau} = \alpha_{10}(\Phi) \quad (1.42)$$

where $\alpha_{10}(\Phi)$ is the value of $\alpha_1(\Phi,x)$ when $\Omega \to \infty$. Eq.-(1.42) governs the deterministic evolution of the process. Expanding $\alpha_k[\Phi+\Omega^{\nu-1}x]$, in the neighborhood of Φ_∞^c, the steady-state value of Φ at the critical point of the system, we find

$$\alpha_k\left[\Phi_\infty^c+\Omega^{\nu-1}\right]x=\sum_{j\ge 0}\frac{1}{j!}\left[\frac{\partial^j \alpha_k}{\partial \Phi^j}\right]_{\Phi=\Phi_\infty^c}[\Omega^{\nu-1}x]^j \quad \text{.......} (1.43)$$

Eq.-(1.43) forms the foundation of the extension of van Kampen's method as suggested by Fox [41] and Dekker [42]. If the first non-vanishing derivatives of the first second jump moments α_k $(k=1,2)$, at $\Phi=\Phi_\infty^c$ are of order m_1 and m_2 respectively, the fluctuations in the critical region will be determined by the equation :

$$\frac{\partial q}{\partial \tau}=-\frac{1}{m_1!}\alpha_1{}^{(m_1)}(\Phi_\infty^c)\Omega^{(1-\nu)(1-m_1)}\frac{\partial}{\partial x}[x^{m_1}q]+\frac{1}{2(m_1)!}\alpha_2{}^{(m_2)}(\Phi_\infty^c)\Omega^{(1-\nu)(1-m_2)-\nu}\frac{\partial^2}{\partial x^2}[x^{m_2}q]+\ldots$$

. ... (1.44)

since $0(x)=1$, the drift and diffusion processes will be of comparable significance only if

$(1-\nu)(1-m_1)=(1-\nu)(1-m_2)-\nu$, that is

$$\nu=\frac{m_1-m_2}{1+m_1-m_2} \quad \text{.......} (1.45)$$

Simultaneously, the critical slowing index μ, which implies that the approach of the system towards its equilibrium state is slowed down by a factor of order Ω^{μ}, is given

$$\mu = \frac{m_1 - 1}{1 + m_1 - m_2} \qquad (1.46)$$

In terms of the slowing index μ, Eq.-(1.44) assumes the form:

$$\frac{\partial q}{\partial \tau} = \Omega^{-\mu}\left[\frac{1}{m_1!}\alpha_1^{(m_1)}(\Phi_\infty^c)\frac{\partial}{\partial x}[x^{m_1} q] + \frac{1}{2(m_1)!}\alpha_2^{(m_2)}(\Phi_\infty^c)\frac{\partial^2}{\partial x^2}[x^{m_2} q] + \ldots\right] \qquad (1.47)$$

This expansion technique leads to a non-linear *FPE* which describes the critical region of the system quite adequately.

It will be worth mentioning that the van Kampen's original method of the system-size expansion corresponds to $m_1 = 1$ and $m_2 = 0$, hence to $\nu = \frac{1}{2}$ and $\mu = 0$. This implies that the original version is not adequate for studying a non-linear system in its critical region.

1.7 CATASTROPE THEORY:

The structural stability of a dynamic system depends on the parameters or structural constants appearing in the system of differential equations describing the system. A system that regains its topological structure after any small perturbation is referred to as a stable or a coarse system. Poincare [46], about a century ago, initiated the study of qualitative properties of the solution of ordinary differential equations involving three fundamental concepts : Structural stability, dynamic stability and critical sets, in terms of the parameters (constant) appearing in the differential equations. Morse [47] investigated the structure for canonical forms of a function near an isolated critical point and Whitney [48] examined canonical forms for mapping at singular points. During 1950's Thom [49] introduced the concept of transversality as a mechanism for discussing structural stability and then employed this tool to describe canonical forms for certain singularities of mapping $f:\Re^n \to \Re^1$, which he called catastrophe. Put succinctly, Thom's elementary catastrophe theory is the out come of the study of the equilibrium of dynamical systems that are derivable from a potential function and attempts to show how the qualitative nature of solutions of equations is dictated by the parameters of the equations.

It is a branch of mathematical topology developed by Rena Thom [50] which is concerned with the way in which nonlinear interactions within systems can produce sudden and dramatic effects.

Therefore catastrophe theory [51] is the mathematical modeling of sudden changes, so called "catastrophe", in the behaviuor of nature systems, which may appear in the split of continuous changes of the system parameters.

Four basic problems related to catastrophe theory are : (i) determinacy (ii) unfolding (iii) classification and (iv) globalization.

Here our limited aim is to convey the spirit of the mathematical aspect, without much rigor. An elaborate exposition may be consulted for finer details from Hirsch [52]. Hirsch and Smale [53], Arnold [54] and Gilmore[55]. Those desiring a more 'classical' language should consult Poston and Stewart [56] and Saunders[57].

1.8. RUDIMENTS OF CATASTROPHE THEORY :

A system of equation of the type

$$F_i = \frac{dx_1}{dt} - f_i(x_j; c_\alpha, t) = 0 \qquad \begin{matrix} i, j = 1, 2, \ldots, n. \\ \alpha = 1, 2, \ldots, k. \end{matrix} \qquad \ldots\ldots (1.48)$$

is called a dynamical system. Here x_i's describe the state of the system and are called state variables, c_α's are control variables and t stands for v the time variable.

The dynamical system Eq.-(1.48) is said to be autonomous if t does not appear explicitly, that is

$$F_i = \frac{dx_1}{dt} - f_i(x_j; c_\alpha) = 0 \qquad (1.49)$$

Further, if all the functions f_i can be obtained as a negative gradient of a potential function $V(x_i, c_\alpha)$, so that

$$f_i = -\frac{\partial V}{\partial x_{i\ i}}(x_j; c_\alpha) \qquad i = 1, 2, \ldots, n. \qquad (1.50)$$

then Eq.-(1.57)

$$\frac{dx_1}{dt} + \frac{\partial v}{\partial x_{i\ i}}(x_j; c_\alpha) = 0 \qquad i = 1, 2, \ldots, n. \qquad (1.51)$$

The resulting system represented by Eq.-(1.51) is called a gradient system. The equilibrium states or simply equilibriums are the solutions $\bar{x}_i$ of the equations

$$\frac{dx_1}{dt} = 0$$

or equivalently of

$$\frac{\partial v}{\partial x_{i\ i}}(x_j; c_\alpha) = 0 \qquad (1.52)$$

We would like to remark here that Eq.-(1.52) may or may not have a solution. The elementary catastrophe theory is the study of how equilibrium $\bar{x}_i(c_\alpha)$ of $V(x_j; c_\alpha)$ change with variations in the control parameters c_α, $\alpha = 1, 2, \ldots, k$

The local behaviour of a potential function is investigated, in most of the cases, by employing the theorems : Implicit function theorem, Morse lemma, and Thom's splitting lemma and theorem.

When $\nabla V \neq 0$, at a point in the state space, then by implicit function theorem, it is possible to choose a new coordinate system in the neighborhood of the point so that the force $(-\nabla V)$ has only one non-vanishing component, and hence we can transform the potential V in the form:

$$V \cong \xi_1 + \text{Constant} \quad \ldots (1.53)$$

Where the symbol $\cong$ means 'is equal to' after a smooth change of variables from $(x_1, x_2, \ldots, x_n)$ to $(\xi_1, \xi_2, \ldots, \xi_n)$. Let us recall that the constant term is of no importance when dealing with the local properties of a potential function.

If $\nabla V = 0$ at a point (so-called critical point), the implicit function theorem is of no consequence. Nevertheless, if $\det V_{ij} \neq 0$, where

$$(V_{ij}) = \left(\frac{\partial^2}{\partial x_i \partial x_j} \right)$$

is the stability matrix (or say Hessian matrix), then the Morse theorem guarantees the existence of a smooth change of variables

$(x_1, x_2, \ldots, x_n) \rightarrow (\xi_1, \xi_2, \ldots, \xi_n)$ such that the potential V in the neighborhood of the critical point can be written locally as a quadratic form

$$V = \sum_{i=1}^{n} \lambda_i \, \xi_i^2 \quad \text{.......... (1.54)}$$

Where λ_i are the eigen values of the Hessian matrix (V_{ij}) evaluated at the critical point. Further, by observing a 'length scale' into the new coordinate system

$$\eta_i = \sqrt{(|\lambda_1|)}\, \xi_i$$

the quadratic form of Eq.-(1.54) can be written in the Morse canonical form

$$V \cong -\sum_{i=1}^{k} \eta_i^{\;2} + \sum_{i=k+1}^{n} \eta_i^{\;2} \cong M_k^{\;n}(\eta) \quad \text{.......... (1.55)}$$

The potential function

$$V = M_k^{\;n}(\eta)$$

is called Morse k-saddle point. In passing we would like remark that only Morse 0-saddles have local minimum at the equilibrium (critical point), and hence only the 0-saddle are locally stable.

Further, if the potential function V depends on one more control variables $c_1, c_2, \ldots, c_k$, then the Hessian matrix (V_{ij}) and its eigen values $\lambda_1, \lambda_2, \ldots, \lambda_n$ also depend on these control variables. Quite possibly, one more of the eigen values $\lambda_i(c_\alpha)$ may vanish for certain values of the control variables. When this happens, then $\det V_{ij} = 0$, hence in this situation not only the implicit function theorem but the Morse lemma (49) also fails. However, it is still possible, in view of the Thom's splitting lemma, to obtain a canonical form at a non-Morse critical point. If ν eigen values $\lambda_1(\underline{c}), \lambda_2(\underline{c}), \ldots, \lambda_\nu(\underline{c})$, vanish at $\underline{c} = \underline{c}^0$, then Thom's splitting lemma may be used to spilt the potential into a non-Morse part and Morse part:

$$V(\vec{x}, \vec{c}) \cong f_{NM}\left[\xi_1(\vec{x}, \vec{c}), \xi_2(\vec{x}, \vec{c}), \ldots, \xi_\nu(\vec{x}, \vec{c})\right] + \sum_{i=\nu+1}^{n} \lambda_i(\vec{c})\, \xi_i^2(\vec{x}) \quad (1.56)$$

where $\lambda_1(\vec{x}, \vec{c}), \lambda_2(\vec{x}, \vec{c}), \ldots, \lambda_\nu(\vec{x}, \vec{c})$, are ν bad coordinates associated with ν vanishing eigen values $\lambda_1(\vec{c}), \lambda_2(\vec{c}), \ldots, \lambda_\nu(\vec{c})$, in the sense that control variables $c_1, c_2, \ldots, c_k$, while $\xi_{\nu+1}(\vec{x}), \xi_{\nu+2}(\vec{x}), \ldots, \xi_n(\vec{x})$ are $n-\nu$ good coordinates associated with $n-\nu$ non-vanishing eigen values $\lambda_{\nu+1}(\vec{c}), \lambda_{\nu+2}(\vec{c}), \ldots, \lambda_n(\vec{c})$, in the sense that they are sooth functions only in the original state variables $x_1, x_2,..., x_n$. At $(\underline{x}^0, c^0)$, the Hessian matrix $\left[\frac{\partial f_{NM}}{\partial \xi_i \partial \xi_j}\right]$, $(1 \le i, j \le \nu)$ vanishes, as all of its entries are zero at $(\underline{x}^0, \underline{c})$, while the Hessian matrix of the Morse part is non-singular. Under appropriate conditions, Thom's theorem ensures the existence of a smooth change of variables so that the potential can be written in the following canonical form

$$V \cong CG(\nu) + \sum_{i=\nu+1}^{n} \lambda_i \xi_i^2 \quad (1.57)$$

The function $CG(\nu)$ is called a catastrophe germ.

A few remark would be perfectly in order about the critical point. In parlance of mathematics a point at which $\nabla V = 0$, however $\det V_{ij} \neq 0$, is referred to as a critical point or Morse critical point or just normal critical point : A point at which $\nabla V = 0$, also $\det V_{ij} = 0$ is referred to as a Non-Morse critical point. In our Thesis we shall be referring a normal critical point as an equilibrium point and degenerate critical point as a critical point.

Further, in our work we shall be dealing with only one state variable and the number of control variables shall be at most, that is, $n = 1$ and $k \le 2$, therefore, in this case the germ of the potential functions $\pm x^4$, while the perturbation function being ax or $ax+bx^2$ (a and b will be here control variables).

1.9 CONVENTION AND FLAGE OF CATASTROOHE :

In order to discuss the dynamics of real world system, we impose some assumptions of physical of intuitive nature such as quasi-state evolution or adiabatic evolution in the sense that all time derivatives are very small with this assumption two conventions are widely adopted in catastrophe theory. They are : Delay convention and Maxwell's convention.

According to the delay convention, the system stays in a stable or unstable equilibrium state until that state disappears. However, according to the Maxwell's convention the system stays in the state that globally minimizes the potential.

In quantitative terms, suppose that H represents the characteristic height of the potential, which separates a meta-stable state from a nearly state, and N represents the level of intrinsic or extrinsic sources of stochasticity, then the delay convention is applied when $\frac{N}{H} << 1$, that is, noise is far less than the characteristic height of separation; however, the Maxwell's convention is to observed when $\frac{N}{H} \cong 1$, that is the noise level is of comparable significance with separation height.

In passing, we would like to remark that these conventions are not intrinsic to catastrophe theory; they rather provide with means by which the canonical mathematics of elementary catastrophe theory is made available to applications in various disciplines. In the word of Gilmore [55]. “The convention are adhoc, the price is incompleteness in the description and the rewards may be enormous”.

There are some remarkable features of a physical which immediately suggest the presence of a catastrophe. These features have been referred to as flags of a catastrophe, and are as follows:

(F1) Modality,

(F2) In accessibility,

(F3) Sudden Jumps,

(F4) Divergence,

(F5) Hysteresis,

(F6) Divergence of Linear Response,

(F7) Critical Slowing down or Model Softening and

(F8) Anomalous Variances.

The first five generally occur in conjunction with each other. These are manifested when two distinct regions are available in the parameter space wherein the potential function becomes locally minimum. Hysteresis may not be observed if the Maxwell's convention is observed, however, even in this, it is sometimes possible by careful experimental techniques to observe it.

For example in physical, science super cooling or b super heating may yield the result. The remaining three flags may be observed even when the potential has only one local minimum. These three flags are of paramount importance in decision making. In numerous cases related to different disciplines safety margins on the control parameters may be achieved and there by sudden jumps (catastrophe) may be avert. We have invariably used these flags to suggest to recipe for managing forestry.

CHAPTER-TWO

QUALITATIVE ANALYSIS OF SPRUCE BUDWORM PEST AND FOREST

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2.1 INTRODUCTION:

Forests constitutes an important category of renewable resources. They are important not only for agricultural and industry but also for several other activities and ecological balance. They cover a large proportion of the land surface of the earth. On one hand, though photo-bio-chemical processes, they entrap enormous amount of solar energy and store in the form of valuable resources for the humanity, on the other hand, they consume carbon-dioxide (a challenging pollutant growing with industrialization) and liberate oxygen for the sustenance of the aerobic world. They enrich the fertility of the soil through a constant supply of decaying matter, make the soil structure porous and conducive to retention and seepage of the precipitation received and thereby augment the underground water supply. Put succinctly, forests have a great bearing on the ground water supplies, soil erosion, climate regulation and flood control. Some potential impact on some environmental factors are discussed in the next section.

2.2 POTENTIAL IMPACTS OF FORESTRY ACTIVITIES WITH EMPHASIS ON THE TROPICS: BY ENVIRONMENTAL ASPECT OR SOCIO-ECONOMICONCERN

Soil: Soil may have lost through rill, gully or shoot erosion, may become prong to rapid leaching of nutrients; may have rapid initial lose of the organic matter, followed by stabilization as soil organisms responsible for decay decrease in numbers; may become indurated as a result of laterization; micro-flora bad fauna may decrease or may be altered through exposure to full sunlight; in turn, changes in micro-organismic life may detrimentally affect decomposition and nutrient transfer, disappearance of the mycorrhizae may, in particular, retard or prevent the re-establishment of the many tree species that feed symbiotically with these soil fungi; organic matter increase under forest plantations, with beneficial effects on soil structure, infiltration capacity, soil-moisture holding capacity and caution-exchange capacity.

Sedimentation: Sediment load in steams made increase with adverse effect on channel stability, navigation, fish spawning, bottom organisms, light penetration and other aspects of aquatic life, accelerated sedimentation may shorten the useful life span of reservoirs; on sloping ground, sediment from logged over areas may bury the roots of the adjacent uncut forest or the crops on nearby field; re-and afforestation may decrease sediment loads and thus reverse the negative effects described above.

Water Resources: Denuded slopes, compacted soil and decreased infiltration and canopy interception may lead to larger volumes of storm runoff and to quicker responses of runoff to precipitation; despite lower infiltration of rainfall, base flow may increase locally after de-forestation owing to decreased transpiration; however large-scale deforestation usually result in lower down-base low flows because of decreased infiltration and groundwater recharge, greater storm runoff and increase evaporation; smaller discharges may interfere with down-stream withdrawals for agriculture and domestic use. These effects may be reversed with re-or afforestation.

Climate and Air Quality: Logging may increase ground temperatures and lower atmospheric humidity locally which, in turn, may interfere with seedling growth and micro-organismic life in the soil; large-scale deforestation may cause regional desiccation of the climate as transpiration is decreased and local convection patterns.

Wildlife and Fisheries: Logging can injure and kill some animals outrigh but more likely it damages or destroys key habitats such as nesting sites, including old hollow trees, feeding and breeding ground. It can also interrupt or eliminate the aerial pathways of arboreal species that seldom move at ground level. Some endemic species of animals could be eliminated altogether.

Demographic-Economic Expansion: Forestry projects can stimulate the local cash economy through direct and indirect employment and increased demand for goods and service. These projects can also result in improved facilities such as new or better road, medical facilities, schools etc. A larger scale to new settlement created by the influx of people directly or indirectly employed in the forestry sector.

Epidemiology: Forest removal may increase the rates of the incidence of certain diseases or introduce new diseases such as malaria and scrub typhus. Any water impoundment associated with forestry could lead in certain regions, to outbreaks of schistosomiasis or onchocerciasis. Destruction of forestry may bring forest arthropod vectors of the arbovirus diseases into closer contact with man. On the other hand, clearing of riparian forest is used to control trypanosomiasis. The influx of the forest workers and outsiders may increase the incidence of certain diseases such as trypaosomiasis sexually transmitted diseases. On the other hand, medical facilities and standards established in connection with forestry project can significantly improve local or regional health conditions [1] .

2.3 NATURE AND DEVELOPMENT OF SPRUCE BUDWORM:

The spruce budworm (choristoneura fumiferana) is a forest pest insect. It is a inborn defoliator of North American coniferous forest whose size is immense and their activity impact on a regular basis over extensive forest areas at least past three centuries [2-6]. Outbreaks of the spruce budworm occur in intervals of 0-40 years and it is periodically destroy most canopy balsam fir, which is a tall, pointed tree and usually an evergreen that has their needle-like leaves, over large areas and release small tree in the seedling bank [7-11]. Moreover periodic outbreaks of such defoliating insect cause major growth reduction but not destroy most of the trees. Last outbreaks were in 1910, 1940, and 1970. Also there are another type of insect C. biennis has been found in the forest northern British Columbia and further appears about 32 years [12-19].

In every year the spruce budworm gives one generation. The female moth which is as like as similar to butter flies and that flies lay eggs on the flat under surface of balsam fir or spruce needles, generally within 3 inches of the buds or defoliated areas. When the populations are extremely high, eggs may be laid on almost any surface. The eggs hatch in 14 days. Generally, newly hatched larvae immediately search a suitable place to spin their hibernacular. However, during hot season, the larvae begin to move about and feed on needles before spinning a hibernacula. In this stage they may spin down from a branch on silken thread used in sewing dispersed by wind flow. At this time the young larvae transforms into second phase within hibernaculum's and remains dormant all through the winter.

When spring start and weather become hot and balsam fir bud not begin to expand, then the larva emerges from hibernation and start feeding. At the first stage, feeding is confined to the new buds of staminate flowers, in case the new buds of the staminate flowers are available. If the staminate flowers are rare, then the larvae depend on previous year's needle. The new flower buds provide ready source of food before the vegetative buds expand. The larvae which are early emerging that feed on staminate flower buds grow much rapidly and have higher survival rate, than those, which feed on old needle.

In this stage the larvae advance to the end of a twig and bores into a needle or an expanding vegetative bud. On the other hand, some larvae spin down on silken threads and at first instars larvae may be dispersed by wind flow. The larvae feeding on staminate flower buds and flowers stay in until the immediate food supply is depleted and after this stage the larvae feed on the new foliage of developing shoot. In this way, when the larvae is reaches in fifth instars, it start tying the tips of twigs together with silk and finally formatting a small nest. At this stage the new foliage is eaten first. In epidemic situation old needle and bark may also eaten in such a proportion that branch tips and terminal shoots are ruined. During the last days of the June to middle of the July, the larva completes its development and finished its feeding.

At the last, the larvae transforms to pupa and some pupa are found base of needle of the twigs. And the moth emerges about 10 days. Peak moth flight activity occurs from about evening to end of the mid night. From this time the moth may be transforms up to 10 miles or more by wind flow and can be reached of 100 miles by storm fronts.

From the foregoing life history of the spruce budworm, we can understand that, it is a higher reproductive insect. Moreover favorable weather, especially warm and spring, sufficient food and suitable hibernation site can lead to an outbreak.

If the budworm reaches to an epidemic level, environmental factor such as adverse weather, disease, and predator can not stop the outbreak. On the other hand, once spruce budworm outbreak start it usually continuous until the consume much of the available foliage. In this stage the use of biological and chemical insecticides may be urgent to supplement natural control agent. A close scrutiny of the situation reveals that the predation itself follows a natural pattern. Initially the budworms so few that the birds can not easily locate them. At a later stage, the population attains a good size, as the size of the population tends the carrying capacity of the system (the available leaf-area) their rate of reproduction declines due to defoliation, Fig.-2.1 presents a schematic description of this situation.

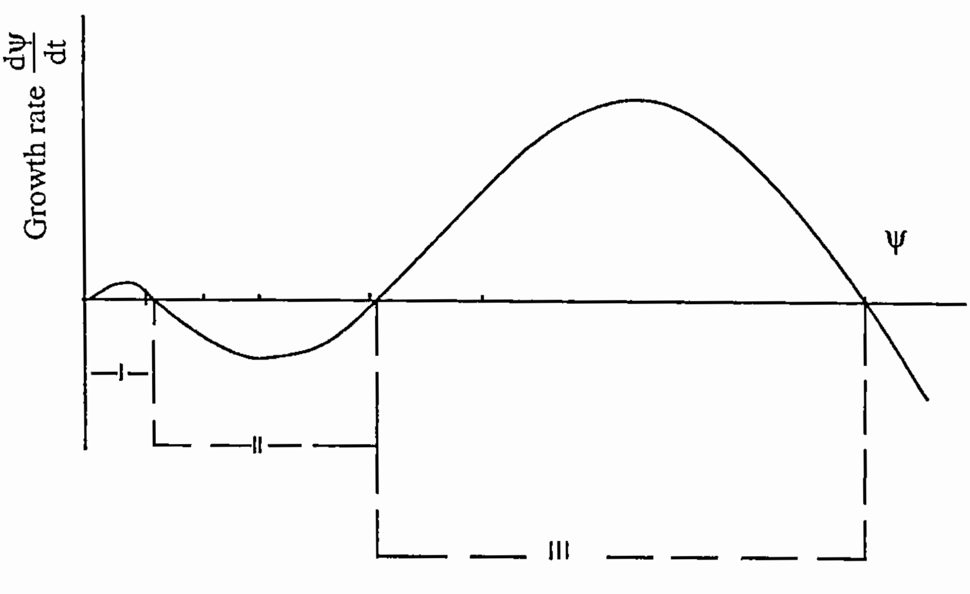

Fig.-2.1: Curve shows the effect of predation on the growth rate $\left[\frac{d\psi}{dt}\right]$ of budworm population at various levels of the population ψ. Regions marked I, II and III represent the three stages discussed in the text. (Not to scale)

2.4 THE BUDWORM EQUATION AND ITS EQUILIBRIA

THE LUDWIG, JONES AND HOLLING'S NON-LINEAR DETERMINISTICS MODLE:

The maintenance, proper management and protect North American coniferous forest from spruce budworms in 1978, Ludwig, Jones and Holling [20] propose a nonlinear deterministic differential equation, which satisfies by local budworm B is

$$\frac{dB}{dT} = rB(1-\frac{B}{K'S}) - \beta\frac{B^2}{(\alpha' S)^2 + B^2} \quad \text{.......................(2.1)}$$

Here the parameters and variable are as follows: K' is the carrying capacity, S is the area of the branch surface, β is the consumption rate of the predators which kill the spruce budworm. The parameters $\alpha' S$ are the density of the budworm.

The right hand side of the Eq.-(2.1) is consisting with two parts. The first part of the Eq.-(2.1) is a logistic term where involves $K'S$ which indicates that K' is proportion to the branch surface area S. The wideness of the surface area is measure of the size of the balsam fir tree and the evergreen foliage is the main source of the food of the budworm.

The second part on the right hand side of the Eq.-(2.1) gives the rate of the consumption of the budworm by the predators, one kind of birds and they kill and eat the spruce budworm. But this predators have a limited numerical response and individual the predators have a fixed eating capacity and for this reason at high levels of budworm B, the second term saturates to the consumption rate β. At low budworm densities, the predators can not locate the budworm easily then the consumption rates drops sharply because the predators switch to alternate pray. The parameter $\alpha's$ gives the density of budworms at which predators consume at half the saturation rate. It will be demonstrated these saturation and switching effects can create several stable equailibriums in the spruce budworm population.

The units of the measurement which use in the Eq.-(2.1) are arbitrary: they will never affect our next dimensionless version. In the Eq.-(2.1), Ludwig, Jones and Holling [20] introduce the units are as follows:

(i) B, the density of the budworm is measured in larvae per acre which feed on staminate flower buds.

(ii) T and $\frac{1}{r}$ are measured in years.

(iii) S is the surface area of branches which is measured in branches per acre. Therefore K' and α' must be measured in larvae per branch.

(iv) β, the consumption rate is measured in larvae per years.

These units suggest several possible scaling for the budworm density. The combination $K'S$, $\alpha'S$ and rB all of these are used in the Eq.-(2.1) are measured in larvae per acre.

In this stage we introduce the dimensionless quantities:

$$\left.\begin{aligned} \psi &= \frac{B}{\alpha' S} \\ or,\quad B &= \alpha' S \psi \\ or,\quad dB &= \alpha' S d\psi \end{aligned}\right] \qquad \left.\begin{aligned} R &= \frac{r\alpha' S}{\beta} \\ \beta &= \frac{r\alpha' S}{R} \end{aligned}\right]$$

$$\left.\begin{aligned} rT &= t \\ or,\quad T &= \frac{1}{r}t \\ or,\quad dT &= \frac{1}{r}dt \end{aligned}\right] \qquad \left.\begin{aligned} Q &= \frac{K'}{\alpha'} \\ or,\quad K' &= Q\alpha' \end{aligned}\right]$$

Put these values in the Eq.-(2.1) we have ,

$$\alpha' Sr\frac{d\psi}{dt} = r\psi\alpha' S(1-\frac{\psi\alpha' S}{\alpha' SQ}) - \frac{r\alpha' S}{R}\cdot\frac{\psi^2\alpha'^2 S^2}{(\alpha' S)^2+\psi^2\alpha'^2 S^2}$$

or, $$\alpha' Sr\frac{d\psi}{dt} = \alpha' Sr\psi(1-\frac{\psi}{Q}) - \frac{r\alpha S}{R}\cdot\frac{\psi^2}{1+\psi^2}$$

or, $$\frac{d\psi}{dt} = \psi(1-\frac{\psi}{Q}) - \frac{1}{R}\cdot\frac{\psi^2}{1+\psi^2}$$

or, $$\frac{d\psi}{dt} = \phi(\psi;R,Q) \qquad (2.2)$$

Where $$\phi(\psi;R,Q) = \psi - \frac{\psi^2}{Q} - \frac{1}{R}\cdot\frac{\psi^2}{1+\psi^2} \qquad (2.3)$$

In the Eq.-(2.1), we introduce the parameters R and Q and the variable ψ. The variable ψ have been used in the above equation for the purpose to make predation part most simply. If the surface area S is increase or decrease then R is also increase or decrease. So R varies with the surface area S. However, the parameters Q depends upon the properties of the budworm and predators. The qualitative nature of the solution of the Eq.-(2.1) or Eq.-(2.2) will be observed by the parameters R and Q, though Q does not depend Eq.-(2.1) upon the forest condition.

As we are interested about the qualitative nature of the solution of the Eq.-(2.1) or Eq.-(2.2), and for this purpose we first locate and describe the equilibrium of the Eq.-(2.2). It is clear to us that the equilibrium depends upon ψ for which $\phi(\psi;R,Q)=0$ and it is to be mentioned that one such equilibrium is for $\psi=0$, however it is always unstable because $\phi(\psi;R,Q)>0$ for all sufficiently small positive value of ψ. The remaining zeros of ϕ satisfy

$$R(1-\frac{\psi}{Q}) = \frac{\psi}{1+\psi^2} \qquad (2.4)$$

This is an equation of curve and we indicate it by C, given by

$$v = k(\psi)$$

where $k(\psi) = \dfrac{\psi}{1+\psi^2}$

and $v = R(1-\dfrac{\psi}{Q})$ ....(2.5)

It is straight forward that v is a equation of straight line and we denoted it by L_R. However Eq.-(2.5), ψ and v are variables and R and Q are parameters. Eq.-(2.4) is cubic equation and the curve C and straight line L_R intersect at most three points and these are the roots of the Eq.-(2.4). We graphically solve the Eq.-(2.4). The straight line L_R depend upon the value of Q and R. Q depend upon the properties of the budworm and predators but we omitted this, since we are specially interested to investigate in the behavior of the roots of the cubic Eq.-(2.4) keeping Q fixed and R varies since R is related with the forest condition. Now multiplying the Eq.-(2.4) by $(1+\psi^2)$ and rearrange it we obtain

$$\psi^3 - Q\psi^2 + \psi(1+\frac{Q}{R}) - Q = 0 \quad \text{..........(2.6)}$$

Eq.-(2.6) is a cubic equation that will provide three roots which are positive or negative and real or complex and be discussed in later.

2.5 THE BEST FIT OF REGRESSION LNNE:

If we draw a graph of $k(\psi)$ vs ψ where ψ is the explanatory variable represents the population size following estimated parameters value $Q=17.72$ and $R=.28$, the corresponding estimated regression line is

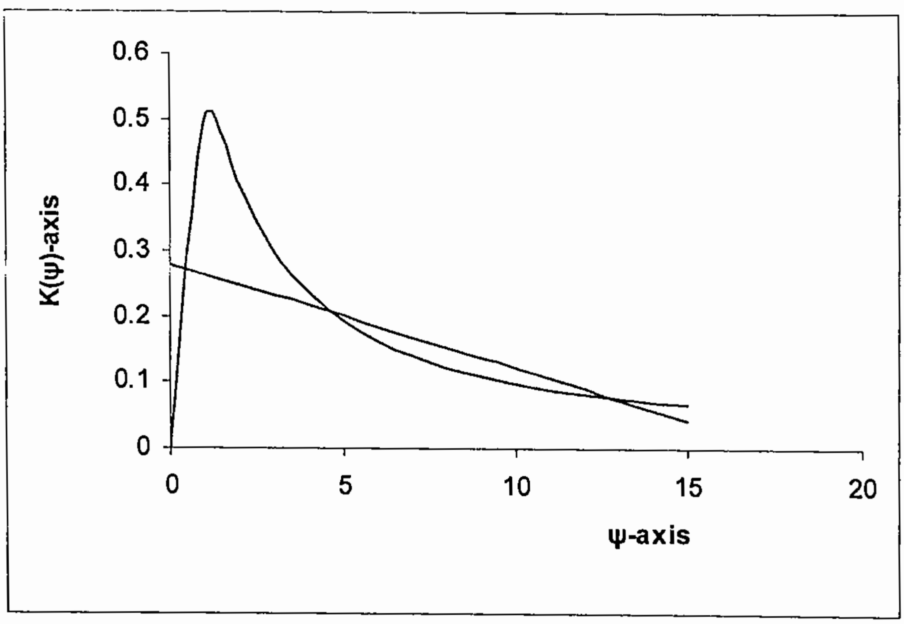

Fig-2.2: Depicts the relationship between line L_R and the curve C for the value of control parameter $Q=17.72$ and $R=0.28$.

$y=-0.0158x+0.28$ ....(2.7a)

and $R^2=0.3763$.

Another set of parameters value $Q = 17.0578$ and $R = 0.2951$ and

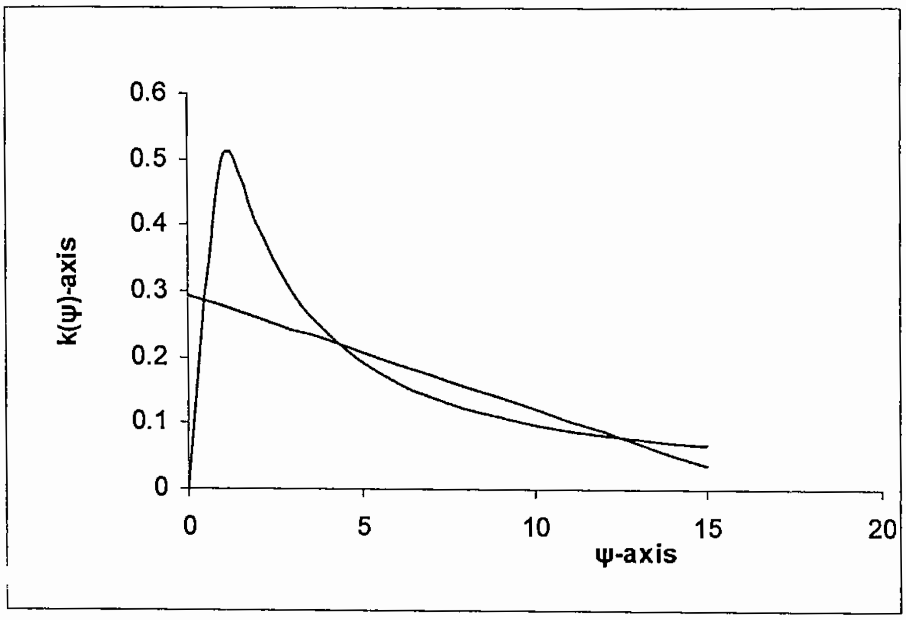

Fig-2.3 : Depicts the relationship between line L_R and the curve C for the value of control parameter $Q = 17.0578$ and $R = 0.2951$.

the estimated regression line is

$$y = -0.173x + 0.2951 \quad \text{...(2.7b)}$$

and $R^2 = 0.3801$.

Further when the parameters value $Q = 16.85393$ and $R = 0.30$,

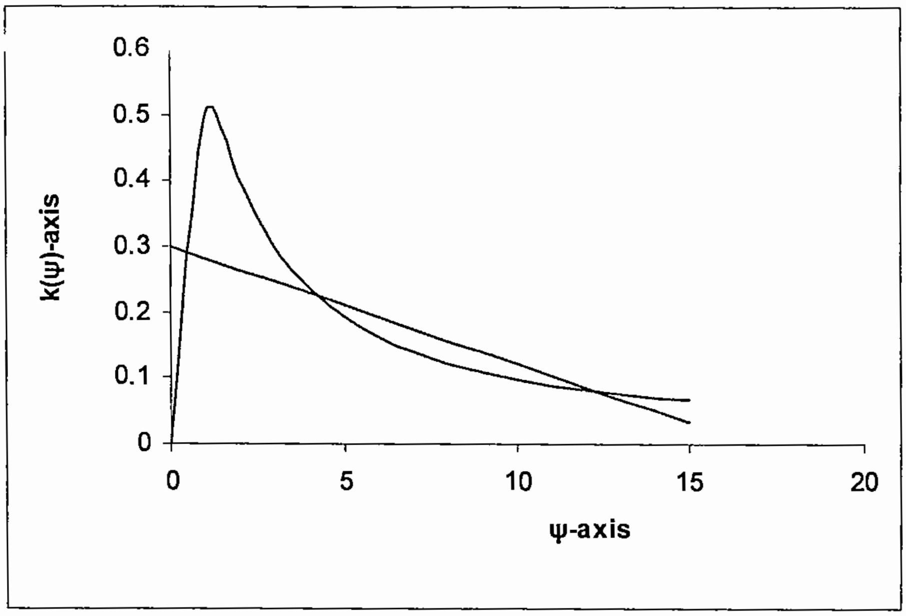

Fig-2.4 : Depicts the relationship between line L_R and the curve C for the value of control parameter $Q = 16.85393$ and $R = 0.30$.

the estimated regression line is

$$y = -0.0178x + 0.3 \qquad (2.7c)$$

and $R^2 = 0.3797$.

We summarized the value of Q, R and R^2 in the Table-2.1.

Table-2.1: Coefficient of determination R^2 for different set of control parameter (Q, R) is cited in table.

Fig. No.	Control parameter Q	Control parameter R	Coefficient of determination R^2
2.2	17.72	0.28	o.3763
2.3	17.0578	0.2951	0.3801
2.4	16.85393	0.30	0.3797

From the Fig.-(2.2) to Fig.-(2.4) we have $R^2 = 0.3763$, $R^2 = 0.3801$ and $R^2 = 0.3797$ and the highest value of R^2 received from Fig.-(2.3) which is 0.3801. From the properties of best fit of regression line, we may conclude that when the control parameters $Q = 17.0578$ and $R = 0.2651$ the regression line is best. Moreover line is intersects the ψ-axis at $A(17.0578,\ 0)$ and $k(\psi)$ axis at $B(0,\ 0.2951)$.

The Fig.-(2.3) represents the regression line L_R intersect the curve at three points clearly. So the Eq.-(2.4) must have three roots and either one or three nonzero. Again the line L_R and the curve C intersect at three point in the first quadrant and the curve C is always situated in the first quadrant, so the Eq.-(2.4) must have three nonzero positive roots.

2.6 DETERMINATION OF THE NATURE OF ROOTS OF THE CUBIC EQUATION:

From the Eq-(2.6) we have,

$$\psi^3 - Q\psi^2 + \psi S - Q = 0 \qquad (2.8)$$

Where, $S = 1 + \frac{Q}{R}$

We have from the best fit of the regression line, $Q = 17.0578$ and $R = 0.2951$.

So $S = 58.80436$.

Substitute the values of Q, R and S in Eq.-(2.8) we have,

$$\psi^3 - 17.0578\psi^2 + 58.80346\psi - 17.0578 = 0 \qquad (2.9)$$

Compare Eq-(2.9) with the general cubic equation,

$$ax^3 + bx^2 + cx + d = 0 \qquad (2.10)$$

we get $a = 1$, $b = -17.0578$ $c = 58.80346$, $d = -17.0578$

Put $x = y - \frac{b}{3a}$ in Eq.-(2.10) this is a transformation that increases the roots by $\frac{b}{3a}$.

Hence,

$$a\left(y - \frac{b}{3a}\right)^3 + b\left(y - \frac{b}{3a}\right)^2 + c\left(y - \frac{b}{3a}\right) + d = 0 \qquad (2.11)$$

Now simplify Eq.-(2.11) we have,

$$y^3+\frac{3ca-b^2}{3a^2}y+\frac{2b^3-9abc+27a^2d}{27a^3}=0$$

or, $y^3+py+q=0$ ....(2.12)

where, $p=\frac{3ca-b^2}{3a^2}$, $q=\frac{2b^3-9abc+27a^2d}{27a^3}$

The discriminate of the cubic equation, Eq.-(2.12) is denoted by D and define by $D=\frac{p^3}{27}+\frac{q^2}{4}$.

If the coefficients of the given cubic equation are real numbers then,

(i) the equation have one real root and two imaginary roots if $\frac{p^3}{27}+\frac{q^2}{4}>0$.

(ii) the equation have three real and distinct roots if $\frac{p^3}{27}+\frac{q^2}{4}<0$.

(iii) the equation have three real roots but two are equal if $\frac{p^3}{27}+\frac{q^2}{4}=0$.

and $p^3+q^2\neq 0$.

(iv) the equation have three real roots but all are equal if $\frac{p^3}{27}+\frac{q^2}{4}=0$.

and $p=q=0$.

Therefore, $p=\frac{3ca-b^2}{3a^2}=-38.1861$ and $q=\frac{2b^3-9abc+27a^2d}{27a^3}=-50.3559$

Finally, we have, $\frac{p^3}{27}+\frac{q^2}{4}=-1428.37$.

Which is negative and it holds the property (*ii*), thus the cubic Eq.-(2.9) provides three real and distinct roots. Which informs that best fit of regression provided the nature of the roots of the cubic equation [21].

2.7 ANALYSIS OF THE MODEL:

It is natural to define the zeros of Eq.-(2.4) are the points of intersection of the curve C and the straight line L_R. Thus Eq.-(2.4) can be solved graphically, as shown in Fig.-2.4 to Fig.-(2.6). Of course, the straight line Eq.-(2.5) depends upon Q and R are both. In fact we avoid Q, as we are specially interested the role of the roots of the designed cubic function $\phi(\psi;R,Q)$ in agreement with Eq.-(2.1) when Q is fixed and R varies. This operation is because R changes with the forest conditions but Q does not depend upon the forest conditions, rather it depends upon the behaviours of the attitude of the budworm and predators. In this connection, for various values of Q we have drawn lines L_R which intersect the curve C at one, two or three points. This situations arise for a fixed value of Q, when R varies. But if Q moves to left and fixed for 3.655 then for any value of R, the line L_R cut the curve C in one point only, which has been shown in Fig.-2.4. The drawn line through the points (3.655, 0) and $(0, 0.9)$ for the fixed value $Q = 3.655$ provides $R = 0.9$.

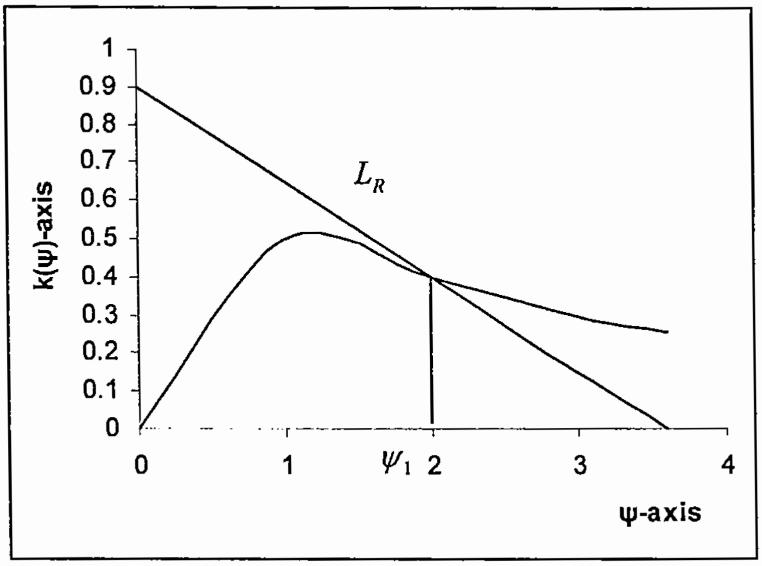

Fig.-2.5: Depicts the relationship between the straight line L_R and the curve C, where $Q = 3.655$ and $R = 0.9$. Graph shows that the line intersect the curve C at only one point.

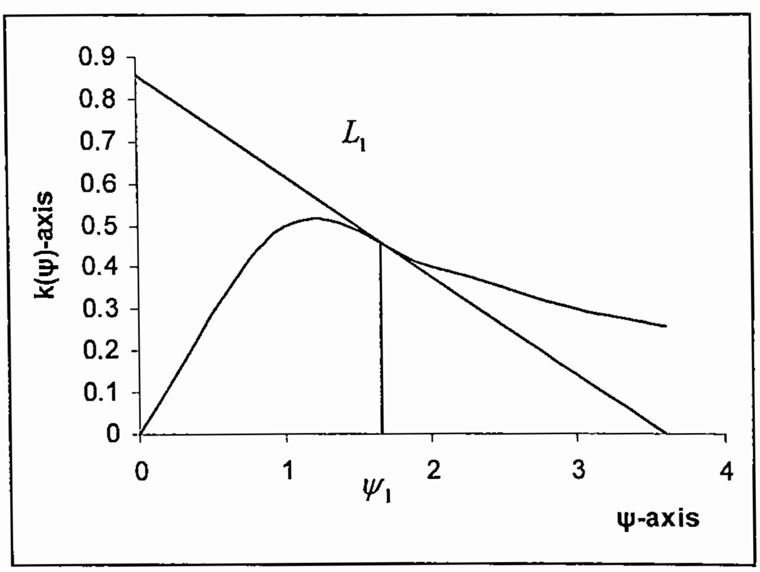

Fig.-2.6: Depicts the relationship between the straight line L_1 and the curve C, where $Q = 3.655$ and $R = 0.855$. Graph shows that the line intersect the curve C at only one point.

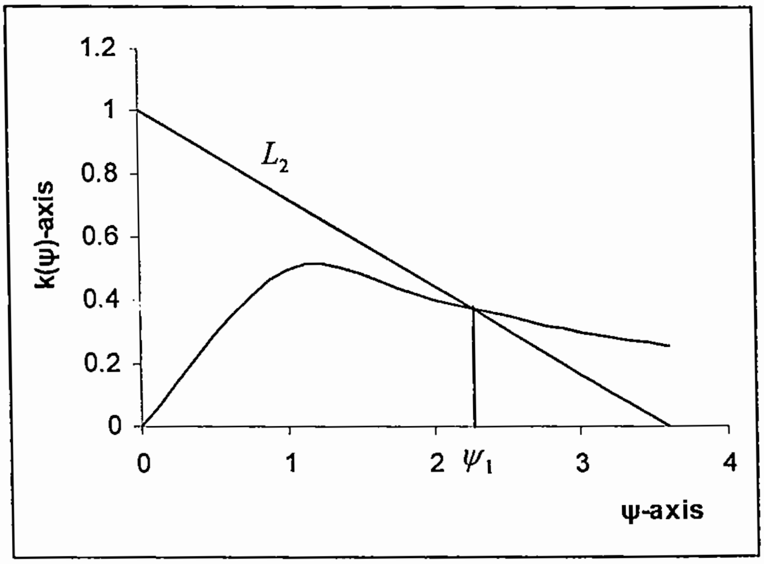

Fig.-2.7: Depicts the relationship between the straight line L_2 and the curve C, where $Q = 3.655$ and $R = 1$. Graph shows that the line intersect the curve C at only one point.

Impelled by this analogy, we thought it worthwhile to investigate $\phi(\psi\,;R,Q)$ along similar lines. The findings of this investigation turn out to be quite striking. The Fig.-2.5 indicates that $\psi_1 = 2$ and on the basis of this observed value, $\phi(\psi\,;R,Q)$ have been listed in Table-(2.2) for various values of ψ.

Table-2.2: $\phi(\psi\,;R,Q)$ is positive when $\psi<\psi_1$ and $\phi(\psi\,;R,Q)$ is negative when $\psi_1<\psi$ for a particular set of control parameters (R,Q) when the lower root, ψ_1 of the cubic equation is to be exercised. This table shows that ψ_1 is a stable equilibrium.

$\psi_1<\psi$	$\phi(\psi;R,Q)$	$\psi<\psi_1$	$\phi(\psi;R,Q)$
2.2	-0.09353	1.8	0.019799
2.4	-0.17250	1.6	0.058536
2.6	-0.26839	1.4	0.08929

Where $R=0.855$, $Q=3.655$ and $\psi_1=2$

Keeping fixed $Q=3.655$ and the two lines L_1 and L_2 are obtained through the points $(0,0.855)$ and $(0,1)$. Therefore from Fig.-(2.5) two Fig.-(2.7) we see that the line L_R cut the curve one point when minimum and maximum values of R are considered. It is clear that for fixed value $Q=3.655$ and any value of R, $0<R<\infty$, the line L_R cut the curve C at one point only and never form a tangent to C. In this circumstance there is exactly one positive real root of the Eq.(2.4), denoted by $\psi_1(R,Q)$ and $\psi_1<Q$ and $Q^*=3.655$ Ludwig and et al. [20] argued that if $Q<3\sqrt{3}$, then there will not any tangencies between the curve C and the line L_R, however our analysis states that $Q^*=Q=3.655$ (a particular value) only then tangencies will not formed between C and L_R.

It is remarkable to note that when ψ moves to the right of $\psi_1 = 2$, that is, $\psi_1 < \psi$, $\phi(\psi\,;R,Q)$ is negative and when ψ decreases below 2, that is, $\psi < \psi_1$, $\phi(\psi\,;R,Q)$ become positive. These findings suggest that the root $\psi_1(R,Q)$ is a stable equilibrium.

Further to monitor the qualitative behaviour of the system, the imposed conditions on control parameters may be determined when R varies and Q fixed. Moreover, when interpreted graphically the maximum and minimum values of the cubic expression cut axes at separate points:

(a) **When Q is Sufficiently Large:**

For a fixed value of Q, there are two distinct values of R for which the line L_R is tangent to the curve C. Suppose $R_1(Q)$ and $R_2(Q)$ are the values for which the tangencies formed at the upper and lower positions in Fig.-(2.8). It is clear that $R_1(Q)$ is greater than $R_2(Q)$, while lines are drawing through tangencies to the $\psi - axis$, they cut two points, namely $\overline{\psi}_1(Q)$ and $\overline{\psi}_2(Q)$ which are the ψ-coordinates of the corresponding points of the tangency and shows $\overline{\psi}_1(Q) < \overline{\psi}_2(Q)$.

The analysis argue that higher value of $R = R_1(Q)$, forced lower value of $\psi(Q) = \overline{\psi}_1(Q)$ and conversely, the lower value of $R = R_2(Q)$ provides higher value of $\psi(Q) = \overline{\psi}_2(Q)$. It follows that for each value of ψ there is a unique line L which passes through the points $(\psi, k(\psi))$ on C and the corresponding value be $R(\psi)$. Following earlier argument, further Fig.-2.8 states that,

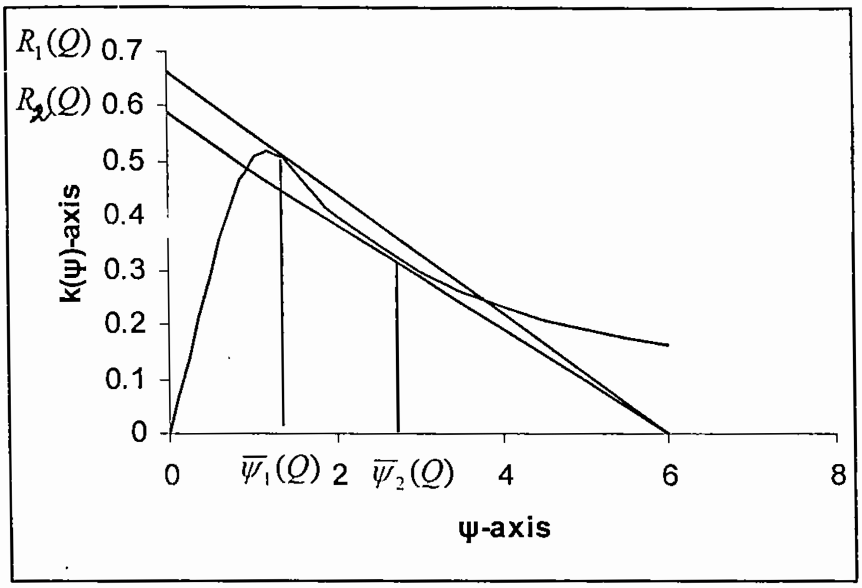

Fig-2.8: Drawn the upper and lower tangent from the fixed point on the curve and the upper tangent passes through $(6, 0)$ and $(0, 0.664)$ and the lower tangent passes through $(6, 0)$ and $(0, 0.584)$.

(i) $R(\psi)$ increases from 0 to $R_1(Q)$ as ψ increases from 0 to $\overline{\psi}_1(Q)$.

(ii) $R(\psi)$ decreases from $R_1(Q)$ to $R_2(Q)$ as ψ increases from $\overline{\psi}_1(Q)$ to $\overline{\psi}_2(Q)$ and finally,

(iii) $R(\psi)$ increases from $R_2(Q)$ to infinity as ψ increases from $\overline{\psi}_2(Q)$ to Q.

(b) When Q is Sufficiently Small:

In this situation the straight line L, never tangent to the curve, C and R is a monotonically increasing function of the population size, ψ.

(c) **When Q is Moderate:**

In this context, Fig.-2.8 reflects that the curve, C is convex up at $\psi(Q)=\overline{\psi}_1(Q)$ and convex down at $\psi(Q)=\overline{\psi}_2(Q)$. Suppose Q moves to the left until the upper and lower tangents coalesce, that is, our interest is to determine a value of Q, say $\overline{Q}$ such that $\overline{\psi}_1(Q)=\overline{\psi}_2(Q)$, which depends upon the nature of the roots of cubic equation.

Since we have,

$$k(\psi)=\frac{\psi}{1+\psi^2}$$

so, $$k'(\psi)=\frac{1-\psi^2}{(1+\psi^2)^2}$$

and $$k''(u)=\frac{-2\psi(3-\psi^2)}{(1+\psi^2)^3} \qquad \text{.........................(2.13)}$$

It is clear that $\overline{\psi}_1(Q)$ is a limit point as the second derivative is negative and simultaneously the second derivative is positive when $\overline{\psi}_2(Q)$ is also a limit point. However, $k''(\psi)=o$ at that is, roots are repeated and the curve touches the ψ-axis.

Further, the equation which we have constructed in Eq.-(2.4), provides $\overline{\psi}_1(Q)=\overline{\psi}_2(Q)=\sqrt{3}.$ With this aid we have $k(\psi)=\frac{\sqrt{3}}{4}$. As we have achieved the most important point $(\sqrt{3},\frac{\sqrt{3}}{4})$, so we can easily construct the tangent line at this point, $\overline{Q}=3\sqrt{3}\cong 5.2$ and accordingly $R_1(\overline{Q})=R_2(\overline{Q})=\frac{3\sqrt{3}}{8}\cong 0.65$.(see Fig.-2.8)

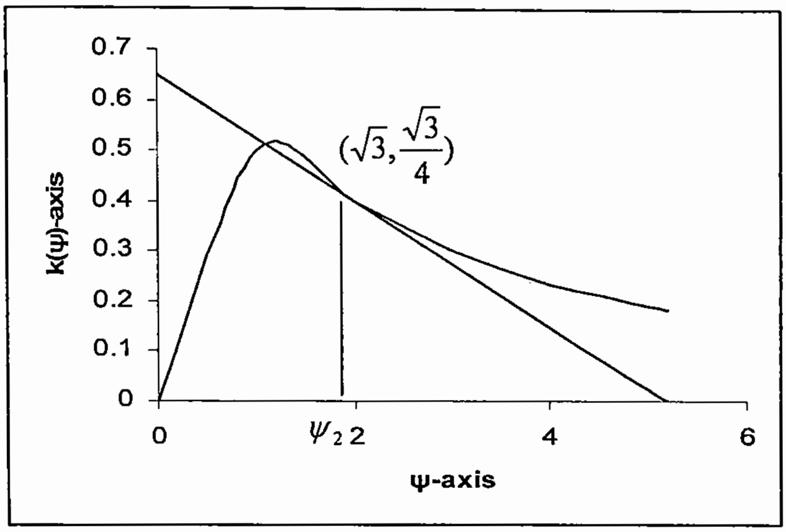

Fig.-2.9: Depicts the relationship between the curve C and straight line L_R which is tangent at $\left(\sqrt{3},\frac{\sqrt{3}}{4}\right)$.

It is obvious that through the point $(5.2, 0)$ and $(0, .65)$ the tangent line touches the curve C at $(\sqrt{3},\frac{\sqrt{3}}{4})$. For $Q>3\sqrt{3}$ we have drawn two tangent for $\psi=\overline{\psi}(Q)$ when $R=R_1(Q)$ and $R=R_2(Q)$.

Further, from Fig-2.7 we see that if $R < R_2(Q)$ then the Eq.-(2.5) has only one positive root which we denote by $\psi_1(R,Q)$. This root is stable for the same reason as discussed in Table-2.2.

The graphical construction of Fig.-(2.10) asserts that $\psi_1(R,Q) < \overline{\psi}(Q) < \sqrt{3}$, it is another reason that $\psi_1(R,Q)$ may be thought of as a low endemic state. Following Fig.-(2.9) we observe that if $R_2(Q) < R < R_1(Q)$, three real and distinct roots of the Eq.-(2.5) appeared, so that $0 < \psi_1(R,Q) < \overline{\psi}_1(Q) < \psi_2(R,Q) < \overline{\psi}_2(Q) < \psi_3(R,Q) < Q$. Our graphical representation shows that as Q increases from $3\sqrt{3}$ to ∞, $R_1(Q)$ decrease from $\frac{3\sqrt{3}}{8}$ to $\frac{1}{2}$ while $R_2(Q)$ decreases from $\frac{3\sqrt{3}}{8}$ to zero. Beside these we also observe that Q increases from $3\sqrt{3}$ to ∞, $\psi_1(Q)$ decreases from $\sqrt{3}$ to 1, while $\psi_2(Q)$ increases from $\sqrt{3}$ to ∞ (see Fig.-2.10 and 2.11).

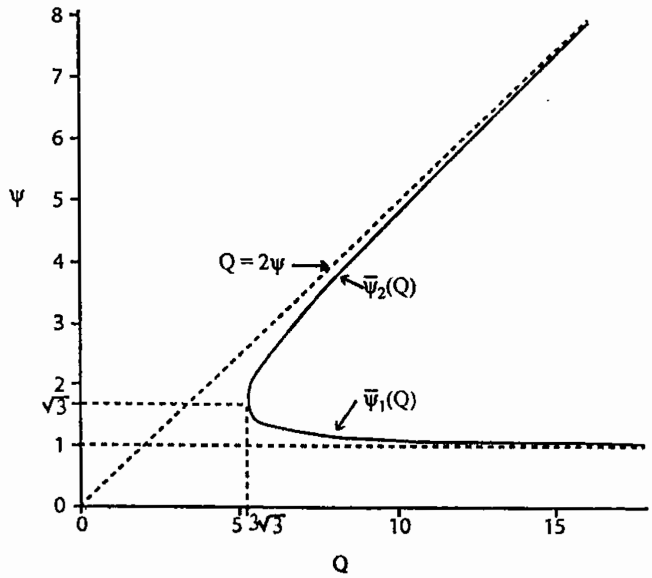

Fig.-2.10: Depicts the relationship between control parameter Q and the population size ψ for various values of control parameter R.

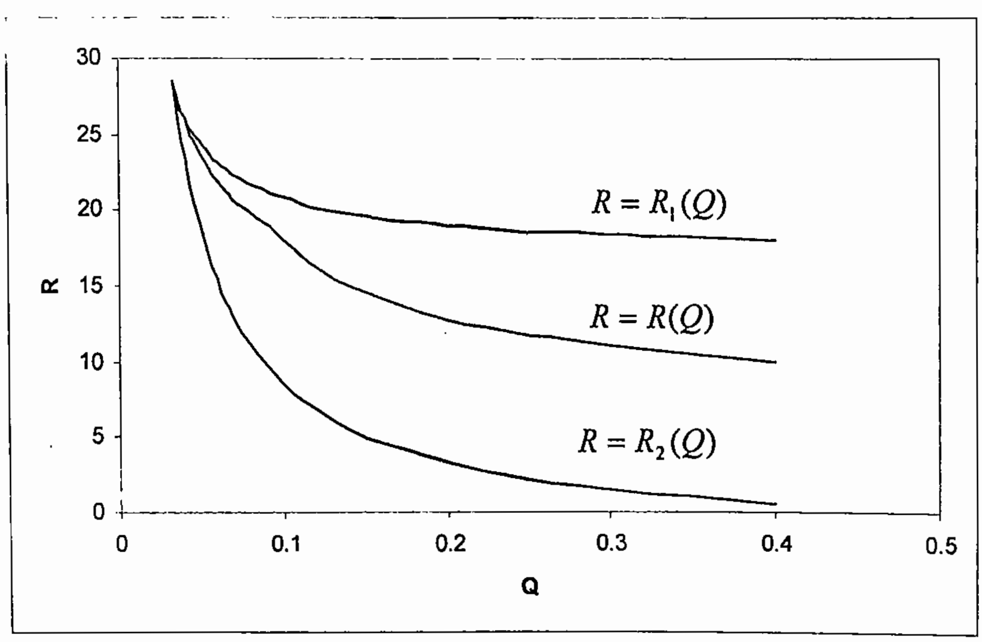

Fig.-2.11: Depicts the relationship between control parameter Q and control parameter R

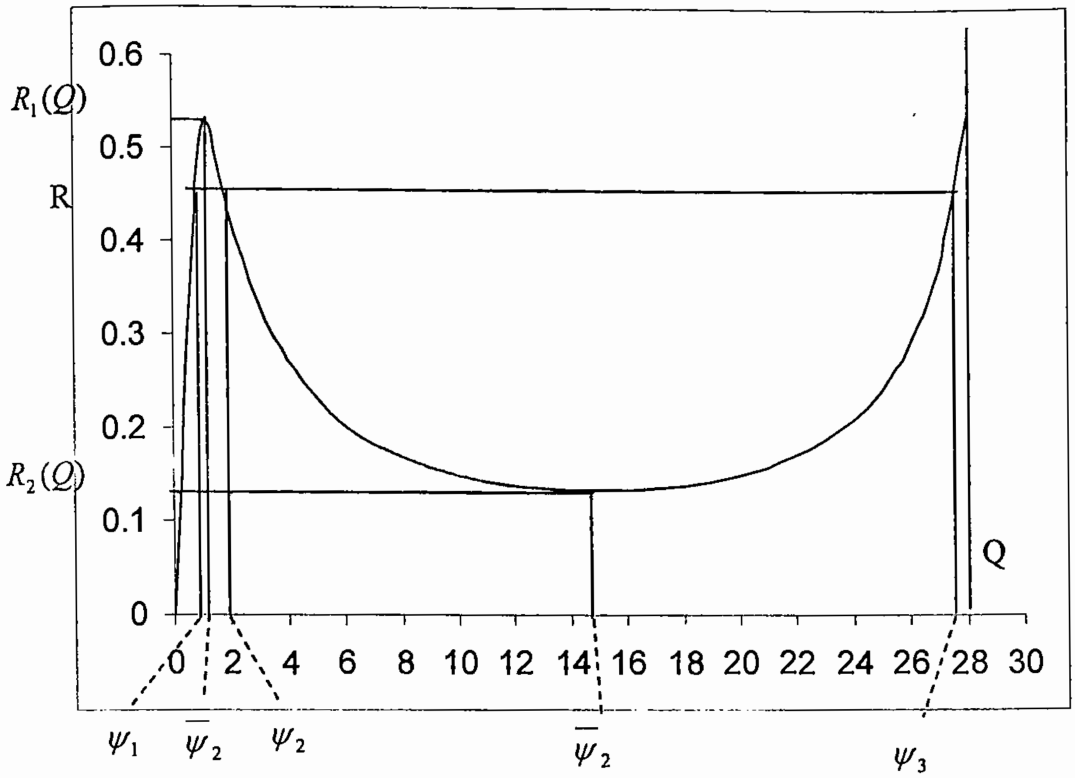

Fig.-2.12: Depicts the relationship between the population size ψ and the control parameter R for fixed value of the control parameter Q.

When $R > R_1(Q)$ another line can be drawn through the point $(5,0)$ and $(0,1.5)$ which cut the curve at $\psi_3 = 4.3$ is shown in Fig.-(2.13). Now the value of $\phi(\psi\,; R, Q)$ for different values of ψ for which $\psi < \psi_3$ and $\psi_3 < \psi$ are tabulated in the Table-2.3 .

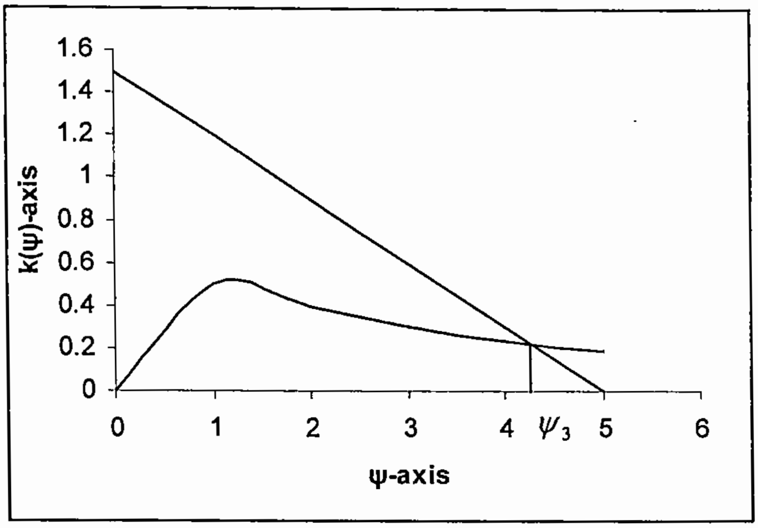

Fig-2.13: Depicts the relationship between the straight line L_R and the curve, C and straight line intersect the curve at one point for large value of Q.At intersecting point $\psi_3 = 4.3$ and it is a stable equilibrium.

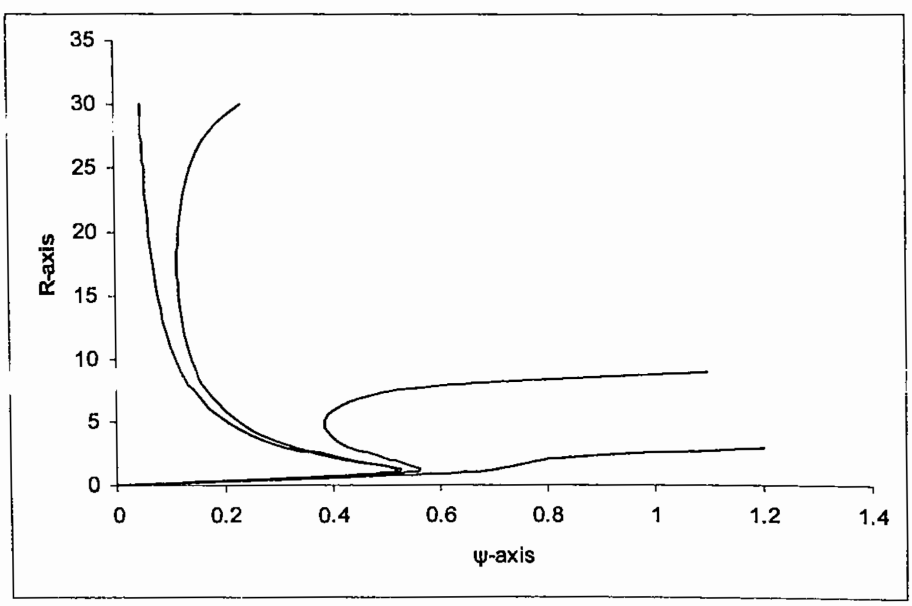

Fig-2.14 : Depicts the relationship between the control parameter R and population size ψ for different values of Q.

Table2.3: $\phi(\psi;R,Q)$ is positive when $\psi < \psi_3$ and $\phi(\psi;R,Q)$ is negative when $\psi_3 < \psi$ for a particular set of control parameters (R,Q) when the higher root, ψ_3 of the cubic equation is to be exercised.

$\psi_3 < \psi$	$\phi(\psi;R,Q)$	$\psi < \psi_3$	$\phi(\psi;R,Q)$
4. 4	-0.10592	4.0	0.172549
4.8	-0.44694	3.6	0.389089
5.2	-0.85089	3.2	0.544645
5.6	-1.13807	2.8	0.640748

Where $R = 1.5$, $Q = 5$ and $\psi_3 = 4.3$

From Table-2.3 we observe that for the root $\psi_3(R,Q)$, $\phi(\psi;R,Q)$ is negative while $\psi_3 < \psi$ and $\phi(\psi;R,Q)$ is positive when $\psi < \psi_3$. Therefore conveniently we can say that the root $\psi_3(R,Q)$ is another stable equilibrium. Graph of the roots for various Q are shown in Fig.-(2.14)

2.8 CONCLUDING REMARK:

In this chapter we have thoroughly investigate the potential impact of parameters on environment as well as discussed and represented them physically.

(i) If $Q < 3\sqrt{3} = 5.2$, the tangencies between the straight line through $(Q, 0)$ and curve C gives exactly one positive root of Eq.-(2.4) which has been represented by $\psi_1 < Q$ and it follows that $\phi(\psi\,; R < Q) > 0$ for each value of R.

(ii) $\phi(\psi\,; R, Q) > 0$ if $0 < \psi < \psi_1$ and $\phi(\psi\,; R, Q) < 0$ if $\psi_1 < \psi$.

Therefore in the same sprit, the root $\psi_1(R, Q)$ is also stable equilibrium for Eq.-(2.2).

(iii) If $Q > 3\sqrt{3} = 5.2$ then we observed two tangencies, namely $\psi = \overline{\psi}_1(Q)$, $R = R_1(Q)$ and $\psi = \overline{\psi}_2(Q)$, $R = R_2(Q)$. In this situation exactly one positive root of Eq.-(2.4), represented by $\psi_1(R, Q)$ will be obtained for $R < R_2(Q)$. Thus the root $\psi_1(R, Q)$ encountered here does indeed obey stability.

(iv) It is straight forward from Fig.-2.12 that $\psi_1(Q) < \overline{\psi}(Q) < \sqrt{3}$. Therefore, $\psi_1(Q)$ may be considered as a low endemic state.

(v) If $R_2(Q) < R < R_1(Q)$ then there are three roots of Eq.-(2.4). The following relationship $0 < \psi_1(R, Q) < \overline{\psi}_1(Q) < \psi_2(R, Q) < \overline{\psi}_2(Q) < \psi_3(R, Q) < Q$, which indicates, how closely the forest condition parameter R with the predators behaviour parameter Q are related. In view of this fact we carryout further analysis of Eq.-(2.3) and monitor the signs of $\phi(\psi\,; R, Q)$ that reveals the lower root ψ_1 and upper root ψ_3 are stable equilibrium. However the middle root ψ_2 is unstable. Impelled by this mathematical development we may interpret these three striking roots ψ_1, ψ_2 and ψ_3 as low endemic threshold point and outbreak states of the system.

Also for the nature of the roots of the Eq.(2.6) which depends on Q and R, we may encountered three distinct possibilities:

(i) By making time $t \to \infty$ in Eq.-(2.2), we find that the system shifted to low endemic state, that is, $\psi \to \psi_1$ as $t \to \infty$ when $0 < \psi < \psi_2$ at $t = 0$.

(ii) Further the system tends to outbreak state which is beyond control, that is, $\psi \to \psi_3$ as $t \to \infty$ when $\psi > \psi_2$ at $t = 0$ and finally

(iii) if $R > R_1(Q)$, then we have only one root $\psi_3(R,Q)$ remain. It is to be mentioned that this outbreak state will be reached from any positive initial density of the budworm population.

CHAPTER-THREE

CRITICAL WIDTH FOR BUDWORM OUTBREAK

CHAPTER THREE

CRITICAL WIDTH FOR BUDWORM OUTBREAK

3.1 INTRODUCTION:

A very important aspect pertaining to a physical system is its stability, which is determined by the solution process modeling system. Intuitively speaking, one may say that a system is stable if its response to an impulse function approaches zero as time approaches infinity. The stability of the system, local or global, is judged in terms of the stability of the moments of the random variables. Here our limited aim is to convey the sprit of mathematical aspect without much rigor. An elaborate exposition may be consulted for finer details from Thom, R [1] and J. Wu and Freedman [2]. Murdoch, Crawford, H. S. and D.T.

Jennings [3-4] introduces the concept of stability effects of heterogeneous system also. Those describing a more classical language should consulted Skellam [5] and Kiersteat and Slobpkin [6].

In present chapter, we shall investigate the evolution of the budworm population which inhabits a strip of dimensionless width, where it is assumed that no member of the population can survive outside the strip. In this context we shall search a region called refuge, which is a patch of favorable environment and is surrounded by an area where survival is not possible. If the population is diffusing, some of the populations will be lost around the boundary because the survival is impossible near the patch also.

3.1.1 LINEAR GROWTH

First of all we have considered the linear growth model. Let π and L^* be the strip size and critical patch size of the budworm population. By the comparison method we will show that if $L^* < \pi$ then every solution of the diffusion equation come close to zero as time increases. On the hand, we will prove that there will be arbitrarily small initial populations which will be grown without bound when $L^* > \pi$. Thus in the case of linear growth $L^* < \pi$ will be the critical strip width.

3.1.2 LOGISTTIC GROWTH

Secondly we have considered the logistic and using the method of first integrals, we will prove that the logistic diffusion equation has a unique positive equilibrium solution if $L^{*} > \pi$ Further if $L^{*} < \pi$ the equation has only the zero solution. As narrated in the first cases, for the logistic growth model critical width will be $L^{*} = \pi$

3.2 SCALING OF LENGTH AND TIME:

Coniferous forest is situated in North America and area of this forest is very large. The forest is ever green but it has a inborn defoliator for pest which is spruce budworm (choristoneura fumiferana). To maintenance, management and protect this forest from spruce budworm, D. Ludwig. D. G. Aronson and H. F. Wienberger [7-12] propose a diffusion equation

$$\frac{\partial B}{\partial T} = \frac{\delta^2}{2}\left(\frac{\partial^2 B}{\partial X^2} + \frac{\partial^2 B}{\partial Y^2}\right) + f(B) \qquad (3.1)$$

Where

(*i*) X and Y be rectangular coordinates.

(*ii*) (ii) B is the population density and

(*iii*) $f(B)$ is the rate of change of density for uniformly distributed population with density B.

Skellam (1951), Okubo (1975), Levin (1976,1978) and McMutrie (1978) have discussed about this topics at some length. [5,13-16]

Propose Eq.-(3 .1), for purpose of illustration, D. Ludwig. D. G. Aronson and H. F. Wienberger [7] have introduce a linear or logistic form for f. In linear case they propose

$$f(B) = rB \qquad (3.2)$$

Since f is a rate of change of density, r must have dimensions of reciprocal time. For the same reasons δ^2 has dimensions of $(length)^2 / time$. If X and Y are measured in kilometer then δ^2 has units of $km^2 / year$.

By Eq.-(3.2), we get from Eq.-(3.1)

$$\frac{\partial B}{\partial T} = \frac{\delta^2}{2}\left(\frac{\partial^2 B}{\partial X^2} + \frac{\partial^2 B}{\partial Y^2}\right) + rB \qquad (3.3)$$

If we introduce the dimensionless time $t = rT$, then we get from Eq.-(3.3)

$$\frac{\partial B}{\partial t} = \frac{\delta^2}{2r}\left(\frac{\partial^2 B}{\partial X^2} + \frac{\partial^2 B}{\partial Y^2}\right) + B \qquad (3.4)$$

In Eq.-(3.4) the coefficient $(\delta^2 / 2r)$ has dimensions of $(length)^2$. Thus we can introduce the dimensionless length,

$$x = \sqrt{\frac{2r}{\delta^2}}X \text{ and } y = \sqrt{\frac{2r}{\delta^2}}Y \qquad (3.5)$$

Then we get from the Eq.-(3.4)

$$\frac{\partial B}{\partial t}=\frac{\partial^2 B}{\partial x^2}+\frac{\partial^2 B}{\partial y^2}+B \quad \text{.......(3.6)}$$

Which is the linear growth model.

In the same scaling, if we put $f(B)=rB\left(1-\frac{B}{K}\right)$ in Eq.-(3.1) similarly we obtain

$$\frac{\partial B}{\partial t}=\frac{\partial^2 B}{\partial x^2}+\frac{\partial^2 B}{\partial y^2}+B\left(1-\frac{B}{K}\right) \quad \text{.......(3.7)}$$

Here K is the parameter. This parameter may be removed from the Eq.-(3.7) if we introduce the dimensionless population density $\psi=\frac{B}{K}$. Therefore finally we obtain from Eq.-(3.7)

$$\frac{\partial \psi}{\partial t}=\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}+\psi(1-\psi) \quad \text{.......(3.8)}$$

Which is the logistic model.

Eq.-(3.6) and Eq.-(3.8) have less number parameters and it is very easy to interpreate.

3.3 CRITICAL PATCH WIDTH FOR THE LINEAR MODEL:

The linear growth model of the population in Section -3.2 is given by

$$\frac{\partial B}{\partial t}=\frac{\partial^2 B}{\partial x^2}+\frac{\partial^2 B}{\partial y^2}+B \quad \text{.......(3.9)}$$

Here B is population size, t is time, and x & y are X and Y coordinate. For the sake of clarity, let the patch size is the infinite strip:

$$\Delta: -\frac{1}{2}L < x < \frac{1}{2}L, \ -\infty < y < \infty$$

Here the population density assumes to be independent of the y coordinate. By use of comparison method we are able to get global stability and convergence result [5-6].

For the strip Δ, the scaled linear growth model Eq.-(3.9) become

$$\frac{\partial B}{\partial t} = \frac{\partial^2 B}{\partial x^2} + B \quad \text{..........(3.10)}$$

Here B will be zero if $x = -\frac{1}{2}L$ or $x = \frac{1}{2}L$, that is the condition that the survival is not possible out side Δ, which implies that,

$$B = 0 \text{ if } x = -\frac{1}{2}L \text{ or, } x = \frac{1}{2}L \quad \text{..........(3.11)}$$

Let the solution of Eq.-(3.10) and Eq.-(3.11) be

$$B_n = \exp[(1 - \frac{\pi^2 n^2}{L^2})t] . \sin[\frac{n\pi}{L}(x + \frac{1}{2}L)] \quad \text{.......... (3.12)}$$

For any positive integer value of n.

3.3.1 VERIFICATION:

We have

$$\frac{\partial B}{\partial t} = (1-\frac{\pi^2 n^2}{L^2}).\exp[(1-\frac{\pi^2 n^2}{L^2})t]\sin\frac{n\pi}{L}(x+\frac{1}{2}L) \quad \text{.........................(3.12a)}$$

$$\frac{\partial B}{\partial x} = \exp[(1-\frac{\pi^2 n^2}{L^2})t].\cos\frac{n\pi}{L}(x+\frac{1}{2}L).\frac{n\pi}{L} \quad \text{.........................(3.12b)}$$

and

$$\frac{\partial^2 B}{\partial x^2} = \exp[(1-\frac{\pi^2 n^2}{L^2})t].-\sin\frac{\pi n}{L}(x+\frac{1}{2}L).(\frac{\pi^2 n}{L^2}) \quad \text{.........................(3.12c)}$$

So

$$\frac{\partial^2 B}{\partial x^2} + B = (1-\frac{\pi^2 n}{L^2}).\exp[(1-\frac{\pi^2 n^2}{L^2})t].\sin\frac{\pi n}{L}(x+\frac{1}{2}L) \quad \text{.........................(3.12d)}$$

From Eq.-(3.12a) and Eq.-(3.12d) we get,

$$\frac{\partial B}{\partial t} = \frac{\partial^2 B}{\partial x^2} + B$$

So the Eq.-(3.10) is verified.

Again if $x = \frac{1}{2}L$,

So

$$B = \exp[(1-\frac{\pi^2 n^2}{L^2})t].\sin[\frac{n\pi}{L}(\frac{1}{2}L+\frac{1}{2}L)].$$

or, $B = \exp[(1-\frac{\pi^2 n^2}{L^2})t] .\ \sin n\pi$

or, $B = 0$

And if $x = -\frac{1}{2}L$,

So

$$B = \exp[(1-\frac{\pi^2 n^2}{L^2})t] .\sin[\frac{n\pi}{L}(-\frac{1}{2}L+\frac{1}{2}L)].$$

or, $B = \exp[(1-\frac{\pi^2 n^2}{L^2})t] .\ \sin 0$

or, $B = 0$

So Eq.-(3.11) is also verified.

Therefore our assumed solution (3.12) satisfy Eq.-(3.10) and Eq.-(3.11). So the value of B in Eq.-(3.12) is a solution of Eq.-(3.10) and Eq.-(3.11).

By the use of Fourier analysis, any smooth preliminary density $B(x,0)$ can be written as superposition of such a sine function it follows that the general solution of Eq.-(3.10) and Eq.-(3.11) can be obtained as a superposition of the solution B. Now let $B(x,t)$ be any solution of Eq.-(3.10) and Eq.-(3.11) and let U be a number such that $B(x,0) \le U$ for $-\frac{1}{2}L \le x \le \frac{1}{2}L$. If $\overline{B}(x, t)$ denotes the solution of Eq.-(3.11) and Eq.-(3.12) with $\overline{B}(x, 0) = U$ then by the Fourier analysis

$$\overline{B} = \tfrac{4U}{\pi}\sum_{j=1}^{\infty}\frac{1}{2j+1}\exp[(1+(2j+1)^2\frac{\pi^2}{L^2})t].\sin\frac{(2j+1)\pi}{L}(x+\frac{1}{2}L) \quad\text{.................(3.13)}$$

If we carefully consider the value U, at the first stage B is not greater than $\overline{B}$, however the two population follows the same growth law as well as satisfy the same condition at the boundary of identical habitats. This is why B always remain below $\overline{B}$; that is

$$0 \le B(x,t) \le \overline{B}(x,t) \qquad \text{for } -\frac{1}{2}L \le x \le \frac{1}{2}L, \quad t \ge 0 \text{ ....(3.14)}$$

From the solution, Eq.-(3.12), we observe that if $L < \pi$ then $1 - \frac{n^2\pi^2}{L} < 0$ for any positive integer value of n and all the equation Eq.-(3.12) decay exponentially to zero as $t \to \infty$. If we observe the Eq.-(3.13), it is clear that $\overline{B}$ is also decays exponentially to zero as $t \to \infty$. In this circumstances we can say; if $B(x, t)$ denotes the density of a population in Δ which evolves according to the Eq.-(3.10) and Eq.-(3.11) from an arbitrary smooth initial density and if $L < \pi$, then

$$\lim_{t \to \infty} B(x,t) = 0 \qquad \text{if } -\frac{1}{2}L \le x \le \frac{1}{2}L.$$

Therefore Δ dose not act as a refuge.

Now we shall show that if L is sufficiently large, then Δ can act as a refuge. In fact if $L > \pi$ then populations with arbitrarily small initial density can grow indefinitely in Δ. To prove this we consider the function $\xi B_1(x,t)$, where B_1 is defined by Eq.-(3.12) and ξ is any pre-assign positive constant. It is remarkable to note that $\xi B_1(x,0)$ can be made as small as one can choose ξ to be sufficiently small. Therefore, if $L > \pi$, it follows from Eq.-(3.12) that for any ξ

$$\lim_{t\to\infty} \xi B_1(x,t) = \infty \quad \text{if} \quad -\frac{1}{2}L \le x \le \frac{1}{2}L.$$

Therefore we may conclude that π is a critical patch width for the proposed linear model.

3.4 CRITICAL PATCH SIZE FOR THE LOGISTIC MODEL:

Under proper assumption for the strip Δ, we have from the Section-3.2 another model which called logistic model is,

$$\frac{\partial \psi}{\partial t} = \psi(1-\psi) + \frac{\partial^2 \psi}{\partial x^2} \quad \text{.......(3.15)}$$

The boundary condition is

$$\psi = 0 \quad \text{if} \quad |x| = \frac{1}{2}L \quad \text{.......(3.16)}$$

At first we investigate for steady solution of Eq.-(3.15) and Eq.-(3.16) and for this purpose we let the positive solution $\eta = \eta(x)$ of the ordinary differential equation

$$\eta'' + \eta(1-\eta) = 0 \quad \text{if } -\frac{1}{2}L < x < \frac{1}{2}L, \qquad (3.17)$$

which satisfy the boundary conditions

$$\eta = 0 \text{ if } \quad |x| = \frac{1}{2}L \qquad (3.18)$$

Let η is a solution of the Eq.-(3.17) and Eq.-(3.18) such that $\eta(x) > 0$ for $-\frac{1}{2}L < x < \frac{1}{2}L$. At the end point $x = \frac{1}{2}L$ and $x = -\frac{1}{2}L$ the solution $\eta = 0$. So η must take on its maximum value μ at some point $x = a$ within the interval

$$-\frac{1}{2}L < x < \frac{1}{2}L.$$

Thus in the interval $-\frac{1}{2}L < x < \frac{1}{2}L$, we have $0 < \eta(x) \le \eta(a) \equiv \mu$ with

$$\eta'(a) = 0, \quad \eta''(a) \le 0 \qquad (3.19)$$

Using the Eq.-(3.18), we get from Eq.-(3.17) $\eta''(a) = -\mu(1-\mu)$. Thus $\mu''(a) \le 0$ means that $0 < \mu \le 1$. At first we choice that $\mu = 1$ then $\eta(x) = 1$ is the only one solution of the initial value problem $\eta(a) = 1$, $\eta'(a) = 0$ but if so then η does not satisfy the Eq.-(3.18). Therefore we can say that $0 < \mu < 1$. Thus the equilibrium density will be below the carrying capacity everywhere [17].

Therefore from the Eq.-(3.17), if $-\frac{1}{2}L \le x \le \frac{1}{2}L$, then $\eta''(x) = 0$. It follows that $\eta(x) < \mu$ if $x \ne a$.

Now we find out the solution of Eq.-(3.17) and Eq.-(3.18), by means of the method of the first integral. After multiplication by η' Eq.-(3.17) have

$$\eta''\eta' + \eta'(\eta - \eta^2) = 0$$

or, $$\frac{d}{dx}\left(\frac{1}{2}\eta'^2 + \frac{\eta^2}{2} - \frac{\eta^3}{3}\right) = 0$$

or, $$\frac{d}{dx}\left[\frac{1}{2}\eta'^2 + F(\eta)\right] = 0$$

where

$$F(\eta) = \frac{\eta^2}{2} - \frac{\eta^3}{3} \quad \text{...(3.19a)}$$

Since the first derivative of $[\frac{1}{2}\eta'^2 + F(\eta)]$ is zero, so $[\frac{1}{2}\eta'^2 + F(\eta)]$ is constant. In the view of Eq.-(3.18) this constant must be equal to $F(\mu)$. Therefore if η is the solution of Eq.-(3.17) and Eq-(3.18) then η and η' are related by the first integral

$$\frac{1}{2}\eta'^2 + F(\eta) = F(\mu) \quad \text{...(3.20)}$$

It is interesting to note that F is strictly increasing function of η for $\eta < 1$. Therefore when x is not equal to a, we can solve Eq.-(3.20) for η'

$$\eta'(x)=\begin{cases}\sqrt{2}\sqrt{F(\mu)-F(\eta)}, & if \quad -\frac{1}{2}L\le x\le a\\ -\sqrt{2}\sqrt{F(\mu)-F(\eta)}, & if \quad a\le x\le \frac{1}{2}L\end{cases} \qquad (3.21)$$

For any x within the interval $-\frac{1}{2}L\le x\le a$, both side of the Eq.-(3.21) divided by $\sqrt{F(\mu)-F(\eta)}$ and integrate from x to a in order to obtain

$$\int_{v(x)}^{\mu}\frac{dz}{\sqrt{F(\mu)-F(z)}}=\sqrt{2}(a-x) \qquad (3.22)$$

which gives η implicitly as function of x for $x<a$. Similarly integrate from a to x in order to obtain

$$\int_{v(x)}^{\mu}\frac{dz}{\sqrt{F(\mu)-F(z)}}=\sqrt{2}(x-a) \qquad (3.23)$$

The Eq.-(3.22) and Eq.-(2.23) contains a parameter a which is not appearing in the original formulation in Eq.-(3.17) and Eq.-(3.18). In order to eliminate the parameters, we apply the boundary condition, Eq.-(3.18) and then the parameters will be eliminated. So using the boundary conditions, Eq.-(3.22) and Eq.-(3.23) we have,

$$\int_{0}^{\mu}\frac{dz}{\sqrt{F(\mu)-F(z)}}=\sqrt{2}(\frac{1}{2}L+a) \qquad (3.24a)$$

$$\int_{0}^{\mu}\frac{dz}{\sqrt{F(\mu)-F(z)}}=\sqrt{2}(\frac{1}{2}L-a) \qquad (3.24b)$$

By subtracting one of these equation from the other we conclude that $a=0$, thus the unique maximum of η must be occurs at the midpoint of the interval. Therefore two equations reduce to the single equation

$$L=\sqrt{2}\int_{0}^{\mu}\frac{dz}{\sqrt{F(\mu)-F(z)}} \quad (3.25)$$

which determines L in terms of μ. As we shall see in the next the integral in Eq.-(3.24) is an increasing function of μ. In this connection Eq.-(3.25) can be solved for μ in terms of L.

To summarize we have shown that the positive solution of Eq.-(3.17) and Eq.-(3.18) is given implicitly by the formula

$$\int_{0}^{\mu}\frac{dz}{\sqrt{F(\mu)-F(z)}}=\sqrt{2}|x| \quad (3.26)$$

where $\mu=\mu(L)$ is defined in Eq.-(3.25). In order to study the relationship between L and μ, let $h(L)$ represents the right-hand side of Eq.-(3.25) and suppose

$$w=\frac{z}{\mu}$$

therefore,

$\mu\, dw=dz$ and the limit $w=0$ and $w=1$.

Substitute μ and $w\mu$ in Eq.-(3.19a) and subtract second from first we get

$$F(\mu)-F(\mu w)=\frac{\mu^2}{2}-\frac{\mu^3}{3}-\frac{\mu^2 w^2}{2}+\frac{\mu^3 w^3}{3}$$

$$=\mu^2\left[\frac{1}{2}(1-w^2)-\frac{1}{3}\mu(1-w^3)\right]$$

$$=\frac{\mu^2}{6}\left[3(1-w^2)-2\mu(1-w^3)\right]$$

put the above values in Eq.-(3.25)

$$h(\mu)=\sqrt{2}\int_0^{\mu}\frac{\mu dw}{\sqrt{F(\mu)-F(\mu w)}}=2\sqrt{3}\int_0^1\frac{dw}{\sqrt{3(1-w^2)-2\mu(1-w^3)}} \quad \text{..................(3.27)}$$

The singularity at $w=1$, and this function is integrable if $\mu<1$, but it is not integrable when $\mu=1$.

The following facts can be derived from Eq-(3.27);

(i) h is an increasing function of μ for $0\le\mu<1$,

(ii) $h(\mu)\to\infty$ as $\mu\uparrow 1$,

(iii) $h(\mu)\to\pi$ as $\mu\downarrow 0$.

The value of μ is determinate by the equation $h(\mu)=L$. According to property (i) there exist at best one value of μ for each value of L. Because of properties (ii) and (iii) there is no positive value of μ which satisfies $h(\mu)=L$ if $L<\pi$, while for $L\ge\pi$ the corresponding value of μ increase from 0 to 1 as L increase from π to ∞. Thus for $L\le\pi$ Eq.-(3.17) and Eq.-(3.18) have exactly one trivial solution $\eta=0$. For $L>\pi$, there is an extra solution given by Eq.-(3.26). This information is summarized in the Fig.-(3.1), which is a graph of Eq.-(3.27). It has been cleared that as μ approaches to zero $h(\mu)$, resulted π which is equivalent to 3.142. Alternatively when μ approaches to one $h(\mu)$ provided infinity.

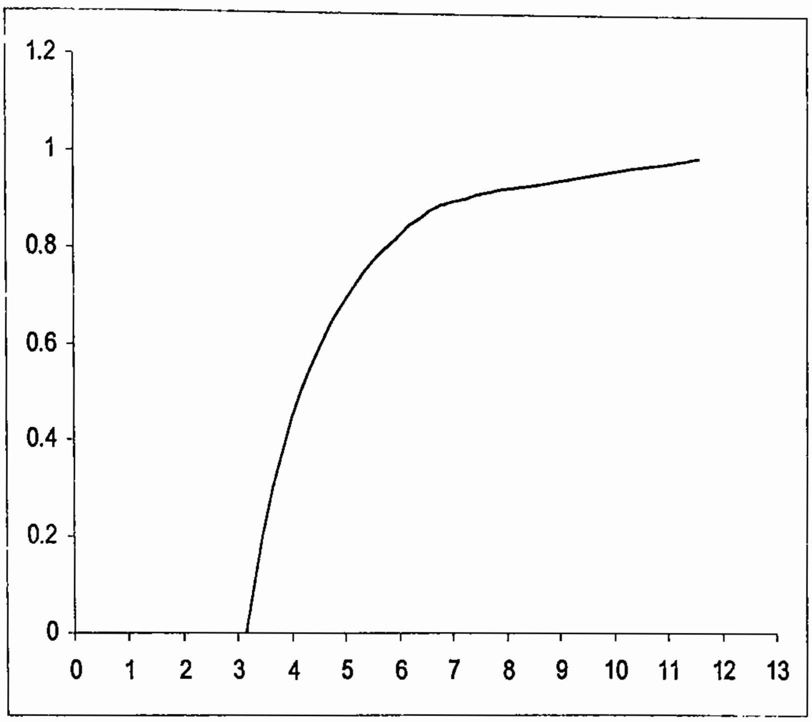

Fig.-3.1: Depicts the faster increasing trend at the initial stage and slow increasing tendency at the upper values of μ.

Further the graph in Fig.-(3.1) is called bifurcation diagram for the Eq.- (3.17) and Eq.-(3.18). Because it shows a branching or bifurcation in the solution set in the critical value $L^{*} = \pi$. It is also a critical value of the linear model.

On the other hand if η is very small, the quadratic term in the Eq.- (3.17) is negligible and it satisfy the linear model,

$$\eta'' + \eta = 0 \quad \text{.......(3.28)}$$

This problem is so much complicated and it is not easy to solve in such a non-linear problem as we have done as before. Thus it is usually not possible to obtain complete bifurcation diagram which has shown in Fig.-(3.1). However its local shape may obtain near the bifurcation point L^* by exploiting the relationship between the non linear and it linearization problem near to zero.

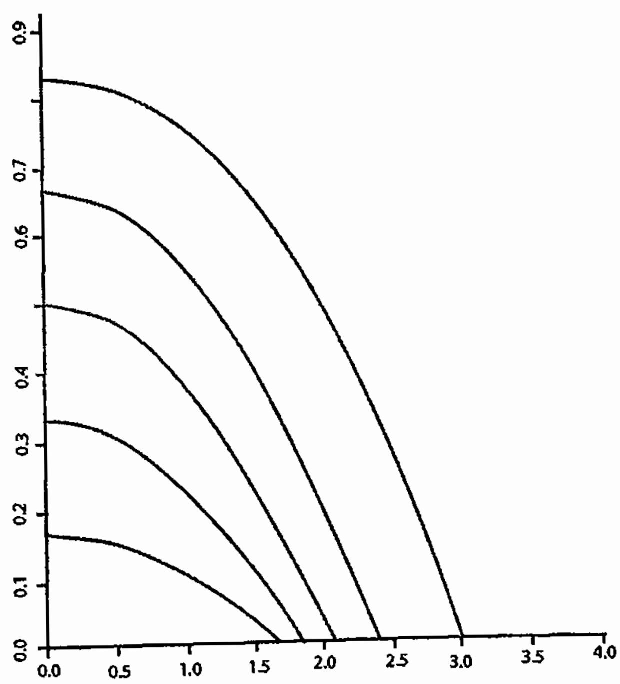

Fig.-3.2 : Depicts the solution processes of $h(\mu)$

In the Fig.-(3.1) only maximum value μ of the solution η as a function of L. The vertical segments which join every point of the graph to the $L-axis$ are the projection of the actual solution. For the various values of L solution plotted in the Fig.-(3.2). Here it is be mentioned that η increases while increasing value of L.

3.5 STABILITY ANALYZE :

Let $\psi^{*}(x)=\psi(x,0)$, where ψ satisfy Eq.-(3.15) and Eq.-(3.16). Also assume that $\psi^{*}(x)\geq 0$ and $\psi^{*}(x)\neq 0$. It is clear that

(i) if $0<L<\pi$ then $\psi(x,t)\to 0$ as $t\to\infty$

(ii) if $L>\pi$ then $\psi(x,t)\to\eta(x)$ as $t\to\infty$

Here η is given by Eq.-(3.25) and Eq.-(3.26). In this direction we can say that that the strip Δ is a stable refuge if its width is greater than π and Δ is not a refuge at all when $L<\pi$. The following arguments will be established through (i), however, the proof of (ii) is somewhat complicated [18-20]. In view of our assumption $\psi^2<0$ shows that $\psi(x,t)\geq 0$. Moreover Eq.-(3.15) can be written as

$$\frac{\partial\psi}{\partial t}-\frac{\partial\psi^2}{\partial x^2}-\psi=-\psi^2 \quad\text{.........(3.29)}$$

Eq.-(3.29) can be explained as follows: the left-hand side of Eq.-(3.29) is the difference between the time rate of the change in ψ and the sum of the contribution to the rate of change of ψ from diffusion and the linear component of the logistic growth term however, $-\psi^2 \leq 0$. Further Eq.-(3.29) implies that the density of a population which is governed by the logistic law grows less rapidly than the density of a population which is governed by the corresponding linear law. Now we can imagine an experiment in which we begin with identical population in identical habitats in which one population P evolves according to the logistic law in Eq.-(3.15), while the other population $\overline{P}$ obeys the linear law of equation, Eq.-(3.10). As we have observed the density ψ of P grows less rapidly than the density $\overline{\psi}$ of $\overline{P}$. It follows that

$$0 \leq \psi(x,t) \leq \overline{\psi}(x,t) \qquad \text{....(3.30)}$$

for all x in Δ and $t > 0$. However in our analysis of linear growth model at the beginning of this Chapter, we have shown that $\overline{\psi}(x,t) \to 0$ as $t \to \infty$ if $L < \pi$. Therefore (ii) follows from Eq.-(3.30).

3.6 OUTBREAK PATCH SIZE FOR THE BUDWORM EQUATION:

In the previous section we have dealt with the critical width of a strip for a population which obeys the logistic growth law with diffusion. In this section our concerns with analogous problem for a scatter ness budworm population.

The budworm has two critical width. The smaller of these provides a lower bound for the width of a strip which can support a nonzero population. On the other hand the larger critical width is a lower bound for the width of a strip which can support an outbreak.

According to Ludwig, Jones and Holling [21] the non-linear diffusion equation is

$$\frac{\partial B}{\partial T}=\frac{\delta^2\partial^2 B}{2\partial X^2}+rB(1-\frac{B}{K'S})-\frac{\beta B^2}{(\alpha'S)^2+B^2} \qquad (3.31)$$

Now we want to introduce the scaled variable, as

$$x=\frac{\sqrt{2r}}{\sigma}X \qquad \psi=\frac{B}{\alpha'S}$$

$$or,\quad x^2=\frac{2r}{\sigma^2}X^2 \qquad or,\quad B=\alpha'S\psi$$

$$or,\quad \partial B=\alpha'S\partial\psi$$

$$\therefore\quad \partial X^2=\frac{\sigma^2}{2r}\partial x^2 \qquad \therefore\quad \partial^2 B=\alpha'S\partial\psi^2$$

$$t=\frac{1}{2}T \qquad R=\frac{r\alpha'S}{\beta}$$

$$\therefore\quad \partial T=\frac{1}{r}\partial t \qquad \therefore\quad r\alpha'S=R\beta$$

and

$$Q=\frac{K'}{\alpha'}$$

Then Eq.-(3.31) becomes of the form,

$$\frac{r\alpha'S\partial\psi}{\partial t}=\frac{\sigma^2}{2}\cdot\frac{2r\alpha'S\partial^2\psi}{\sigma^2\partial x^2}+r\alpha'S\psi(1-\frac{\alpha'S\psi}{Q\alpha'S})-\beta\frac{\alpha^2S^2\psi^2}{(\alpha'S)^2+(\alpha'S\psi)^2}$$

or. $$\frac{R\beta.\partial\psi}{\partial t}=\frac{R\beta.\partial^2\psi}{\partial x^2}+R\beta.\psi(1-\frac{\psi}{Q})-\beta\frac{\psi^2}{1+\psi^2}$$

or. $$\frac{\partial\psi}{\partial t}=\frac{\partial^2\psi}{\partial x^2}+\psi(1-\frac{\psi}{Q})-\frac{\beta}{R\beta}.\frac{\psi^2}{1+\psi^2}$$

or, $$\frac{\partial\psi}{\partial t}=\frac{\partial^2\psi}{\partial x^2}+\psi(1-\frac{\psi}{Q})-\frac{1}{R}.\frac{\psi^2}{1+\psi^2}$$

or $$\frac{\partial\psi}{\partial t}=\frac{\partial^2\psi}{\partial x^2}+\phi(\psi\ ;R,Q)\quad\text{.......(3.32)}$$

where

$$\phi(\psi\ ;R,Q)=\psi(1-\frac{\psi}{Q})-\frac{1}{R}.\frac{\psi^2}{1+\psi^2}$$

As before the Eq.-(3.32) is hold in the strip $|x|<\frac{1}{2}L$. Outside the strip, we let that the environment is not favorable, since ψ obeys a non-linear growth law with non-positive net growth rate. However for the sake of sincerity, we confined ourselves to the limiting case where the survival is not possible outside the strip, so that

$$\psi=0 \text{ if } x=\frac{1}{2}L \text{ or } x=-\frac{1}{2}L \text{ for } t>0\quad\text{.......(3.33)}$$

The problem is same as Eq.-(3.10) and Eq.-(3.11). If we apply the method of first integral as in Section- 3.4 to the problem, Eq.-(3.32) and Eq.-(3.33), the result explained that there is a critical size L^* for the strip which can support a stationary non-zero population. For the small population density, the length

L^* may be computed from the approximating linear problem. In this context we can say $L^* = \pi$

Till now, the above discussions does not achieve our main objective to calculate a critical length for the strip in order to support an outbreak [22-24]. By definition, we know that an out break involves large population density [25-26]. Therefore we can not reach near the Eq.-(3.33) by linear equation. Instead of this we can apply the first integral method, which was employed in Section-3.4. We assume that $\eta(x)$ denote a steady-state population density, which satisfy

$$\eta''(x) + \phi(\psi \; ; R, Q) = 0 \qquad \text{.........(3.34a)}$$

and the boundary condition

$$\psi = 0 \text{ if } x = -\frac{1}{2}L \text{ or, } x = \frac{1}{2}L \qquad \text{.........(3.34b)}$$

Now set

$$F(\eta) = F(\eta\,; R, Q) = \int_0^\eta \phi(\psi; R, Q)\, d\psi$$

$$= \frac{1}{2}\eta^2 - \frac{1}{3Q}\eta^3 - \frac{\eta}{R} + \frac{1}{R}\arctan\eta \qquad \text{.........(3.35)}$$

In the interval $-\frac{1}{2}L \le x \le \frac{1}{2}L$, let μ denote the maximum value of η where $\mu = \eta$ we must have $\eta_x = 0$. From the first integral we have

$$\frac{1}{2}\eta'^2 + F(\eta) = F(\mu) \quad \text{......(3.36)}$$

The integral then leads to the formula

$$\int_0^\mu \frac{dw}{\sqrt{F(\mu) - F(w)}} = \sqrt{2}x \quad \text{......(3.37)}$$

If we apply the boundary condition, Eq.-(3.34b) which are satisfied, then μ and L are related as follows

$$h(\mu) = \sqrt{2}\int_0^\mu \frac{dw}{\sqrt{F(\mu) - F(v)}} = \sqrt{2}.\sqrt{2}x$$

or, $h(\mu) = L$ ......(3.38)

It is remarkable to note that η varies between 0 and μ whenever x varies from $-\frac{1}{2}L$ to 0. Therefore we have from Eq.-(3.34)

$$F(w) < F(\mu) \text{ if } 0 < w < \mu \quad \text{......(3.39)}$$

For a certain value of μ, the condition, Eq.-(3.39) is a restriction. The integral of Eq.-(3.38) is defined if the condition, Eq.-(3.39) is satisfied. Now we have to consider, how F varies following the changes in η and R. It has been shown in Section-2.7 of Chapter-Two that $Q > \acute{Q}$, $R_2(Q) < R_1(Q)$ and also $\psi_1 < \psi_2 < \psi_3$ with the following properties:

(i) $R \le R_2$, if $0 < \eta < \psi_1$, and $\phi(\eta) < 0$ if $\eta > \psi_1$. For our convenience we have used $\phi(\eta)$ instead of $\phi(\eta; R, Q)$.

Table-3.1(a) : The relationship between $\phi(\eta\,; R, Q)$ and η for a particular set of control parameters (R, Q) when the lower root, ψ_1 of the cubic equation is to be exercised and η lies between 0 and ψ_1.

$0 < \eta < \psi_1$	$\phi(\eta\,; R, Q)$
0.9	0.03013
0.8	0.041037
0.7	0.047396
0.6	0.048839
0.5	0.045388

Where $R = 0.5$ and $Q = 6$

Table-3.1(b) : The relationship between $\phi(\eta\,; R, Q)$ and η for a particular set of control parameters (R, Q) when the lower root, ψ_1 of the cubic equation is to be exercised and η greater than ψ_1.

$\eta > \psi_1$	$\phi(\eta\,; R, Q)$
1.2	-0.02388
1.5	-0.09691
1.8	-0.17660
2	-0.23015
2,2	-0.28322

Where $R = 0.5$ and $Q = 6$

(ii)For $R_1 < R < R_2$, $\phi(\eta) > 0$ if $0 < \eta < \psi_1$ or $\psi_2 < \eta < \psi_3$ and $\phi(\eta) < 0$ if $\eta > \psi_3$

Table-3.2(a) : The relationship between $\phi(\eta\,; R, Q)$ and η for a particular set of control parameters (R, Q) when the lower root, ψ_1 of the cubic equation is to be exercised and η lies between 0 and ψ_1.

$0 < \eta < \psi_1$	$\phi(\eta\,; R, Q)$
0.9	0.094847
0.8	0.089525
0.7	0.081954
0.6	0.071902
0.5	0.059423

Where $R = 0.62$ and $Q = 6$

Table-3.2(b) : The relationship between $\phi(\eta\,; R, Q)$ and η for a particular set of control parameters (R, Q) when the middle and higher roots, ψ_2 and ψ_3 of the cubic equation are to be exercised and η lies between ψ_2 and. ψ_3

$\psi_2 < \eta < \psi_3$	$\phi(\eta\,; R, Q)$
2.0	0.115473
2.5	0.144509
3.0	0.175880
3.5	0.182566
4.0	0.131247

Where $R = 0.62$ and $Q = 6$

Table-3.2(c) : The relationship between $\phi(\eta\,;R,Q)$ and η for a particular set of control parameters (R,Q) when the higher root, ψ_3 of the cubic equation is to be exercised and η is greater than. ψ_3

$\eta>\psi_3$	$\phi(\eta\,;R,Q)$
5.5	-0.74557
5.8	-1.11623
6.0	-1.42025
6.4	-2.12259
6,8	-3.01815

Where $R=0.62$ and $Q=6$

(iii)For $R>R_1$, $\phi(\eta)>0$ if $0<\eta<\psi_3$ and $\phi(\eta)<0$ if $\eta>\psi_3$

Table-3.3(a) : The relationship between $\phi(\eta\,;R,Q)$ and η for a particular set of control parameters (R,Q) when the higher root, ψ_3 of the cubic equation is to be exercised and η is smaller than ψ_3.

$\eta<\psi_3$	$\phi(\eta\,;R,Q)$
2.0	0.563499
2.5	0.801711
3.0	1.054495
3.5	1.290274
4.0	1.556899

Where $R=0.9$ and $Q=6$

Table-3.3(b) : The relationship between $\phi(\eta\,;R,Q)$ and η for a particular set of control parameters (R,Q) when the higher root, ψ_3 of the cubic equation is to be exercised and η is greater than. ψ_3

$\eta > \psi_3$	$\phi(\eta\,;R,Q)$
8.0	-3.72616
8.5	-5.82229
9.0	-8.37762
9.6	-11.4337
10.0	-15.0321

Where $R = 0.9$ and $Q = 6$

The properties (i), (ii) and (iii) have been verified in Table-3.1 to Table-3.3. Further, as F is the integral of ϕ, this information can be expressed into facts about F:

(i) If $R < R_2$, then F is increasing for $o < \eta < \psi_1$ and again F is decreasing for $\psi_1 < \eta$. Thus F has a local maximum at ψ_1 and this situation is shown in Fig.-(3.3).

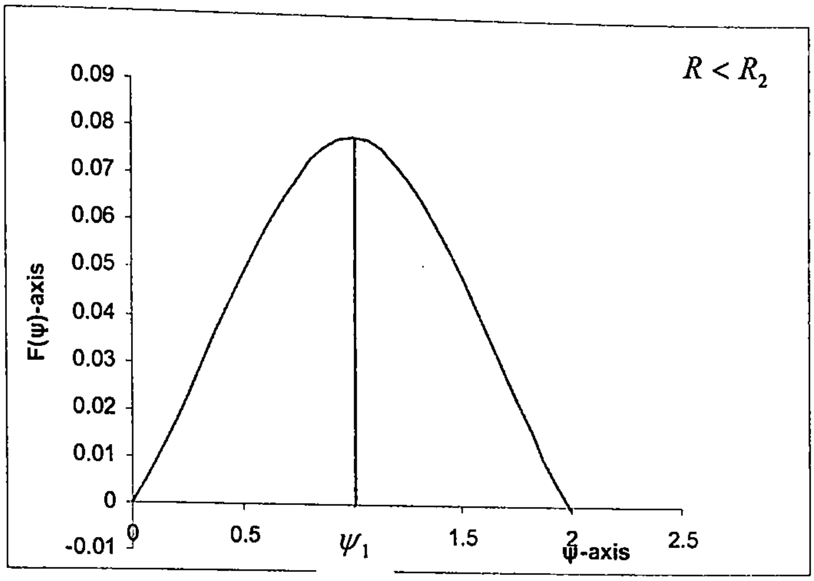

Fig.-3.3: Increasing and decreasing trends of F for the steady-state populations density within $(0, \psi_1)$ and ψ_1 represents the local maximum.

(ii) If $R_2 < R < R_1$ then at beginning F is increasing for $\eta < \psi_1$ and later on decreasing for $\psi_1 < \eta < \psi_2$. Further, F is increasing for $\psi_2 < \eta < \psi_3$ and after that decreasing further for $\psi_3 < \eta$. Thus F has two local maxima at ψ_1 and ψ_3 and a local minimum at ψ_2. If $R_2 \downarrow R$ then ψ_2 and ψ_3 are coincide and $F(\psi_1) > F(\psi_3)$. So the highest maximum is observed at ψ_1. On the other hand $R \uparrow R_1$ then ψ_1 and ψ_2 are coincide and $F(\psi_1) < F(\psi_3)$. So the highest maximum is observed at ψ_3.

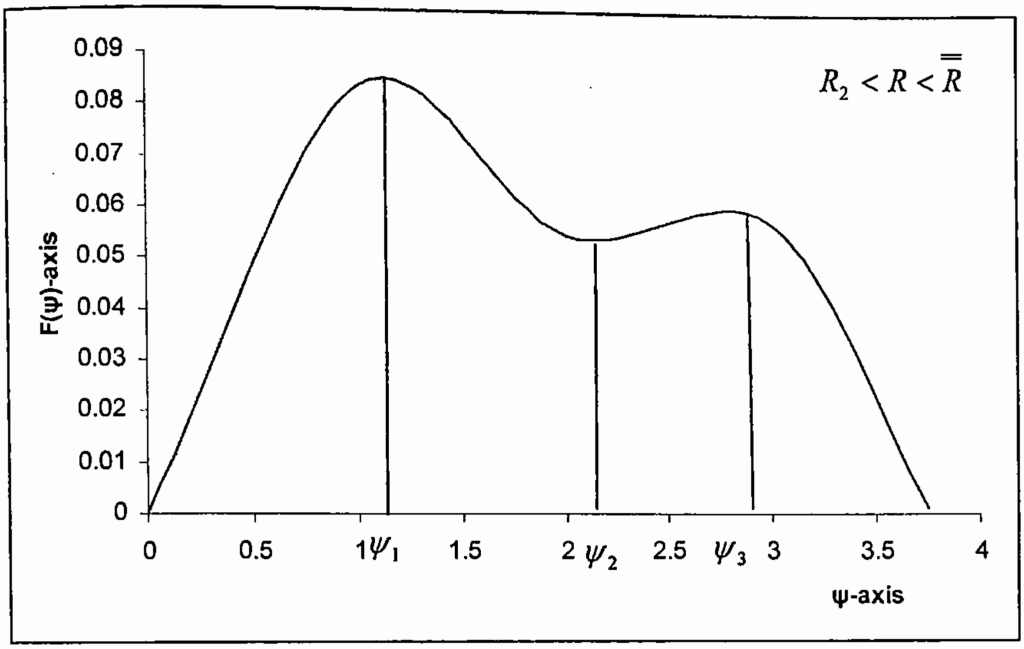

Fig.-3.4 : Depicts the variation in the shape of the F curves with the changes in the control parameters $Q = 6$ and $R = 0.595$ (which is close to R_2 but away from R_1), being kept constants leads to the sudden jump: occurrence of catastrophe.

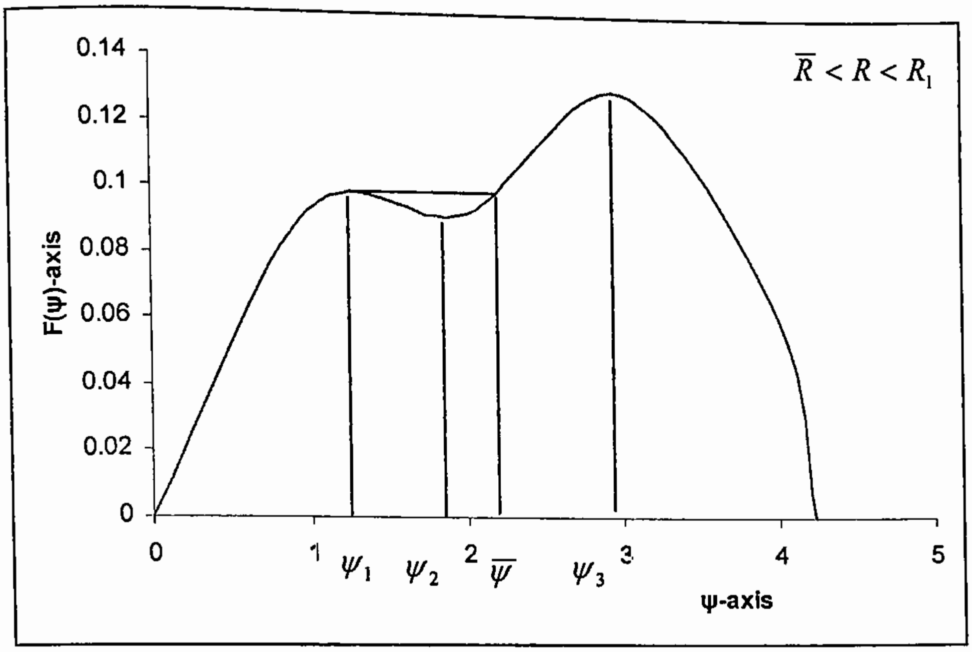

Fig.-3.5 : Depicts the variation in the shape of the F curves with the changes in the control parameters $Q = 6$ and $R = 0.61$ (which is close to R_1 but away from R_2), being kept constants leads to the sudden jump: occurrence of catastrophe.

By the way if we define $\overline{\overline{R}}(Q)$ with the condition

$$F(\psi_1, R, Q) = F(\psi_3, R, Q) \qquad \text{if } R = \overline{\overline{R}}(Q)$$

we have obtained for a large value of Q

$$\overline{\overline{R}} = \frac{16}{3Q} + 0(\frac{1}{Q^2}) \ .$$

Now if $R_2 < R < \overline{\overline{R}}$ then $F(\psi_1) > F(\psi_3)$ and the graph of such situation is depicted in Fig.-(3.4). Condition imposed in Eq.-(3.33) implies that if $R_2 < R < \overline{\overline{R}}$, $h(\psi)$ is defined only if $0 < \mu < \psi_1$. On the other hand if $\overline{\overline{R}} < R < R_1$, then $F(\psi_1) < F(\psi_3)$ and the graph of such situation is depicted in Fig.-(3.5). Suppose $\overline{\psi}$ be the unique value of η between ψ_2 and ψ_3, for which $F(\overline{\psi}) = F(\psi_1)$.

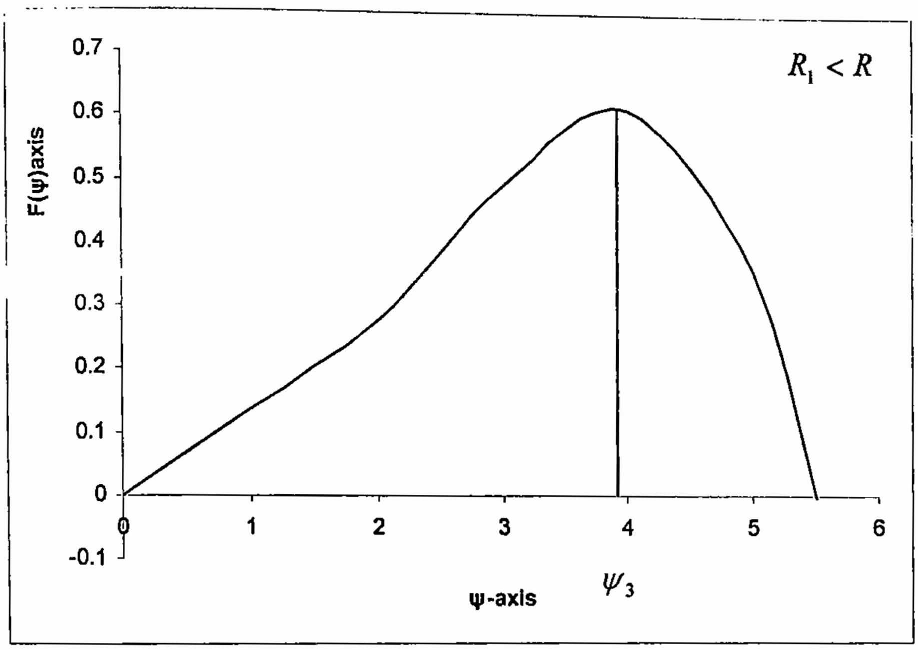

Fig.-3.6: Depicts the variation in the shape of the F curves with the changes in the control parameters $Q = 6$ and $R = 0.9$ (which is upper neighborhood of R_1), being kept constants leads to the sudden jump: occurrence of catastrophe.

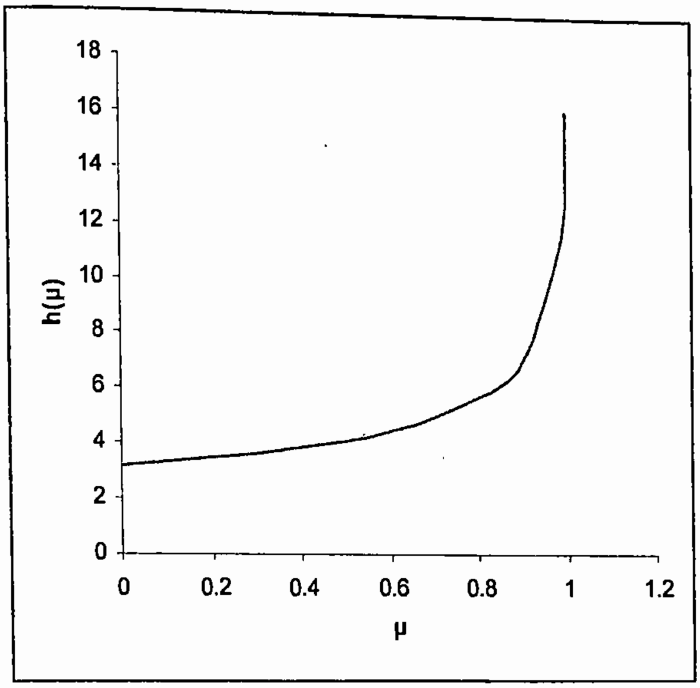

Fig-3.7 : Depicts the relationship between μ and $h(\mu)$ which is reversed of the Fig.-3.2.

Now we can define on two intervals : h is defined for $0 < \mu < \psi_1$ and for $\overline{\psi} < \mu < \psi_3$. It is to be noted that as $R \uparrow R_1$, these intervals coalesce, since, $\psi_2 - \psi_1 \to 0$

(iii) If $R > R_1$, then F has a local maximum at ψ_3 and h is defined for $0 < \mu < \psi_3$ and the whole situation is depicted in Fig.-(3.6).

From Fig.-(3.3) to Fig.-(3.6), the difference, $F(\mu) - F(w) \to 0$ to the first order as $w \uparrow \mu$, if $\mu < \psi_1$. However, this difference vanishes to the second order if $\mu = \psi_1$. Thus the integral, Eq.-(3.39) is finite if $\mu < \psi_1$ but $h \uparrow \infty$ as $\mu \uparrow \psi_1$ if $R < R_1$.

Further, a quadratic approximation to $F(w)$ shows that $h(0)=\pi$ and this represents that h is monotone increasing when $0<\mu<\psi_1$. Therefore, if $R<\overline{\overline{R}}$ the graph of $L=h(\mu)$ will be alike Fig.-(3.7).

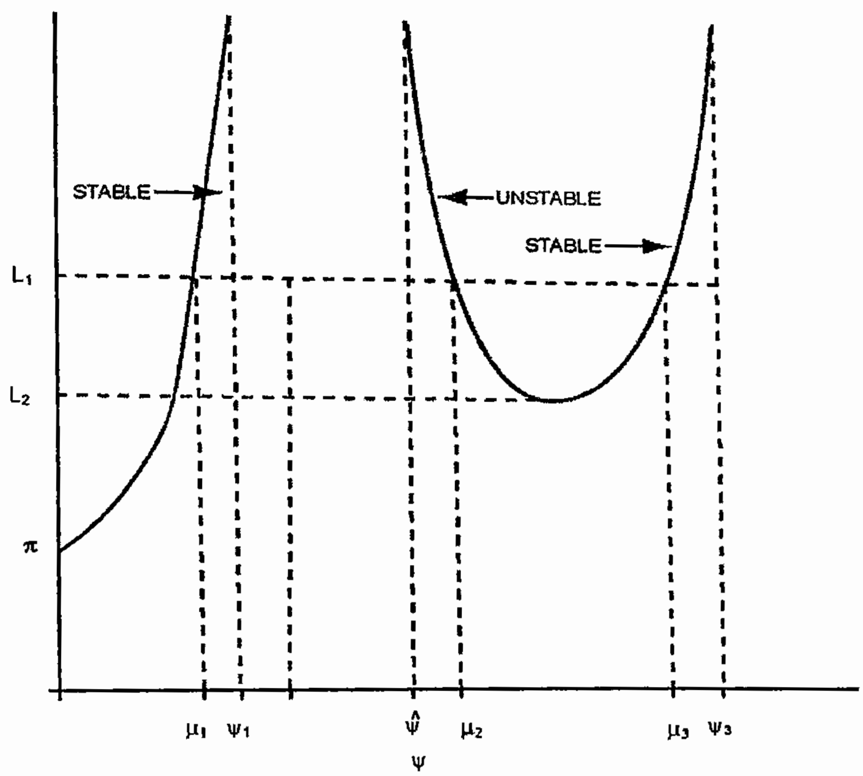

Fig.-3.8 : Depicts the relationship between L and μ and represents the asymptotes ψ_1, ψ_2 and ψ_3.

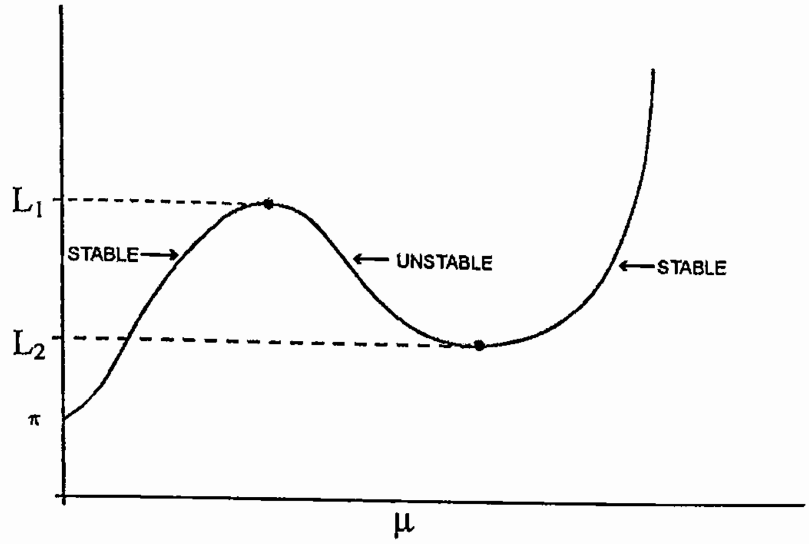

Fig.-3.9 : Depicts schematic representation of the stability of the model depends upon roots of the cubic equation.

If R approaches to R_1 and coincide to R_1 then $\psi_2 = \overline{\psi} = \psi_3 = \overline{\psi}_2$. Thus two vertical asymptotes of the left of the Fig.-(3.8) coincide. If $R > R_1$, for $0 < \mu < \psi_3$ $h(0) = \pi$ and $h(\psi_3) = \infty$. By the definition of continuity, when R is above and not too far from R_1, $h(\mu)$ has a local maximum near $\overline{\psi}_2$, which has been observed in Fig.-(3.9). Again it can be shown that h is monotone increasing when R is sufficiently large.

3.7 PHYSICAL INTERPRETAION OF THE RESULTS:

All the previous information's are summarized in Fig.-(3.10), in which $\log_{10}\mu$ have been considered in the ordinate and the length, L in x-axis for several values of R, keeping $Q = 302$ fixed.

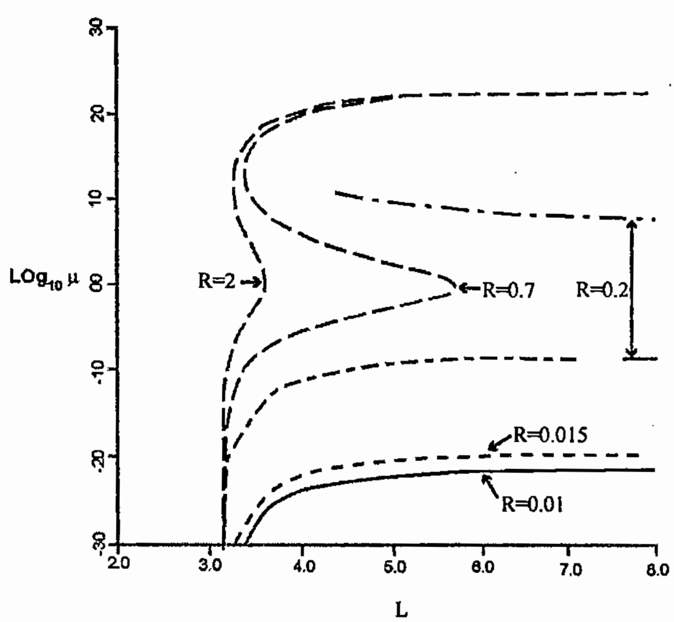

Fig.-3.10: Depicts the relationship between L and $\log_{10}\mu$ for the highest value of the control parameter $Q = 302$, following various values of R.

The result may be interpreted as follows: if $R < \overline{\overline{R}}$, then outbreak steady state is impossible. If $L > \pi$, a low endemic state persist, but the maximum scaled budworm density is below ψ_1. That is, ψ is less than the spatially uniform solution. Every point on the graph, Fig.- (3.10) corresponds to a solution of the Eq.-(3.35). On the other hand if $L < \pi$, then all solution of Eq-(3.32) and Eq.-(3.33) tends to zero as $t \to \infty$. If $L > \pi$ and if the initial density is not identically zero, then the solution of Eq.-(3.32) and Eq.-(3.33) tends to the steady solution $\eta(x)$ whose maximum μ is represented in Fig.-(3.8) or the corresponding lower curves of Fig.-(3.10). So that solutions are globally attractive.

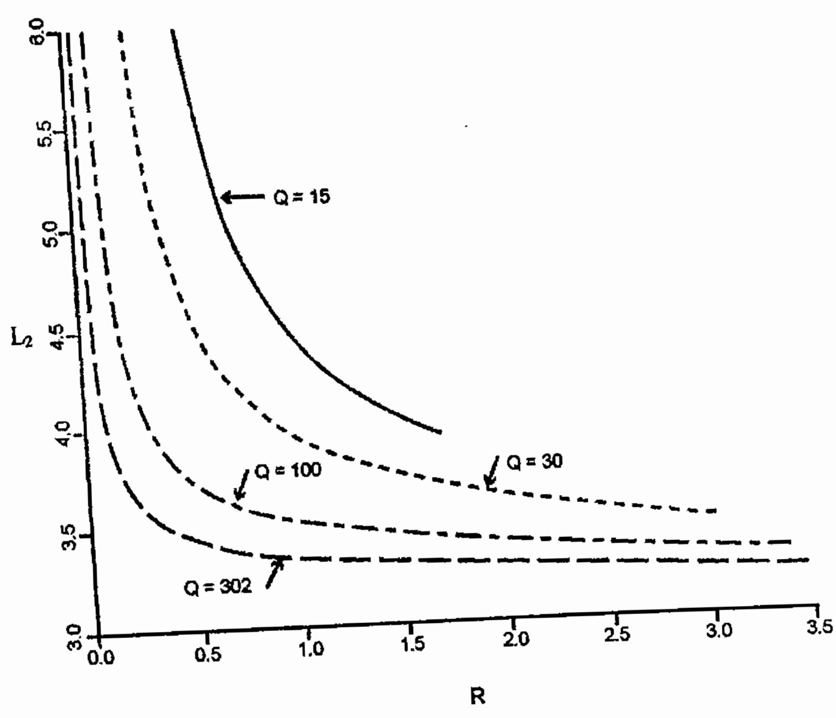

Fig.-3.11: Depicts the relationship between R and L_2 following various values of Q.

For $\overline{\overline{R}} < R < R_1$, the situation will be more completed because an additional length $L_2(R)$ forces an outbreak in the strip whose width $L > L_2(R)$. For different values of Q (15,30, 100 and 302), $L_2(R)$ are depicted in Fig.-(3.11). As mentioned earlier, only the low endemic steady state is possible if $\pi < L < L_2(R)$. On the other hand, there are three steady states as shown in Fig.-(3.8), if $L > L_2$. The left-most part of Fig.-(3.8) corresponds to the low endemic state and the right-most one is an outbreak. The initial budworm density in the strip will be determined by these three states the budworm will finally be reached. The methods we have discussed is not sufficient to determine whether or not an outbreak will occur for arbitrary initial budworm densities, however, the development is sufficient that the low endemic state and the outbreak state are both asymptotically stable. Moreover, the solution processes which correspond to the middle branch in Fig.-(3.8) is unstable. If we choose the initial density, which is everywhere greater than the unstable one, then the budworm population will tends to outbreak state. On the other hand if we choose the initial density which is everywhere less than the unstable one, then the budworm population will tends to the low endemic state.

It is more simple, if the initial density $\psi(x,0)$, which is bounded above by the function $\eta(x)$ defined in Eq.-(3.37) with a value of $\mu < \mu_2$, then there will be no outbreak.

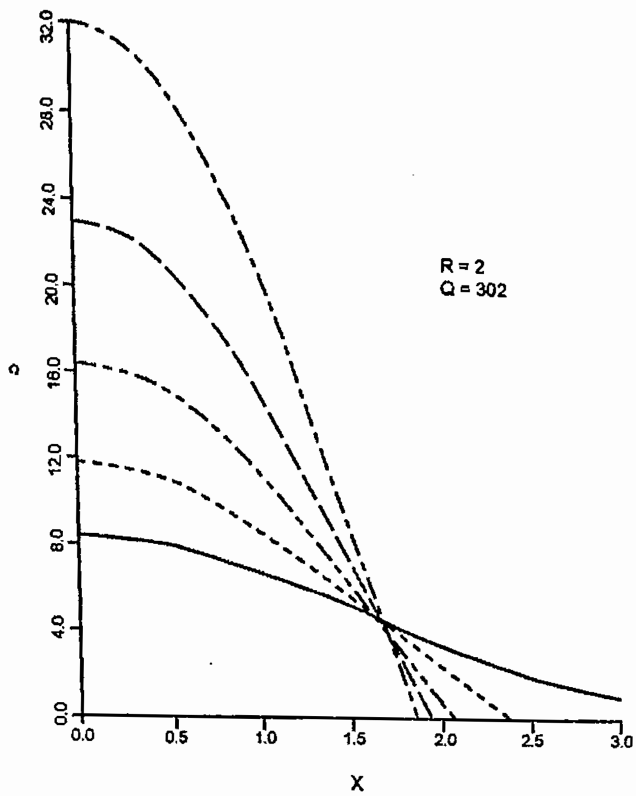

Fig.-3.12: Graphical representation of $\eta(x)$ and μ for $R = 2$ and $Q = 302$

For $R = 2$, $Q = 302$, the graph of function $\eta(x)$ and different values of μ is depicted in Fig-(3.12). By the way it is to be mentioned that an outbreak will be occurred either by a very high but narrow peak in the initial population density or by a somewhat lower but broader initial population density.

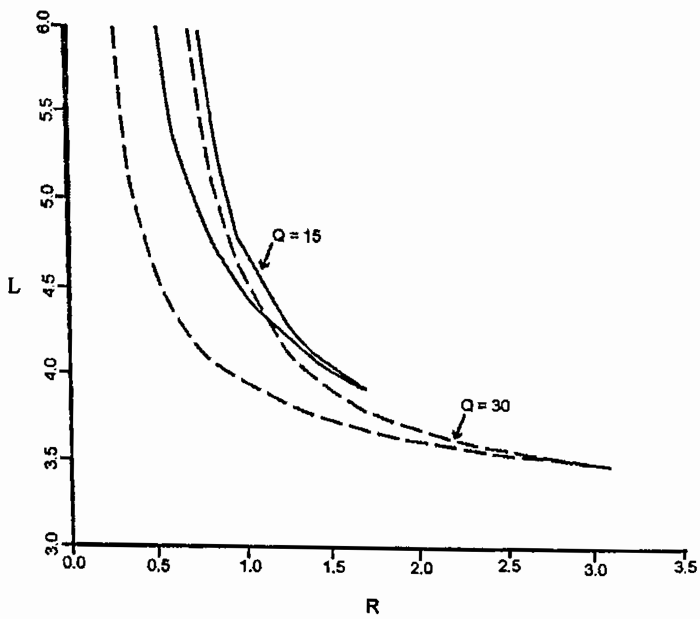

Fig.-3.13: Graphical representation of $L_1(R, Q)$ and $L_2(R, Q)$ instead of $Q = 302$

When R is increased through R_1, the two branches of the graph in Fig.-(3.8) coalesces which has also been established in Fig-(3.9). Two critical lengths $L_1(R,Q)$ and $L_2(R,Q)$ have been monitored. When $L_1 < L_2$, only then the low endemic state exists. Further when $L_2 < L < L_1$, then both the low endemic and outbreak states exist, however, for $L > L_1$, outbreak will exist. Moreover, R continuously increases, then the maximum and minimum in Fig.-(3.9) ultimately coincide. In all these situations the graph becomes monotone increasing and there are no critical length. From Fig.-(3.13), we have

$L_1(R,Q)$ and $L_2(R,Q)$ which are the functions of R for different small changes in Q and Fig.-(3.14) depicted a partial part of the graph of $L_1(R,Q)$ and $L_2(R,Q)$ are also the function of R, for a particular value of $Q = 302$.

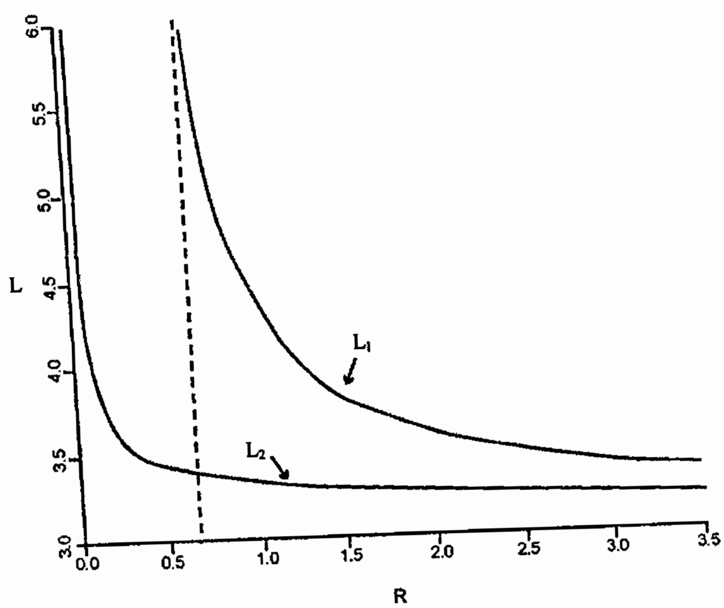

Fig.-3.14: Graphical representation of L_1 and L_2 for highest value $Q = 302$, which a part of the previous figure.

3.8 CONCLUDING REMARKS:

In this chapter we have investigated the evolution of budworm pest population which inhabits a strip of dimensionless width L, where it is considered that no member of the pest population can survive outside the strip. First of all we have considered the linear growth model and shown that if $L < \pi$, where π

be the strip size, then every solution of the diffusion equation approaches zero as time increases. On the other hand, if $L > \pi$ there are arbitrarily small populations which will grow without bound. Secondly we have considered the logistic growth model for $L = \pi$ is the critical width and in order to prove this concept we have used the first integrals method. A unique positive solution is observed when $L > \pi$, and has zero solution for $L < \pi$. In this context a bifurcation is occurred at $L = \pi$. Further, Eq.-(3.9) has been verified with the assumption made by Eq.-(3.11) and Eq.-(3.12). The positive and negative values of $\phi(\eta\,;R,Q)$ have been presented in Table- 3.1 to Table- 3.3 to meet the properties of roots corresponding to the vital parameters R and Q. Two local maxima and a local minimum have been cited at ψ_1 and ψ_3 and at ψ_2 for F.

CHAPTER-FOUR

ANALYSIS THE CRITICAL BEHAVIOUR OF SPRUCE BUDWORM PEST POPULATION

CHAPTER FOUR

ANALYSIS THE CRITICAL BEHAVIOUR OF SPRUCE BUDWORM PEST POPULATION

4.1 INTRODUCTION:

Forests constitute an important category of renewable resources. They are important not only for agricultural industry but also for several other activities and ecological balance. They cover a large proportion of the land surface of the earth. On the hand, though photo-bio-chemical processes they entrap enormous amount of solar energy and store in the form of valuable resources for the humanity, on the other they consume carbon dioxide (a challenging pollutant growing with industrialization), and liberate oxygen for the sustenance of the aerobic world. They enrich the fertility of the soil through a constant supply of decaying matter, make the soil structure porous and conducive to retention and seepage of the precipitation received, and thereby augment the underground water supply. Put succinctly, forests have a great bearing on ground water supplies, soil erosion, climate regulation, flood control which have been elaborately discussed in chapter two, section 2.2.

Forestry management involves a number of interesting economic and biological problem. From the economic point of view, a standing forest is just one particular form of growing capital, growing with the growth of the timber. An obvious and important objective in the management of forestry is to improve the growth and quality of the trees. To this end, clear cutting and replanting, and thinning from and/or below are the common strategies and practice adopted in the forestry management.

Leaves prepare food for the plant, and thus they play a cardinal role in the healthy and rapid growth of the forestry. The conservation and protection of leaves from the insect and pests is a very important aspect in the forestry management.

Recently, Wright [1] has carried out a detailed qualitative analysis of management problems related to forestry by employing concepts of Thom`s Catastrophe theory [2,3]. Regarding forestry management, he has examined the Holling`s model [4] and has prescribed a management strategy for controlling the pest using a combination of felling trees and spraying with a pesticides. The model relates to the occasional population burst of the spruce budworm at an epidemic level in the Coniferous forests of Canada.

In this chapter, we propose and analyze a deterministic version of the Ludwig, Jones, and Holling`s model [5]. The model has been formulated as a birth-and-death process. The purpose of this work is to explore possible mathematical structures in the model by incorporating 'fluctuation' components. The mean evolution of the process and the stead-state solution along with the minimum and maximum values of the dominant control parameters have been analyzed in this direction.

4.2 DETERMINISTIC FORMULATION OF THE MODEL:

Left to itself, the growth of the population of budworm is governed by two factors, the leave-area, leaves being the food for the budworm, the predation by the birds. In the absence of the predation, the population growth would follow the logistic law. However, the predation by the birds puts a control on the growth.

A close scrutiny of the situation reveals that the predation itself follow a natural pattern. Initially the budworms are so few that the birds can not easily locate them. At the later stage, the population attains a good size, as the size of the population tends towards the carrying capacity of the system (the available leaf-area) their rate of reproduction declines due to defoliation. Fig.-(4.1) presents a schematic description of the this situation.

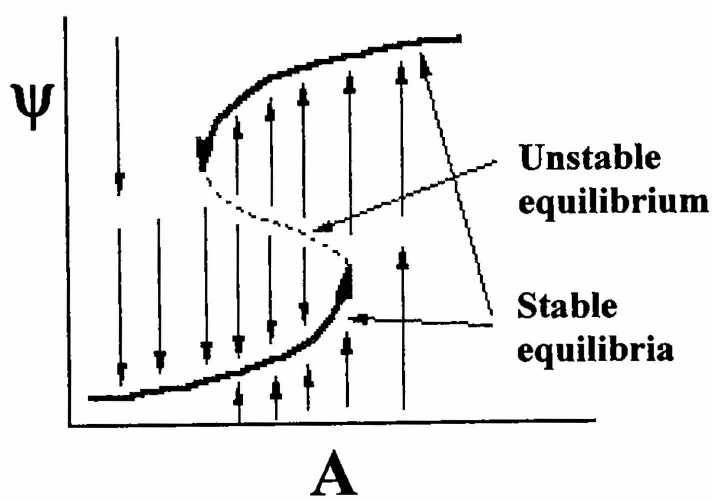

Fig.-4.1: With increasing age of tree, the leaf area (A) available as food increases and result into catastrophe growth of budworm population (ψ). The broken part of the curve represents the unstable population.

It has been empirically observed that when sufficiently large leaf-area becomes available with aging of trees, the two distinct stable states become accessible to the system (see Fig.-4.2). In the view of these facts Ludwig et al. [5] described the population growth of budworm by deterministic differential equation

$$\frac{d\psi}{dt} = r\psi(1-\frac{\psi}{K}) - \frac{b\psi^2}{a^2+\psi^2} \quad \text{.......(4.1)}$$

where $\psi = \psi(t)$ is the population size at time t, r the intrinsic growth rate of the pest, K the carrying capacity of the system, 'b' a measure of the intensity of the predation, and 'a' as measure of the discouragement of the predator when only a few prays are left. The solution to the Eq.-(4.1) with the initial condition

$$\psi(0) = \psi_0 \quad \text{.......(4.2)}$$

describe the deterministic temporal evolution of the budworm population that we shall frequently call as the system in the sequel.

4.3 THE STOCHASTIC VERSION OF THE MODEL :

As already mentioned in the introduction, in reality $\psi(t)$ can not be a deterministic variable. So we consider $\psi(t)$ as a non-negative integer-valued random variable with $p_{\psi}(t)$ as the probability of there being $\psi(t)$ budworm at time t. The phenomenon of the pest growth may be considered as a birth-and- death process, characterized by the following transition probabilities during the time interval $(t, t+\delta t)$:

$$pr[\psi \to \psi+1 \text{ Interval}] = \lambda_{\psi}\delta t + 0(\delta t),\ \psi \geq 0, \quad \text{.......(4.3)}$$

$$pr[\psi \to \psi-1 \text{ Interval}] = \mu_{\psi}\delta t + 0(\delta t),\ \psi \geq 0, \quad \text{.......(4.4)}$$

with slightly modified birth and death rate constants λ_{ψ} and μ_{ψ} given

$$\lambda_{\psi} = \gamma\psi(1-\frac{\psi}{K})$$

and

$$\mu_{\psi} = \frac{\beta\frac{\psi^2}{K}}{\delta^2 + \frac{\psi^2}{K^2}} \quad \text{.......(4.5)}$$

The coefficients 'a' and 'b' of the deterministic model have been so related as to incorporate the effect of spraying and felling the trees thereby matching the control measures competitive with the intrinsic growth rate r. Thus

$$r=\gamma\ , \quad b=K\beta \quad \text{and} \quad a=K\delta \qquad (4.6)$$

4.4 ANALYSIY OF THE MODEL:

The master equation of the growth process is given by

$$p(\psi,t)=\lambda_{\psi-1}\,p(\psi-1,t)+\mu_{\psi+1}\,p(\psi+1,\ t)-(\lambda_{\psi}+\mu_{\psi})\,p(\psi,t) \qquad (4.7)$$

with initial distribution

$$p_{\psi}(0)=\delta\,(\psi-\psi_0)$$

where $\delta\,(\psi-\psi_0)$ is the usual Dirac's delta function.

The non-linear character of transition probabilities renders the exact solution of Eq.-(4.7) well neigh impossible. The non-linearity leads to a rather intractable hierarchy of relationship among the moments of the probability distribution [6,7]. though an exact analysis of the model, valid for all times is not possible, nevertheless we can get significant insight into the stochastic evaluation of the process in the asymptotic regime $K>>1$. The model can be represented approximately by a continuous Markov process $\psi(t)$ and the analytic solutions can be obtained by employing 'diffusion approximation technique', developed and used by several researchers [8-11].

The time evaluation of the system can, for large K, be approximately described by the stochastic differential equation (SDE)

$$d\psi(t)=\left[\gamma\psi(1-\frac{\psi}{K})-\beta\frac{\psi^2/K}{\delta^2+\left(\frac{\psi}{K}\right)^2}\right]dt$$

$$+\left[\gamma(1-\frac{\psi}{K}\right]^{\frac{1}{2}}dw_1(t)-\left[\beta\frac{\psi^2/K}{\delta^2+(\frac{\psi}{K})^2}\right]^{\frac{1}{2}}dw_2(t)\text{..........}(4.8)$$

to be interpreted in Ito sense [12]. Here $[w_1(t),\ w_2(t)]$ is two dimensional Wiener process whose components are statistically independent.

In our model, the birth-and-death conceived involves non-linear transition probabilities, and in steady-state, multiple states may become available to our system. Therefore as already discussed in the First Chapter, Section 1.4, the usual law of scaling fluctuations as of order $0(K)^{\frac{1}{2}}$ would not work. At the non-Morse critical point, and at Morse-critical points the scaling index $\nu \neq \frac{1}{2}$. Consequently, keeping with the modified van Kampen's system size expansion, developed by Dekker [10] and Fox [13] independently, for large K, we split the stochastic variable $\psi(t)$ into a deterministic component and a purely stochastic component by setting

$$\psi(t) = K\varphi(t) + K^{\nu} z(t), \quad 0 < \nu < 1 \qquad (4.9)$$

Here the first term on the right hand side represents the mean evolution of the system and the second term $K^{\nu} z(t)$, where $z(t) = 0(1)$, provides stochastic fluctuations around the mean. Further $\varphi(t)$ is a smoothly varying function that governs the macroscopic development of process, and $o < \nu < 1$ is so-called the scaling index for fluctuations. It will not be out of place to mention here that in the region far away from the Morse-critical points and non- Morse critical points of the system, and the fluctuations are again to be scaled as of order $0(K^{\frac{1}{2}})$, accordingly $\nu = \frac{1}{2}$ therein. However, at the critical points of both types, and in their close neighborhood, the fluctuations are enormously enhanced. Consequently, the Central limit theorem (CLT) does not hold in the close neighborhood of the critical point [14] and thus the system can not be subdivided into uncorrected subset.

Substituting Eq.-(4.9) in to Eq.-(4.8) and setting $\varepsilon = K^{\nu-1}$, we obtain

$$d\varphi + \varepsilon\, dz = \left[\gamma(\varphi+\varepsilon z)(1-\varphi-\varepsilon z) - \frac{\beta(\varphi+\varepsilon z}{\delta^2+(\varphi+\varepsilon z)^2}\right]dt$$

$$+K^{-\frac{1}{2}}\left[\gamma(\varphi+\varepsilon z)(1-\varphi-\varepsilon z)\right]dw_1$$

$$+K^{-\frac{1}{2}}\left[(\varphi+\varepsilon z)\{\delta^2+(\varphi+\varepsilon z)^2\}\right]^{\frac{1}{2}}dw_2$$

$$=\left[f_0(\varphi)+f_1(\varphi)\varepsilon z+f_2(\varphi)\varepsilon^2 z^2+f_3(\varphi)\varepsilon^3 z^3+.....\right]dt$$

$$+K^{-\frac{1}{2}}\left[g_0(\varphi)+g_1(\varphi)\varepsilon z+g_2(\varphi)\varepsilon^2 z^2+....\right]dw_1$$

$$-K^{-\frac{1}{2}}\left[h_0(\varphi)+h_1(\varphi)\varepsilon z+h_2(\varphi)\varepsilon^2 z^2+......\right]dw_2 \quad \text{.......... (4.10)}$$

where

$$f_0 = \gamma\varphi(1-\varphi) - \frac{\beta\varphi^2}{\delta^2+\varphi^2} \quad \text{.......... (4.11)}$$

$$..f_1(\varphi) = \frac{df_0(\varphi)}{d\varphi} = \gamma(1-2\varphi) - \frac{2\beta\delta^2\varphi}{(\delta^2+\varphi^2)^2} \quad \text{..........(4.12)}$$

$$f_2(\varphi)=\frac{1}{2}\frac{d^2 f_0(\varphi)}{d\varphi^2}=-\left[\gamma+\frac{\beta\delta^2(1-3\varphi^3)}{(\delta^2+\varphi^2)^3}\right] \qquad (4.13)$$

$$f_3(\varphi)=\frac{1}{6}\frac{d^3 f_0(\varphi)}{d\varphi^3}=\frac{\beta\delta^2(1+\delta^2-2\varphi^2)\varphi}{(\delta^2+\varphi^2)^4} \qquad (4.14)$$

$$g_0(\varphi)=[\gamma\varphi(1-\varphi]^{\frac{1}{2}} \qquad (4.15$$

$$g_1(\varphi)=\frac{1}{2}(1-2\varphi)\left[\frac{\varphi}{\varphi(1-\varphi)}\right]^{\frac{1}{2}} \qquad (1.16)$$

$$h_0(\varphi)=\varphi\left[\frac{\beta}{\delta^2+\varphi^2}\right]^{\frac{1}{2}} \qquad (1.17)$$

$$h_1(\varphi)=\delta^2\left[\frac{\beta}{(\delta^2+\varphi^2)^2}\right]^{\frac{1}{2}} \qquad (1.18)$$

Equating the coefficients of the leading term on both sides, we obtain

$$d\varphi(t)=f_0(\varphi)dt$$

or.

$$\frac{d\varphi}{dt}=\gamma\,\varphi(1-\varphi)-\frac{\beta\varphi^2}{\delta^2+\varphi^2} \qquad (4.19)$$

The term of the next lower yield the stochastic differential equation

$$dz(t) \cong \left[f_1(\varphi)\varepsilon z + f_2(\varphi)\varepsilon^2 z^2 + f_3(\varphi)\varepsilon^3 z^3 + ...\right]dt$$

$$+ K^{\frac{1}{2}-\nu}\left[g_1(\varphi)\varepsilon z + g_2(\varphi)\varepsilon^2 z^2 + ...\right]dw_1$$

$$- K^{-\frac{1}{2}}\left[h_1(\varphi)\varepsilon z + h_2(\varphi)\varepsilon^2 z^2 + ...\right]dw_2 \qquad (4.20)$$

Eq-(4.19) governs the mean evolution of the growth process. While Eq.-(4.20) describes the corresponding fluctuations. The non-linear Fokker-Planck equation (FPE) corresponding to the SDE, Eq.-(4.20) can be written as

$$\frac{\partial p(z,t)}{dt} = -\frac{\partial}{\partial z}\left[A(\varphi,z)p(z,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial z^2}\left[B(\varphi,z)p(z,t\right] \qquad (4.21)$$

where

$$A(\varphi,z) = f_1(\varphi)z + f_2(\varphi)\varepsilon z^2 + f_3(\varphi)\varepsilon^2 z^3 \qquad (4.22)$$

and

$$B(\varphi,z) = K^{1-2\nu}g(\varphi) \qquad (4.23)$$

with

$$g(\varphi) = g_0^{\,2}(\varphi) + h_0^{\,2}(\varphi) = \gamma\varphi(1-\varphi) + \frac{\beta^2}{\delta^2+\varphi^2} \qquad (4.24)$$

In Eq-(4.21), the term $A(\varphi,z)$ is referred to as drift velocity (transportation or convection term) or simply the drift coefficient; the term $B(\varphi,z)$ as conduction term or diffusion coefficient.

4.5 THE MEAN EVOLUTION OF THE PROCESS AND THE STEADY-STATE SOLUTION:

The differential Eq.-(4.19) governs the mean evolution of the process and can be written as

$$\frac{(\delta^2+\varphi^2)d\varphi}{\varphi[\varphi^3-\varphi^2+(\delta^2+\frac{\beta}{\gamma})\varphi-\delta^2]}=-\gamma\,dt \quad (4.25)$$

Setting $0=\varphi_1\le\varphi_2\le\varphi_3\le\varphi_4$, where $\varphi_i=2,3,4$ are the roots of the cubic equation.,

$$\varphi^3-\varphi^2+(\delta^2+\frac{\beta}{\gamma})\varphi-\delta^2=0 \quad (4.26)$$

Let $\varphi^3-\varphi^2+(\delta^2+\frac{\beta}{\delta})\varphi-\delta^2=(\varphi-\varphi_2)(\varphi-\varphi_3)(\varphi-\varphi_4)$ ....(4.26a)

and let $\frac{(\delta^2+\varphi^2)}{\varphi(\varphi-\varphi_2)(\varphi-\varphi_3)(\varphi-\varphi_4)}=\frac{A}{\varphi}+\frac{B}{\varphi-\varphi_2}+\frac{C}{\varphi-\varphi_3}+\frac{D}{\varphi-\varphi_4}$ ....(4.26b)

Multiplying both sides of the Eq.-(4.26b) by $\varphi(\varphi-\varphi_2)(\varphi-\varphi_3)(\varphi-\varphi_4)$ we get

$$(\delta^2+\varphi^2)=A(\varphi-\varphi_2)(\varphi-\varphi_3)(\varphi-\varphi_4)+B\varphi(\varphi-\varphi_3)(\varphi-\varphi_4)$$
$$+C\varphi(\varphi-\varphi_2)(\varphi-\varphi_4)+D\varphi(\varphi-\varphi_2)(\varphi-\varphi_3) \quad \text{...........(4.26c)}$$

put $\varphi=0$, φ_2, φ_3, and φ_4 in Eq.-(4.26c) respectively, we have

$$A=\frac{-\delta^2}{\varphi_2\varphi_3\varphi_4}, \qquad B=\frac{\delta^2+\varphi_2{}^2}{\varphi_2(\varphi_2-\varphi_3)(\varphi_2-\varphi_4)},$$

$$C=\frac{\delta^2+\varphi_3{}^2}{\varphi_3(\varphi_3-\varphi_2)(\varphi_3-\varphi_4)} \quad \text{and} \quad D=\frac{\delta^2+\varphi_4{}^2}{\varphi_4(\varphi_4-\varphi_2)(\varphi_4-\varphi_3)}$$

Therefore $$\frac{\delta^2+\varphi^2}{\varphi(\varphi-\varphi_2)(\varphi-\varphi\ )(\varphi-\varphi_4)}=\frac{-\delta^2}{\varphi\varphi_2\varphi_3\varphi_4}+\frac{\delta^2+\varphi_2{}^2}{\varphi_2(\varphi-\varphi_2)(\varphi_2-\varphi_3)(\varphi_2-\varphi_4)}$$

$$+\frac{\delta^2+\varphi_3{}^2}{\varphi_3(\varphi-\varphi_3)(\varphi_3-\varphi_2)(\varphi_3-\varphi_4)}+\frac{\delta^2+\varphi_4}{\varphi_4(\varphi-\varphi_4)(\phi_4-\varphi_2)(\varphi_4-\varphi_3)} \quad \text{....(4.26d)}$$

Now from Eq.-(4.25) we get,

$$\int\left[\begin{array}{l}\dfrac{-\delta^2}{\varphi\varphi_2\varphi_3\varphi_4}+\dfrac{\delta^2+\varphi_2{}^2}{\varphi_2(\varphi-\varphi_2)(\varphi_2-\varphi_3)(\varphi_2-\varphi_4)}+\dfrac{\delta^2+\varphi_3{}^2}{\varphi_3(\varphi-\varphi_3)(\varphi_3-\varphi_2)(\varphi_3-\varphi_4)}\\+\dfrac{\delta^2+\varphi_2}{\varphi_4(\varphi-\varphi_4)(\varphi_4-\varphi_2)(\varphi_4-\varphi_3)}\end{array}\right]d\varphi=-\int\gamma dt$$

$$\text{...........(4.26}e\text{)}$$

Setting $\delta^2 = \dfrac{-\varphi}{(\varphi-\varphi_1)(\varphi_3-\varphi_2)(\varphi_4-\varphi_3)(\varphi_4-\varphi_2)}$ in Eq.-(2.26e) we obtain,

$$\int\left[\begin{array}{l}\dfrac{1}{\varphi_2\varphi_3\varphi_4(\varphi-\phi_1)(\varphi_3-\varphi_2)(\phi_4-\varphi_3)(\varphi_4-\varphi_2)}+\dfrac{\delta^2+\varphi_2{}^2}{\varphi_2(\varphi-\varphi_2)(\varphi_2-\varphi_3)(\varphi_2-\varphi_4)}\\+\dfrac{\delta^2+\varphi_3{}^2}{\varphi_3(\varphi-\varphi_3)(\varphi_3-\varphi_2)(\varphi_3-\varphi_4)}+\dfrac{\delta^2+\varphi_2}{\varphi_4(\varphi-\varphi_4)(\varphi_4-\varphi_2)(\varphi_4-\varphi_3)}\end{array}\right]d\varphi=-\int\gamma dt..$$
...............(4.26f)

Multiplying both sides of Eq.-(2.26f) by $\varphi_2\varphi_3\varphi_4(\varphi_4-\varphi_3)(\varphi_4-\varphi_2)(\varphi_3-\varphi_2)$ we get,

$$\int\left[\begin{array}{l}\dfrac{1}{(\varphi-\phi_1)}+\dfrac{(\delta^2+\varphi_2{}^2)\varphi_3\varphi_4(\varphi_4-\varphi_3)}{(\varphi-\varphi_2)}+\\\dfrac{(\delta^2+\varphi_3{}^2)\varphi_2\varphi_4(\varphi_4-\varphi_2)}{(\varphi-\varphi_3)}+\dfrac{(\delta^2+\varphi_2)\varphi_2\varphi_3(\varphi_3-\varphi_2)}{(\varphi-\varphi_4)}\end{array}\right]d\varphi=-\int\left[\dfrac{\gamma\varphi_2\varphi_3\varphi_4(\varphi_4-\varphi_3)}{(\varphi_4-\varphi_2)(\varphi_3-\varphi_2)}\right]\gamma dt$$

Integrating on both sides we have,

$$\log(\varphi-\varphi_1)+(\delta^2+\varphi_2{}^2)\varphi_3\varphi_4(\varphi_4-\varphi_3)\log(\varphi-\varphi_2)+(\delta^2+\varphi_3{}^2)\varphi_2\varphi_4(\varphi_4-\varphi_2)\log(\varphi-\varphi_3)+$$
$$(\delta^2+\varphi_4{}^2)\varphi_2\varphi_3(\varphi_3-\varphi_2)\log(\varphi-\varphi_2)=-\gamma\varphi_2\varphi_3\varphi_4(\varphi_4-\varphi_3)(\varphi_4-\varphi_2)(\varphi_3-\varphi_2)t+\log A$$
......(4.26h)

Setting

$$\left.\begin{array}{l}\varphi_0=\varphi(0)\ \geq 0\\a_2=(\delta^2+\varphi_2{}^2)(\varphi_4-\varphi_3)\varphi_4\varphi_3\geq 0\\a_3=(\delta^2+\varphi_3{}^2)(\varphi_4-\varphi_2)\varphi_4\varphi_2\geq 0\\a_4=(\delta^2+\varphi_4{}^2)(\varphi_3-\varphi_2)\varphi_3\varphi_2\geq 0.\end{array}\right]$$
......................................(4.26i)

we obtain from Eq.-(2.26h)

$$\log(\varphi-\varphi_1)+a_2\log(\varphi-\varphi_2)+a_3\log(\varphi-\varphi_3)+$$
$$a_4\log(\varphi-\varphi_2)=-\gamma\varphi_2\varphi_3\varphi_4(\varphi_4-\varphi_3)(\varphi_4-\varphi_2)(\varphi_3-\varphi_2)t+\log A$$

or $\log\left[(\phi-\varphi_1).(\varphi-\varphi_2)^{a_2}.(\varphi-\varphi_3)^{a_3}.(\varphi-\varphi_4)^{a_4}\right]$

$$=-\gamma\varphi_2\varphi_3\varphi_4(\varphi_4-\varphi_3)(\varphi_4-\varphi_2)(\varphi_3-\varphi_2)t+\log A$$

or, $\phi-\varphi_1).(\varphi-\varphi_2)^{a_2}.(\varphi-\varphi_3)^{a_3}.(\varphi-\varphi_4)^{a_4}=A\exp\left[-\gamma\varphi_2\varphi_3\varphi_4(\varphi_4-\varphi_3)(\varphi_4-\varphi_2)(\varphi_3-\varphi_2)\right]$

..............................(2.27)

When $t=0$ then $\varphi=\varphi(0)=\varphi_0$, so we have

$$A=(\phi_0-\varphi_1).(\varphi_0-\varphi_2)^{a_2}.(\varphi_0-\varphi_3)^{a_3}.(\varphi_0-\varphi_4)^{a_4}$$

Thus finally we get,

$$\left(\frac{\varphi-\varphi_1}{\varphi_0-\varphi_1}\right)\left(\frac{\varphi-\varphi_2}{\varphi_0-\varphi_2}\right)^{a_2}\left(\frac{\varphi-\varphi_3}{\varphi_0-\varphi_3}\right)^{a_3}\left(\frac{\varphi-\varphi_4}{\varphi_0-\varphi_4}\right)=\exp\begin{bmatrix}-\gamma\varphi_2\varphi_3\varphi_4(\varphi_4-\varphi_3)\\(\varphi_4-\varphi_2)(\varphi_3-\varphi_2)t\end{bmatrix} \quad \text{......................(4.28)}$$

Since $\gamma\varphi_2\varphi_3\varphi_4(\varphi_4-\varphi_3)(\varphi_4-\varphi_2)(\varphi_3-\varphi_2)\geq 0$ .... (4.29)

We conclude from the structure of Eq.-(4.27), that as $t\to\infty$, the asymptotic value $\varphi(\infty)$ of $\varphi(t)$ remains bounded, and hence system will remain always bounded.

The state $\varphi=\varphi_1=0$ is an absorbing state and is of no relevance for us. In the limit of the large time the steady state value φ tend to φ_2 or φ_3 or φ_4, according to as φ_0 is less than or equal to greater than φ_3 (i. e., according to as $\varphi_0<\varphi_3$ or $\varphi_0=\varphi_3$ or $\varphi_0>\varphi_3$).

To explore the catastrophic behavior of the model, it would be instructive to examine the steady-state solution φ_s of Eq.-(4.19), which describes the eventual population size. To this we begin with Eq.-(4.1). For the sake of clarity we would like to workout with the extensive variable ψ. Eq.-(4.1) can be easily restored by setting $\psi = K\varphi$, $b = K\beta$ and $a = \delta K$, thus we obtain,

$$\frac{d\psi}{dt} = r\psi(1-\frac{\psi}{K}) - \frac{b\psi^2}{a^2+\psi^2}$$

On the carrying out the transformation

$$X = \frac{\psi}{a}, \quad \tau = \frac{bt}{a}, \quad R = \frac{ar}{b} \quad \text{and} \quad Q = \frac{K}{a} \qquad (4.30)$$

Eq.-(4.1) transform to

$$\frac{dX}{d\tau} = RX(1-\frac{X}{Q}) - \frac{X^2}{1+X^2} \qquad (4.31)$$

or,

$$\frac{dX}{d\tau} = -\frac{RX}{Q(1+X^2)}\left[X^3 - QX^2 + (1+\frac{Q}{R})X - Q\right] \qquad (4..32)$$

Here it will be worth noting that Q is a measure of the carrying capacity which increases with the age of trees and can be reduced by felling them. Further, R is a measure of the intrinsic growth rate in the presence of the predation and can be regulated by spraying pesticides. In what follows we shall observe that Q and R would play the role of control variable.

Considering the evolution quasi-static for large time, we note that the equilibriums are given by $X_s = 0$, and by the roots of the cubic equation

$$X_s^3 - QX_s^2 + (1+\frac{Q}{R})X_s - Q = 0 \qquad (4.33)$$

or, $$R(1-\frac{X}{Q}) = \frac{X}{1+X^2} \qquad (4.33a)$$

Ignoring the trivial value $X_s = 0$, which constitutes and absorbing state or a so-called natural boundary of the system, we obtain the necessary quatric potential

$$V(X;Q,R) = \frac{1}{4}X^4 - \frac{1}{4}QX^3 + \frac{1}{2}(1+\frac{Q}{R})X^2 - QX \qquad (4.34)$$

for a cusp catastrophe. Within well defined parametric space, the Eq.-(4.33) possesses three non-negative distinct roots, $0 < X_1 < X_2 < X_3$(say). Further, using the concepts of linear stability, it can be shown that X_1 and X_3 correspond to stable states, while X_2 is unstable; and by changing the control parameters R and Q, it is possible to cause the unstable state is coalesce with one of the stable steady-states.

If $X_2 \to X_1$ or $X_2 \to X_3$ i.e if $X_2 = X_1$ or $X_2 = X_3$ the Eq.-(4.33a) has only two distinct roots and unstable-state for which occurs for X_2 is coalesce with any one stable steady-state. For this reason the Eq.-(4.33a) has only double root. We adopt the following expressions for the two side of Eq.-(4.33a):

$$F(X) = R(1-\frac{X}{Q}) \qquad (4.34a)$$

and

$$G(X) = \frac{X}{1+X^2} \qquad (4.34b)$$

A double root occur if

$$F(X) = G(X)$$

or, $$R(1-\frac{X}{Q}) = \frac{X}{1+X^2}$$

$$\frac{R}{Q}(Q-X) = \frac{X}{1+X^2} \qquad (4.34c)$$

and

$$\frac{dF}{dX} = \frac{dG}{dX}$$

or, $$\frac{d}{dX}\left[R(1-\frac{X}{Q})\right] = \frac{d}{dx}\left[\frac{X}{1+X^2}\right]$$

or, $$\frac{R}{Q} = \frac{X^2-1}{(1+X^2)^2}$$

or, $$R = \frac{X^2-1}{(1+X^2)^2}Q \qquad (4.34d)$$

Putting the value of $\frac{R}{Q}$ into Eq.-(4.34c) we have

$$\frac{X^2-1}{(1+X^2)^2}(Q-X)=\frac{X}{1+X^2}$$

or, $$Q-X=\frac{X+X^3}{X^2-1}$$

or, $$Q=\frac{2X^3}{X^2-1}$$

so we get from Eq.- (4.34d)

$$R=\frac{2X^3}{(1+X^2)^2}.$$

By this way we get,

$$R=\frac{2X^3}{(1+X^2)^2},\ Q=\frac{2X^3}{X^2-1}\quad\text{...(4.35)}$$

which is a parametric curve.

Therefore for the proper changing the control parameters R and Q the unstable state will coalesce with one of the steady- state on the parametric curve which is given by Eq.-(4.35) (See Nisbet and Gurney page 59-60) [15].

With the help of the parametric Eq.- (4.35), it can be seen that slow and small changes in R and/or Q can cause very rapid and large changes (outbreaks or collapse) in the population of budworm, exhibiting a catastrophe[16-20].

To get some deeper inside into the problem, we recast Eq.-(4.33), into

$$X_s^3 - QX_s^2 + PX_s - Q = 0 \qquad (4.36)$$

where

$$P = 1 + \frac{Q}{R} \quad \text{or} \quad Q = R(P-1).$$

we shall obtain condition on P and R for the existence of the non-trivial cusp region. Already we have got an inkling that more than one stable state may be accessible to the system only when leaf-area available for given intrinsic growth rate exceeds a certain critical value. If this condition is fulfilled then the cusp region would demarcate a non-degenerate $S-$shape region.

Doing a little bit of algebra, we can show that if $P = 9$, and $R = \frac{3\sqrt{3}}{8}$ (hence $Q = 3\sqrt{3}$), then Eq.-(4.33) possesses three identical roots equal to $\sqrt{3}$. That is the cusp-region degenerate into a single point in the parameter space given by

$$(Q, R) \cong (3\sqrt{3}, \ \frac{3\sqrt{3}}{8}) \qquad (4.37)$$

This is a non-Morse critical point in the parameter space (called the tip of the cusp), which unfolds into a cusp region extending over the interval $(R_{\min}, R_{\max})$ along the parameter R.

Further, we write Eq.-(4.36)

$$X^3 - QX^2 + PX - Q = 0$$

or, $$X^3 + 3(-\frac{Q}{3})X^2 + 3(\frac{P}{3})X + (-Q) = 0$$

or, $$dX^3 + 3eX^2 + 3fX + g = 0 \qquad (3.37a)$$

where $d = 1$, $e = -\frac{Q}{3}$, $f = \frac{P}{3}$ and $g = -Q$

If we reduce the roots of Eq.-(3.37a) by $-\frac{e}{d}$ the it takes the form

$$d^3X^3 + 3d(df - e^2)X + d^2g - 3def + 2e^3 = 0 \qquad (3.37b)$$

or, $$d^3x^3 + 3dHX + G = 0 \qquad (3.37c)$$

where

$$H = df - e^2 \text{ and } G = d^2g - 3def + 2e^2$$

Therefore Eq.-(3.37c) becomes

$$X^3 + \frac{3HX}{d^2} + \frac{G}{d^3} = 0 \qquad (3.37d)$$

If we multiply the roots of Eq.-(3.37d) by d, we obtained

$$X^3 + 3HX + G = 0 \quad \text{.......... (3.37e)}$$

The discriminate part of Eq.-(3.37e)

$$G^2 + 4H^3 = d^2\left\{d^2g^2 - 6defg + 4df^3 - 3e^2f^2 + 4e^3g\right\} \quad \text{.......... (3.37f)}$$

Putting the value of d, e, f and g in right hand side of Eq.-(3.37f), we get

$$G^2 + 4H^3 = 1^2\left\{1^2(-Q)^2 - 6.1.\left(-\frac{Q}{3}\right)\left(\frac{P}{3}\right)(-Q) + 4.1.\left(\frac{P}{3}\right)^3 3\left(-\frac{Q}{3}\right)^2\left(\frac{P}{3}\right)^2 + 4\left(-\frac{Q}{3}\right)^3(-Q)\right\}$$

$$= Q^2 - \frac{6PQ^2}{9} + \frac{4P^3}{27} - \frac{3P^2Q^2}{81} + \frac{4Q^4}{27} \quad \text{.......... (3.37g)}$$

Put the value of $Q = R(P-1)$ in Eq.-(3.37g), we obtained

$$G^2 + 4H^3 = R^2(p-1)^2 - \frac{2PR^2(p-1)^2}{3} + \frac{4P^3}{27} - \frac{P^2R^2(P-1)^2}{81} + \frac{4R^4(P-1)^4}{27}$$

or, $$G^2 + 4H^3 = \frac{4(P-1)^4}{27}R^4 + \left\{(p-1)^2 - \frac{2P(P-1)^2}{3} - \frac{P^2(P-1)^2}{27}\right\}R^2 + \frac{4P^3}{27} \quad \text{..(3.37h)}$$

which is quadratic in R^2.

Therefore, $G^2 + 4H^3 = 0$ gives

$$\frac{4(P-1)^4}{27}R^4 + \left\{(p-1)^2 - \frac{2P(P-1)^2}{3} - \frac{P^2(P-1)^2}{27}\right\}R^2 + \frac{4P^3}{27} = 0$$

or, $$4(P-1)^4R^4 + 27(P-1)^2R^2 - 18(p-1)^2 - P^2(P-1)^2 + 4P^3 = 0$$

or, $$4(P-1)^4R^4 + 27(P-1)^2\left(27 - 18P - P^2\right)R^2 + 4P^3 = 0$$

Hence

$$R^2 = \frac{-(P-1)^2(27-18P-P^2) \pm \sqrt{(P-1)^4(27-18P-P^2)^2 - 4.4.4(P-1).P^3}}{2.4(P-1)^4}$$

$$= \frac{(P-1)^2(P^2+8P-27) \pm (P-1)^2\sqrt{(27-18P-P^2)^2 - 64P^3}}{8(P-1)^4}$$

$$= \frac{(P^2+(8P-27) \pm \sqrt{(27-18P-P^2)^2 - 64P^3}}{8(P-1)^2}$$

$$= \frac{(P^2+(8P-27) \pm \sqrt{729+324P^2+P^4-972P-54P^2+36P^3-P^2-64P^3}}{8(P-1)^2}$$

$$= \frac{(P^2+(8P-27) \pm \sqrt{P^4-28P^3+270P^2-972P+729}}{8(P-1)^2}$$

$$= \frac{(P^2+(8P-27) \pm \sqrt{P^4-27P^3+243P^2-729P-P^3+27P^2-243P+729}}{8(P-1)^2}$$

$$= \frac{(P^2+(8P-27) \pm \sqrt{P(P^3-27P^2+243P-729)-1(P^3-27P^2+243P-729)}}{8(P-1)^2}$$

Finally,

$$R^2 = \frac{(P^2+!8P-27) \pm \sqrt{(P-1)(P-9)^3}}{8(P-1)^2}$$

Thus

$$R^2{}_{\min} = \frac{\left[(P^2+18P-27) - \{(p-1)(p-9)\}^{\frac{1}{2}}\right]}{\left[8(P-1)^2\right]} \quad \text{.. (4.38)}$$

and $$R^2_{max} = \frac{\left[(P^2+18P-27)+\{(p-1)(p-9)\}^{\frac{1}{2}}\right]}{\left[8(P-1)^2\right]} \qquad \text{.....(4.39)}$$

For $R_{min} < R, < R_{max}$ there exists three realizable steady states; at $R = R_{min}$, X_2 coalesces with X_3, and at $R = R_{max}$, X_2 coalesces with X_1.

For $P < 9$, there exists on any one realizable steady- states. The whole situation is summarized in Fig.-4.2

A very pertinent question arises at this stage : in which of these states does the system settle down ? From the point of view of forestry managers, a straight forward question is : What should be the optimal strategy to save the forest? Here we would address to the first question and would defer the second one till we carryout the stochastic analysis.

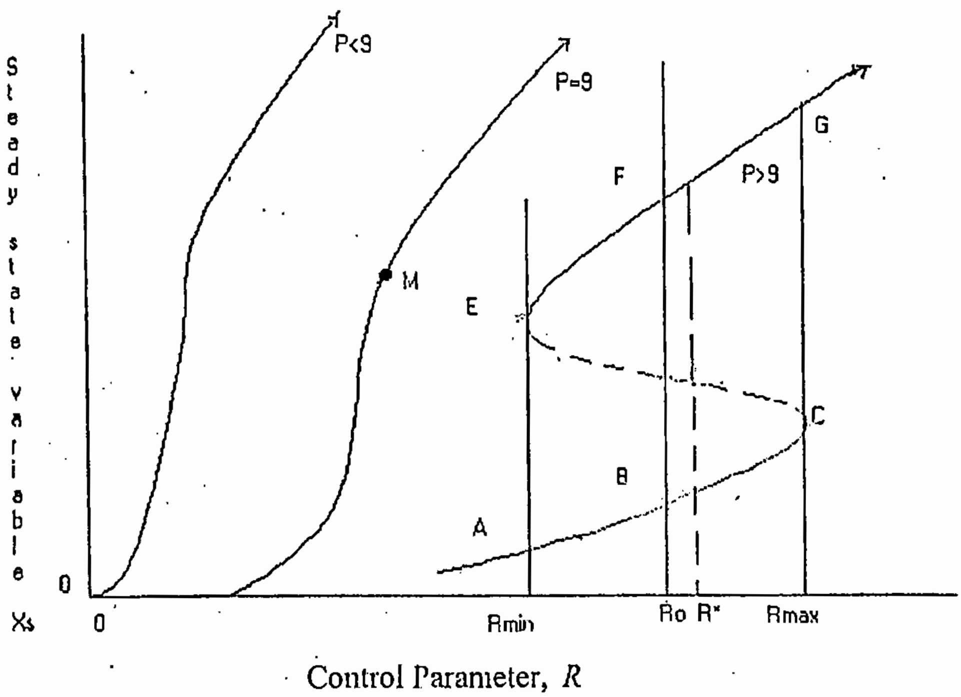

Fig.-4.2: variation of the Steady-State variable X_s with control parameter R is depicted of different values of P. The non-Morse critical point M occurs for $P = 9$ and P exceeds 9, it unfolds into a cusp region (not scaled)

As mentioned earlier, with the help of linear stability analysis, it can be shown that the lower branch ABC and the upper branch EFG in Fig.-4.2 correspond to stable states accessible to the system, the middle branch CDE depicted by the broken line, however, correspond to the unstable state cannot be realized.

From Fig.-4.2, we also find that as parameter R increases, the pest population also increases but remains confined to the lower branch ABC. But once the parameter reaches the threshold point C, the pest population jumps discontinuously from the lower branch ABC to the upper branch EFG. The analysis offers significant policy guidelines to the managers. They should not allow the attainment of the threshold value of the intrinsic growth parameter R. They have to bear in mind that once there is outburst, the control is not possible until R is managed to fall to the threshold level at point E on the upper branch (Fig,-4.2), by felling trees or heavy spray pests for, if it does not reach this level, the pest-size cannot be brought to the lower branch (controlled state), and saving the forest would become next to impossible. This is the well-known hysteresis effect [21] highlighted by Wright [1].

It is instructive to point out that model which involve discontinuities, (that is more than one stable states become accessible to the system represented by the model for a given set of control parameters), can be analyzed within the framework of catastrophe theory [22-24]. catastrophe theory can account for discontinuous behavior and can be employed for modeling first-order phase transition [16,14,25] or sudden change in the behavioral variable due to continuous changes in terms of certain control parameters of the system.

In the realm of catastrophe theory, the behaviour of the system is characterized by the shape of the relevant potential function. for the model under consideration, the potential functions is given by relation (4.34). appropriate scaling and translation reduces this quartic potential to

$$V(\xi : A, B) = \xi^4 + A\xi^2 + B \qquad (4.40)$$

where A and B are functions of the control parameters Q and R, and $\xi = X_s - X_c$. Obviously this canonical form correspond to Cusp Catastrophe which describes the behaviour of the model under consideration. In terms of the A and B, the different regions of the cusp are identified as follows :

(i) If $\frac{A^3}{27} + \frac{B^2}{4} > 0$, the potential function has exactly one minimum.

(ii) If $\frac{A^3}{27} + \frac{B^2}{4} < 0$, the potential function has two minimum.

(iii) When $\frac{A^3}{27} + \frac{B^2}{4} = 0$, it corresponds to the non-Morse critical point or the tip of the cusp beyond which the potential function starts unfolding, resulting into multiple steady-states.

4.6 CONCLUDING REMARKS:

In this chapter we have investigated the deterministic approach of Holling's model applied to a problem of pest control in the preservation and protection of a forest from pests. Specially we have considered the spread of spruce budworm at epidemic level in coniferous forests of Canada. With the increase age of balsam fir trees the leaf area also increases which sense the availability of food for the pest as a result the abnormal growth trends of the budworm population brings catastrophe. We have received a cubic equation which provides three roots and the roots are considered as the three states of the system. From the point of view of forestry managers, a straight forward question is : What should be the optimal strategy to save the forest? Such questions have been addressed through our lengthy calculations.

CHAPTER-FIVE

THE APPLICATION OF CATASTROPHE THEORY IN BUDWORM PEST POPULATION

CHAPTER FIVE

THE APPLICATION OF CATASTROPHE THEORY IN BUDWORM PEST POPULATION

5.1 INTRODUCTION:

In the preceding chapter, we have discussed the qualitative and quantative aspects of budworm pest model by Ludwig et al. [1]. Also the model highlighted and demonstrated by Jones [2], Kendeigh [3], Morris [4] and Mitchell [5]. However, it is very complicated to define the dynamic nature of management problems as it involves an important factor of uncertainty, arising from various sources-intrinsic as well as extrinsic to the management system of the proposed pest model. Jones [2] independent measure shows that among 150 to 200 larvae, approximately 25% of the foliage of the balsam fir trees consumed by the larvae per branch. Environmental factors like temperature, seasonal impact etc. are also responsible for the smooth development of such pest. Therefore, most of the management problem involves very sensitive parameters and the parameters are sensitive in the sense that a very small changes in the parameter lead to the unexpected large effects reflected in the response. It is also true that when sensitive parameters are present, the system under study, may exhibit a catastrophic behavior over certain region in the parameter space. The qualitative aspects like spruce budworm problem using elementary catastrophe theory have discussed by Peterman [6] and Wright [7]. At the very out set, we would like to mention that for the exhibition of a catastrophe, at least two stable states and one intermediatory unstable state must be accessible to the system [8-12]. The stochastic version of the proposed model have been carried out in the present chapter and a part of this chapter has developed in the last chapter, Chapter-Four, which is logical in the sense that the stochastic formulation and its analysis based on the deterministic approach of the solution processes. We observe that besides the trivial solution, the time evolution exhibits accessibility of three states, one of which is 'meta-stable' implying a transition from one stable state to another stable state. The details of such a transition are worked out in this chapter and the population variable has been studied as a function of model parameters. The model also depicts critical behaviour [13-15] in the sense that around the critical region, including the critical point, the relevant probability density function describing the model switches abruptly from uni-model to a bimodal distribution. It has also been shown that the model is related to the cusp catastrophe theory [16].

5.2 STOACHASTIC ANALYSIS :

As pointed out in the introduction, chance plays a vital role in bringing out phase transition. To this end, we shall now examine the stochastic behaviour of the model and we shall examine the model in three distinct region following calculations of previous chapter :

(1) Far From the Critical Point Outside the Cusp

(2) At Critical Point and its Neighborhood within the Cusp Region and

(3) Far From The Critical Point in the Cusp Region.

5.2.1 FAR FROM THE CRITICAL POINT OUTSIDE THE CUSP:

This situation occur when the control parameter, representing the intrinsic growth rate is less than 9. There is only one real root of the Eq.-(4.33), signifying a unique population size of the pest. Under this condition, the dominant non-vanishing term in the drift coefficient is $f_1(\psi_s)$, and consequently the scaling index ν, representing the order of the drift of fluctuation is obtained by equating $1-2\nu$ to zero (so that the drift and diffusion may be of matching signification). This means $\nu=\frac{1}{2}$. Substituting $\nu=\frac{1}{2}$ in Eq.-(4.12) to Eq.-(4.18). then the Fokker Plank Equation (FPE), Eq.-(4.21) becomes

$$\frac{\partial P(z,t)}{\partial t}=-f_1(\varphi)\frac{\partial}{\partial z}\{z\,p(z,t)\}+\frac{1}{2}g(\phi)\frac{\partial^2}{2}p(z.t) \quad \text{(5.1)}$$

where $f_1(\varphi)$ is given by equation Eq.-(4,12) and $g(\varphi)$ by Eq.-(4.24)

Equation Eq.-(5.1) corresponds to the well-know non-stationary Ornstein-Uhlenbeck process [17], whose solution with natural boundaries is the Gaussian distribution given by

$$p(z,t)=(2\pi\sigma_z^{\ 2})^{-\frac{1}{2}}\exp(-\frac{z^2}{2\sigma_z^{\ 2}}) \quad \text{(5.2)}$$

with

$$\sigma_z^{\ 2}=f_1(\varphi)\int_{\varphi_0}^{\varphi(t)}\left[\frac{g(u)}{f_0^{\ 3}(\varphi)}\right]du \quad \text{(5.3)}$$

where $f_0(\varphi)$ is given in Eq.-(4.11). Thus at any time t the mean and variance are given by

$$E[n(t)] = K\varphi(t) \qquad (5.4)$$

$$Var[n(t)] = K\sigma_z^{\,2}(t) \qquad (5.5)$$

with corresponding probability density function

$$P(n,t) = (2\pi K\sigma_z^{\,2})^{-\frac{1}{2}} \frac{\exp\left[-\{n - K\varphi(t)\}^2\right]}{2K\sigma_z^{\,2}} \qquad (5.6)$$

5.2.2 AT CRITICAL POINT AND ITS NEIGHBORHOOD WITHIN THE CUSP REGION:

The critical point of the model corresponds to $P = 9$ and $Q = 3\sqrt{3}$. The usual scaling of the fluctuation given $o(K^{\frac{1}{2}})$ breaks down at the critical point, and its immediate vicinity, wherein the fluctuations are greatly enhanced. Once disturbed from the equilibrium, the system is restored to the equilibrium very slowly. At the critical point, all the three roots of Eq.-(4.36) becomes identical *i, e.,* $X_1 = X_2 = X_3 = \frac{Q}{\sqrt{3}}$, and consequently we find that in the drift term

$A(\varphi,x)$ (see Eq-(4.22)), $f_1(\varphi_s)=f_2(\varphi_s)=0$, leaving $f_3(\varphi_s)$ as the dominant term. Thus the Fokker Plank Equation (FPE) Eq.-(4.21) reduce to

$$\frac{\partial P(z,t)}{\partial t}=-\varepsilon^2 f_3(\varphi_s)\frac{\partial}{\partial z}p(z^3,t)+\frac{1}{2}K^{1-2\nu}\frac{\partial^2 P'}{\partial z^2} \quad \text{.......} (5.7)$$

where

$$f_3(\varphi_s)=-\frac{729}{128}\beta \quad \text{and} \quad g(\varphi_s)=\frac{3}{2}\beta \quad \text{.......}(5.8)$$

Further, since the stochastic variable z is supposed to be $0(1)$, the drift and diffusion processes are of comparable significance only when ε^2 and $K^{1-2\nu}$ are of the same order of magnitude. This implies $2\nu-2=1-2\nu$ or $\nu=\frac{3}{2}$, indicating that the usual scaling law of fluctuations (i, e. $\nu=\frac{1}{2}$) is no more valid at the critical point. Substituting $\nu=\frac{3}{4}$ into Eq.-(5.7), we obtain

$$\frac{\partial p}{\partial t}=K^{-\frac{1}{2}}\left[\frac{729\beta}{128}\frac{\partial}{\partial z}(z^3 p)+(\frac{3\beta}{2})\frac{\partial^2}{\partial z^2}(p)\right] \quad \text{.......} (5.9)$$

which is a non-linear Fokker Plank Equation (FPE) on account of non-linear drift term. The factor $K^{-\frac{1}{2}}$ on the right hand side of Eq.-(5.9) indicates that the approach of the system towards its steady-state is slowed down by a factor $K^{-\frac{1}{2}}$. The corresponding lengthening of the relaxation time is called critical slowing [12], this phenomenon is always associated with the phenomenon of the phase transition [18].

To have an idea about the order of fluctuation, we solve Eq.-(5.9) for large time, with appropriate boundary conditions. Thus setting $\frac{\partial p}{\partial t}=0$, for large time, and carrying out the integration, we obtain

$$p_s(z)=C\exp(-\frac{243}{256}z^4) \qquad (5.10)$$

where C is the normalization constant, obviously, the steady-state mean and variances are :

$$\left.\begin{aligned} &E(z)=0 \\ &Var(z)=\sigma_z^{\,2}=\left(\frac{256}{243}\right)^{\frac{1}{2}}\frac{\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{1}{4}\right)} \end{aligned}\right] \qquad (5.11)$$

Accordingly, at the critical point the mean and variance of the population size at large times will be given by

$$\left.\begin{aligned} E(\psi_{sc}) &= K\varphi_s \\ Var(\psi_{sc}) &= K^{\frac{3}{2}}\left(\frac{256}{243}\right)^{\frac{1}{2}}\frac{\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{1}{4}\right)} \end{aligned}\right] \quad \text{.......... (5.12)}$$

Eq.-(5.12) confirms occurrence of anomalous variance, an important flag of catastrophe. Recalling that the steady–state probability density function $P_s(z)$ and the corresponding catastrophic potential function $V(z)$ of the system are related through

$$p_s(z) = C\exp\left(-\frac{v(z)}{D}\right) \quad \text{.......... (5.13)}$$

we see that

$$V(z) = \frac{729}{512}\beta z^4 \quad \text{.......... (5.14)}$$

Thus we note that at the critical point (the tip of the cusp) the form of the potential function is a quadratic, which starts unfolding as we enter the cusp region.

CRITICAL REGION:

The region around the critical point $(R_c, Q_c) = \left(\frac{3\sqrt{3}}{8}, 3\sqrt{3}\right)$, where fluctuation become of the order $0(K^{2\nu})$ is termed as critical region. Expressions, Eq.-(4.38) and Eq.-(4.39) for $R_{\min}$ and $R_{\max}$, show that when P exceeds 9 even by a small amount, the roots of Eq.-(4.36) become distinct. Suppose small changes in R and P correspond to changes η_1 and η_2 in the value of R and Q around the critical point. Since φ_2, φ_3 and φ_4 become distinct for all R, lying in the interval $(R_{\min}, R_{\max})$, we have $f_i(\varphi_s) \neq 0$, $i = 1, 2, 3$ and $\varphi_s = \varphi_2, \varphi_3, \varphi_4$ in this region. Using Taylor series expansion and following Karmeshu et al. [19] it can be shown that

$$V(z;u,v) = z^4 + 4z^3 + uz \qquad (5.15)$$

where u, v are function of η_1 and η_2. The canonical from Eq.-(5.15) of potential for cusp catastrophe illustrates the transition of uni-model probability density function into a bi-model probability density function.

5.2.3 FAR FROM THE CRITICAL POINT IN THE CUSP REGION:

Within the cusp region, multiple steady-state become accessible to the system. For $R = R_{\min}$ or $R = R_{\max}$, one of the roots of Eq.-(4.36) become repeated, thereby implying that $f_1(\varphi_s)=0$ while $f_2(\varphi_S)\neq 0$. Consequently the scaling index parameter $\nu=\frac{2}{3}$, accordingly the steady-state solution to the Fokker Plank Equation (FPE), at the threshold points E and C (see Fig.-4.2) turn out to be non-Gaussian and are

$$P_{s,E}(z)=C_1\exp\left[\frac{-(\delta^2+\varphi^2)(\varphi_3-\varphi_2)z^3}{3\beta\varphi_3}\right] \qquad (5.16)$$

with

$R=R_{\min}$ and $\varphi_3=\varphi_4$

and

$$P_{s,C}(z)=C_2\exp\left[\frac{-(\delta^2+\varphi^2)(\varphi_4-\varphi_2)z^3}{3\beta\varphi_2}\right] \qquad (5.17)$$

with

$R=R_{\max}$ and $\varphi_2=\varphi_3$.

Where C_1 and C_2 are the normalization constants. In these states, the system is marginally stable. However, at the points A and G on the lower and upper boundary (see Fig.-4.2), $f_1(\varphi_S)\neq 0$. Thus the scaling index $\nu=\frac{1}{2}$, and the steady-state solution of Fokker Plank Equation (FPE) Eq.-(4.21) yields a Gaussian distribution. Further, for R, $R_{\min} \leq R \leq R_{\max}$, again $f_1(\varphi_S) \neq 0$, therefore, as earlier $\nu = \frac{1}{2}$, and the probability density functions are

$$P_{s,l}(z) = C_3 \exp\left(-\frac{z^2}{2\sigma_2^{\ 2}} \right) \text{ at the lower branch} \quad \text{(5.18)}$$

and

$$P_{s,u}(z) = C_4 \exp\left(-\frac{z^2}{2\sigma_4^{\ 2}} \right) \text{ at the upper branch} \quad \text{(5.19)}$$

where

C_3 and C_4 are the normalization constants and

$$\sigma_2^{\ 2} = f_1^{\ 2}(\varphi_2) \int_{\varphi_0}^{\varphi_2} \left[\frac{g(u)}{f_0^{\ 3}(u)} \right] du \quad \text{(5.20)}$$

$$\sigma_4^{\ 2} = f_4^{\ 2}(\varphi_4) \int_{\varphi_0}^{\varphi_4} \left[\frac{g(u)}{f_0^{\ 3}(u)} \right] du \quad \text{(5.21)}$$

when R is slightly greater than $R_{\min}$, the pdf $P_s(z)$ is dominant in the vicinity of $\psi_s = K\varphi_2$. As R increases from $R_{\min}$, towards $R_{\max}$, being held constant, the pdf remains dominant on lower branch till R attains a value

$R_0 \leq R_{max}$. At $R = R_0$, the probability of the system being on any one of the stable branch become the same. As R exceeds R_0 and moves towards R_{max}, the dominance shifts from the lower to the upper branch. The switching over of the pdf from uni-modal to bi-modal. In general, the pdf p(z) may be approximated by a mixture of $P_{s,1}(z)$ and $P_{s,u}(z)$. given by

$$P_S(z) = C_5(R)e^{-\left(\frac{z^2}{2\sigma_2^2}\right)} + C_6(R)e^{-\left(\frac{z^2}{2\sigma_4^2}\right)} \qquad (5.22)$$

where $C_5(R)$ are $C_6(R)$ depend on R, and are called mixing coefficients.

5.3 CONCLUDING REMARKS:

In this chapter we have investigated the deterministic approach of Holling's model applied to a problem of pest control in the preservation and protection of a forest from pests. We have derived the conditions under which the system will exhibit cusp catastrophe. The display of hystereisis effect and the phenomenon of critical slowing down have been explained in a quantitative manner. We have also carried out a detailed stochastic study of the model in three regions of interest and of importance and have explained why and what extra care is to be exercised for the investigation of the associated cusp region.

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